The spectral changes of partially coherent light focused by an apertured lens with chromatic aberration

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The spectral changes of a partially coherent polychromatic light focused by an apertured lens with chromatic aberration are investigated. It is demonstrated that the spectrum in the focused field is different from that in the aperture. Comparing with the spectrum in the aperture, the spectrum in the focused field shifts to lower or higher frequency, which is defined as a spectral shift. The influence of chromatic aberration of the lens, the coherence of the partially coherent light in the aperture, the radius of the aperture, and the spectral width of the spectrum of the aperture on the spectral shift are investigated in detail. The numerical results show that these parameters affect the spectral shift noticeably.

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It has been demonstrated that the spectrum of light radiated from a primary or a secondary partially coherent polychromatic source will generally change even in the case of propagation in free space, which is often referred as the Wolf effect[1−3]. Sometimes this kind of spectral change due to the correlation of the source is termed the correlation-induced spectral changes[4−7]. Since the work by Wolf[1], there has been an extensive work to explore several situations in which this effect plays a fundamental role[4−9]. The possible consequences of the spectroradiommetric measurement for sources that do not have invariant spectrum of propagation have been discussed[10−12]. An interesting experimental work has been done by Kandpal *et al.*[12] in a filter-lens combination system, in which one has observed the appreciable spectral changes and attributed the spectral changes to the correlation-induced spectral changes. However, Foley and Wang[13] calculated the possible spectral changes of the experimental arrangement by Kandpal, and found that the spectral changes predicted by the theoretical analysis are much smaller than that experimentally observed. They attributed the spectral changes to other effects, such as the chromatic aberration of the lenses. But the further study to show how the chromatic aberration of the lenses affects the spectral changes has not been carried out. In a recent study, Pu and Nemoto[14] theoretically investigated the influence of chromatic aberration of a lens on the spectral changes of polychromatic Gaussian Schell model (GSM) beams focused by the lens. It was shown that a moderate chromatic aberration of the lens leads to appreciable spectral changes in the focused field, and the coherence of the GSM beams plays an important role in determining the spectral changes of the beams.

In this letter, the focusing of a partially coherent polychromatic light focused by an apertured lens with chromatic aberration is studied. In particular, we investigate the spectral changes of the light in the focused field. It is shown that the spectral changes of the light in the focused field are strongly dependent on both the chromatic aberration and the coherence of the incident partially coherent light. The influence of the spectrum width of the incident partially coherent light and a limiting aperture in front of the lens on the spectral changes is also discussed.

As shown in Fig. 1, a partially coherent polychromatic light is focused by an apertured lens with chromatic aberration. The cross-spectral density of the partially coherent light in the limiting aperture of radius a is given by

$$W^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z=0,\omega)=S^{(0)}(\omega)\cdot\exp\left\{-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{2\sigma(\omega)^2}\right\}, \quad (1)$$

where $S^{(0)}(\omega)$ is the spectrum of the light in the aperture; $\sigma(\omega)$ is the effective coherence length of the partially coherent light in the aperture, generally it is dependent on the angular frequency ω; $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are respectively the position vectors of two points in the limiting aperture; the focal length of the focusing lens with chromatic aberration at the central frequency ω_0 (or at the central wavelength λ_0) is f_0. The dispersion $\mathrm{d}n/\mathrm{d}\lambda|_0$ (or $\mathrm{d}n/\mathrm{d}\omega|_0$) of the material of the lens results in the chromatic aberration. The focal length due to the chromatic aberration can be expressed as[15]

$$f(\omega)=f_0+\left.\frac{\mathrm{d}f}{\mathrm{d}\omega}\right|_0(\omega-\omega_0)=f_0\left[1+\xi\left(\frac{\omega}{\omega_0}-1\right)\right], \quad (2)$$

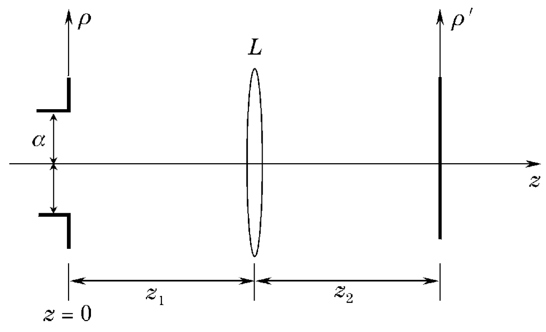

Fig. 1. Illustration of focusing of a partially coherent polychromatic light by an apertured lens with chromatic aberration.

where

$$\xi = \frac{\omega_0}{f_0} \frac{\mathrm{d}f}{\mathrm{d}\omega}\bigg|_0 . \qquad (3)$$

It is assumed that the distance between the limiting aperture and the lens is z_1, and the distance between the lens and the observation plane is z_2. The optical ray matrix from the aperture plane to the observation plane is obtained by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f(\omega)} & 1 \end{pmatrix} \begin{pmatrix} 1 & z_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \frac{z_2}{f(\omega)} & -\frac{z_1 z_2}{f(\omega)} \\ z_1 + z_2 - \frac{1}{f(\omega)} & 1 - \frac{z_1}{f(\omega)} \end{pmatrix}. \qquad (4)$$

Based on the propagation formula for cross-spectral density of partially coherent light, we get the cross-spectral density of the light in the focused field as[12]

$$W(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \omega) = \left(\frac{k}{2\pi B}\right)^2 \iint W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)$$

$$\times \exp\{-\frac{ik}{2B}\left[A\left(\boldsymbol{\rho}_1^2 - \boldsymbol{\rho}_2^2\right) - 2\left(\boldsymbol{\rho}_1' \cdot \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2' \cdot \boldsymbol{\rho}_2\right)\right.$$

$$\left. + D\left(\boldsymbol{\rho}_1'^2 - \boldsymbol{\rho}_2'^2\right)\right]\} \mathrm{d}\boldsymbol{\rho}_1 \mathrm{d}\boldsymbol{\rho}_2, \qquad (5)$$

where the integral is taken over the limiting aperture, and $\boldsymbol{\rho}'$ is the position vector in the observation plane.

It is shown from Eq. (5) that if the cross-spectral density of the partially coherent light in the limiting aperture is given, we can determine the spectrum of the light in the focused field. It is assumed that the spectrum of the partially coherent light in the aperture is

$$S^{(0)}(\omega) = S_0 \cdot \frac{\Gamma^2}{\left(\omega - \omega_0\right)^2 + \Gamma^2}, \qquad (6)$$

here S_0 is a constant, and ω_0 is the central frequency of the spectrum, and Γ is the half width at half maximum of the spectrum. For simplicity, we assume that the effective coherence length ($\sigma(\omega)$) of the partially coherent light in the aperture is expressed as

$$\sigma(\omega) = \sigma_0 \frac{\omega_0}{\omega}, \qquad (7)$$

where σ_0 is the roots mean square (rms) spatial correlation distance at the central frequency ω_0; and the relative spatial correlation distance is

$$\Delta(\omega) = \frac{\sigma(\omega)}{a} = \Delta_0 \frac{\omega_0}{\omega}, \qquad (8)$$

where $\Delta_0 = \sigma_0/a$ is the relative correlation distance at the central frequency ω_0.

We readily find from Eqs. (1) and (7) that the partially coherent light in the aperture satisfies the so-called *scaling law*, which was firstly revealed by Wolf[1].

For simplicity, in this paper we limit our study to the axial spectrum of the focused field. The axial spectrum intensity in the focused field is given by

$$S(z_2, \omega) = S^{(0)}(\omega) \left(\frac{\omega}{\omega_0}\right)^2 \left(\frac{z_0 \cdot f(\omega)}{(z_1 + z_2) f(\omega) - z_1 z_2}\right)^2$$

$$\times \int_0^1 \int_0^1 \exp\left[-\frac{r_1^2 + r_2^2}{2\Delta(\omega)^2}\right]$$

$$\times \exp\left\{-\frac{i\pi z_0 \left[f(\omega) - z_2\right]}{(z_1 + z_2) f(\omega) - z_1 z_2} \cdot \frac{\omega}{\omega_0} \cdot \left(r_1^2 - r_2^2\right)\right\}$$

$$\times I_0\left(\frac{r_1 r_2}{\Delta(\omega)^2}\right) \cdot r_1 r_2 \mathrm{d}r_1 \mathrm{d}r_2, \qquad (9)$$

where

$$r_1 = \rho_1/a, \quad r_2 = \rho_2/a, \qquad (10)$$

and

$$I_0\left(\frac{r_1 r_2}{\Delta(\omega)^2}\right) \qquad (11)$$

$$= \frac{1}{2\pi} \times \int_0^{2\pi} \exp\left[\frac{r_1 r_2}{\Delta(\omega)^2} \cdot \cos(\phi_1 - \phi_2)\right] \mathrm{d}\phi_1$$

is the first kind and zero-order modified Bessel function.

In order to demonstrate how the chromatic aberration of the lens and the coherence of the partially coherent light influence the spectrum of the light in the focused field, we perform numerical calculation according to Eq. (6). To make numerical calculation, some parameters of the chromatic aberration of the lens should be given. A set of the parameters of chromatic aberration is chosen as[13]

$$f_0 = 150\,\mathrm{mm}, \quad \omega_0 = 7.57 \times 10^{15}\,\mathrm{s}^{-1} \quad (\lambda_0 = 249\,\mathrm{nm}),$$

$$\lambda_0 \frac{\mathrm{d}n}{\mathrm{d}\lambda}\bigg|_0 = -0.1375, \quad \mathrm{d}f/\mathrm{d}\lambda|_0 = 163.089\,\mathrm{mm}/\mu\mathrm{m}$$

$$(\text{i.e.,} \ \mathrm{d}f/\mathrm{d}\omega|_0 = -5.3646 \times 10^{-18}\,\mathrm{m}\cdot\mathrm{s},$$

$$\text{or } \xi = \frac{\omega_0}{f_0} \frac{\mathrm{d}f}{\mathrm{d}\omega}\bigg|_0 = -0.2707). \qquad (12)$$

The spectrum of the light in the limiting aperture is of Lorentzian type with the central frequency $\omega_0 = 7.57 \times 10^{15}\,\mathrm{s}^{-1}$, and half width at half maximum $\Gamma = 0.6 \times 10^{15}\,\mathrm{s}^{-1}$, or $0.4 \times 10^{15}\,\mathrm{s}^{-1}$, or $0.2 \times 10^{15}\,\mathrm{s}^{-1}$.

Figure 2 shows the axial spectra in the focused field $z_2/f_0 = 1.2$. The limiting aperture is located at $z_1/f_0 = 2$; the relative correlation distance at the central frequency ω_0 is $\Delta_0 = 1$ (Fig. 2(a)) or $\Delta_0 = 0.5$ (Fig. 2(b)); and the chromatic aberration of the lens are respectively chosen as $\mathrm{d}f/\mathrm{d}\omega|_0 = 0$, -2.5×10^{-18} m$\cdot$ s (i.e., $\xi = -0.126$), and -5×10^{-18} m$\cdot$ s (i.e., $\xi = -0.252$). As shown in Fig. 2, the axial spectrum in the focused field shifts to the lower frequency, compared with the spectrum of the partially coherent light in the aperture. To

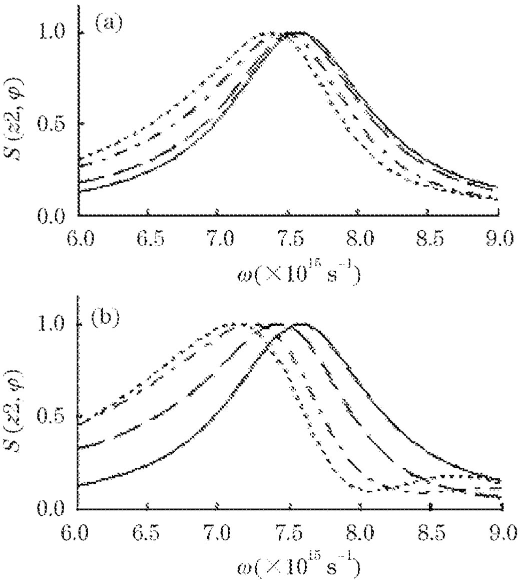

Fig. 2. The axial spectra at the position $z_2/f_0 = 1.2$ for the lens with different chromatic aberration, (a) $\Delta_0 = 0.5$; (b) $\Delta_0 = 1$. Solid curves: the spectra in the aperture; dashed curves: $\mathrm{d}f/\mathrm{d}\omega|_0 = 0$; dashed-dotted curves: $\mathrm{d}f/\mathrm{d}\omega|_0 = -2.5\times10^{-18}$ m·s (i.e., $\xi = -0.126$); dotted curves: $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s (i.e., $\xi = -0.252$). $\omega_0 = 7.57\times10^{15}$ s^{-1}, $\Gamma = 0.6\times10^{15}$ s^{-1}, $z_1 = 2f_0$, $a = 0.5$ mm. The spectra are normalized to be unity at the maximum intensity.

characterize the spectral changes, we define the spectral shifts and the relative spectral shifts respectively as

$$\delta\omega = \omega_m - \omega_0 \tag{13a}$$

and

$$\frac{\delta\omega}{\omega_0} = \frac{\omega_m - \omega_0}{\omega_0}, \tag{13b}$$

here ω_m is the frequency at which $S(z_2,\omega)$ takes its maximum. From Fig. 2, we find that, even if no chromatic aberration, the axial spectrum in the focused field is red-shifted. The existing of the chromatic aberration makes the red-shift larger. For a fixed value of the relative correlation distance Δ_0 (e.g. $\Delta_0 = 1$ in Fig. 2(b)), the larger the chromatic aberration is, the larger the red-shift is. Comparing Figs. 2(a) and (b), we find that, for a fixed chromatic aberration, the larger the relative correlation distance Δ_0, the larger the red-shift.

The influence of the relative correlation distance Δ_0 on the spectral shifts is represented in Fig. 3. As indicated in Fig. 3, when Δ_0 is very small, the spectral shift is red-shift and is negligible. With the increment of Δ_0, the red-shift increases, and the larger chromatic aberration leads to larger red-shift.

In Fig. 4, we plot the relative spectral shifts as a function of the relative correlation distance Δ_0 for the cases of three different widths of the spectra in the limiting aperture $\Gamma = 0.6\times10^{15}$ s^{-1}, or 0.4×10^{15} s^{-1}, or 0.2×10^{15}s^{-1}, and the chromatic aberration $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s (i.e., $\xi = -0.252$). It is found that for a fixed Δ_0, the larger width of the spectrum Γ of partially coherent light is, the larger the red-shift is.

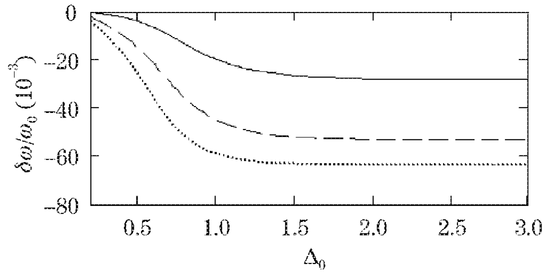

Fig. 3. The relative spectral shifts $\delta\omega/\omega_0$ as a function of the relative coherent distance Δ_0 in the cases of three different chromatic aberrations. Solid curve: $\mathrm{d}f/\mathrm{d}\omega|_0 = 0$ ($\xi = 0$); broken curve: $\mathrm{d}f/\mathrm{d}\omega|_0 = -2.5\times10^{-18}$ m·s ($\xi = -0.126$); dotted curve: $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s ($\xi = -0.252$). The other parameters are the same as those in Fig. 2.

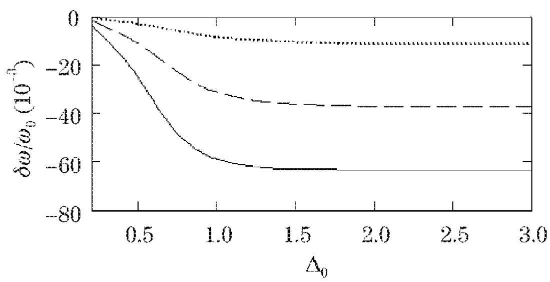

Fig. 4. The relative spectral shifts $\delta\omega/\omega_0$ as a function of the relative coherence distance Δ_0, in the cases of different widths of the source spectra. The chromatic aberration $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s ($\xi = -0.252$), $\omega_0 = 7.57\times10^{15}$ s^{-1}. $\Gamma = 0.6\times10^{15}$ s^{-1} (solid curve), 0.4×10^{15} s^{-1} (broken curve), and 0.4×10^{15} s^{-1} (dotted curve).

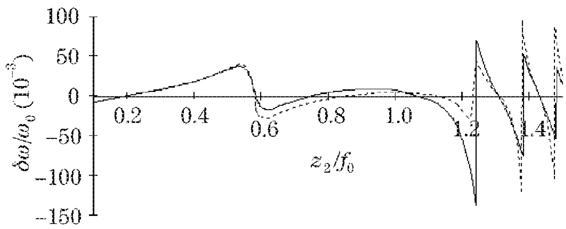

Fig. 5. The relative spectral shifts $\delta\omega/\omega_0$ as a function of z_2/f_0. Solid curve: $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s (i.e., $\xi = -0.252$); dotted curve: non-chromatic aberration. The position at which the spectral shift experiences a sharp transition is a spectral switch. $\omega_0 = 7.57\times10^{15}$ s^{-1}, $\Gamma = 0.6\times10^{15}$ s^{-1}, $z_1 = 2f_0$, $a = 0.5$ mm, $\Delta_0 = 1$.

The spectral shifts from $z_2/f_0 = 0.1$ to $z_2/f_0 = 1.5$ are shown in Fig. 5 in the cases of non-chromatic aberration and the chromatic aberration $\mathrm{d}f/\mathrm{d}\omega|_0 = -5\times10^{-18}$ m·s (i.e., $\xi = -0.252$). It is found from Fig. 5 that when $z_2/f_0 = 0.1$, the spectral shift is slightly red-shifted, and the difference in the spectral shifts between the chromatic aberration case and non-chromatic aberration case is negligible. With the increment of z_2/f_0, the red-shift decreases gradually, and turns to zero at some position. Then as z_2/f_0 continues to increase, a blue-shift occurs, and the blue-shift increases slowly with the increment of z_2/f_0. When z_2/f_0 is equal to a particular value, a sharp transition of the spectral shift happens, i.e., the spectral shift changes from the red-shift to the blue-shift suddenly. This indicates that a spectral switch takes place at this position[7-9]. It is found that when z_2/f_0 is larger than 0.5, the difference in the spectral shift between nonchromatic aberration case and the chromatic aberration case $\mathrm{d}f/\mathrm{d}\omega|_0 = -5 \times 10^{-18}$ m ·s becomes noticeable, and the position at which the spectral switch occurs is slightly different.

In conclusion, when a partially coherent polychromatic light is focused by a filter-lens system with chromatic aberration, the spectral shifts take place in the focused filed, and the spectral shifts are dependent on the coherence, the spectra of the partially coherent light, and the chromatic aberration of the lens. Generally, the larger chromatic aberration results in larger spectral shifts, and the larger spectral width of the spectra leads to larger spectral shifts. The results may be applicable to the high-precision spectral measurements.

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