Implementation of non-local quantum controlled-NOT gate with multiple targets

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Received October 16, 2003

We show how a non-local quantum controlled-NOT (CNOT) gate with multiple targets can be implemented with unit fidelity and unit probability. The explicit quantum circuit for implementing the operation is presented. Two schemes for probabilistic implementing the operation via partially entangled quantum channels with unit fidelity are put forward. The overall physical resources required for accomplishing these schemes are different, and the successful implementation probabilities are also different.

OCIS codes: 270.0270, 000.2690.

The quantum non-locality, i.e. there are non-local correlations among quantum systems, plays a central role in quantum computation and quantum information. In recent years, much attention has been paid to the issue of non-locality of quantum states[1−7]. In fact, an equally important issue is that of non-locality of quantum operations. This situation arises, for example, in the context of distributed quantum computation[8], in which two or more spatially separated computation units are available to solve a computational problem. Other examples where non-local quantum operations are required are quantum network communication[9] and the production of multi-particle entangled states[10]. For the alluring potential application, some fundamental research on non-local quantum operations has been presented[11−16].

"If A is true, then do B". This is one of the most useful types of controlled operation in quantum computation and quantum information. The prototypical controlled operation is the controlled-NOT (CNOT) gate, which acts on two qubits, a control qubit and a target one, and flips the target whenever the control qubit is $|1\rangle$. Another useful and important controlled operation is the CNOT gate with multiple targets $U_{\rm CNOT}(c;t_2,t_3,\cdots,t_N)$. In terms of the computational basis, the action of the operation $U_{\rm CNOT}(c;t_2,t_3,\cdots,t_N)$ is given by

$$U_{\rm CNOT}(c;t_2,t_3,\cdots,t_N)\,|c\rangle\,|t_2\rangle\,|t_3\rangle\cdots|t_N\rangle = |c\rangle\,|t_2\oplus c\rangle\,|t_3\oplus c\rangle\cdots|t_N\oplus c\rangle\,, \tag{1}$$

which flips the $(N-1)$ targets "$t_2,t_3,\cdots,t_N$", conditioned on the control qubit "c" being set one. It is convenient to use, for example, in constructing classical functions such as permutations, or in encoders and decoders for quantum error-correction computing such as the 7-qubit Calderbank-Shor-Steane (CSS) code[8]. In this letter, we firstly present one perfect scheme by which the non-local CNOT gate with multiple targets $U_{\rm CNOT}(c;t_2,t_3,\cdots,t_N)$ can be implemented by using some Einstein-Podolsky-Rosen (EPR) pairs as quantum channels. The consumption of overall physical resources (such as the local unitary operations, prior entanglement, classical communication, and so on) required for accomplishing the scheme is given. Then, we investigate the problem in a slightly practical way. We propose other two schemes for probabilistic implementing the non-local quantum operation $U_{\rm CNOT}(c;t_2,t_3,\cdots,t_N)$ via partially entangled quantum channels, and the probabilities of successful implementation are compared for the two schemes.

Consider N distant users U_1, U_2, $\cdots$, U_N, each of them holds a message particle in an arbitrary state $|\psi\rangle_j = a_j\,|0\rangle_j + b_j\,|1\rangle_j$ $(j=1,2,\cdots,N)$. To begin with, the user U_j $(j=2,3,\cdots,N)$ of the network needs to share an EPR pair with user U_1. The $(N$-1$)$-EPR pairs shared by users U_2, U_3, $\cdots$, and U_N with U_1 are $(2',2'')$, $(3',3'')$, $\cdots$, and (N',N''), respectively (where the particles $(2',3',\cdots,N')$ are held by U_1). Assume that the $(N-1)$-EPR pairs are all in the Bell state $|\phi^+\rangle = (1/\sqrt{2})(|00\rangle+|11\rangle)$. The state of N message particles, which the operation $U_{\rm CNOT}(1;2,3,\cdots,N)$ will perform on, can be expanded as

$$\begin{aligned}|\psi\rangle_{1,2,\cdots,N} &= \prod_{j=1}^{N}|\psi\rangle_j = \sum_{i=1}^{2^N}\lambda_i\prod_{j=1}^{N}|u_{ij}\rangle_j \\ &= \sum_{i=1}^{2^{N-1}}\lambda_i\,|0\rangle_1\,|u_{i2}\rangle_2\,|u_{i3}\rangle_3\cdots|u_{iN}\rangle_N \\ &+ \sum_{i=2^{N-1}+1}^{2^N}\lambda_i\,|1\rangle_1\,|u_{i2}\rangle_2\,|u_{i3}\rangle_3\cdots|u_{iN}\rangle_N,\end{aligned} \tag{2}$$

where $\prod_{j=1}^{N}|u_{ij}\rangle_j = |u_{i1}\rangle_1\,|u_{i2}\rangle_2\cdots|u_{iN}\rangle_N$ is the ith basis vector in the 2^N-dimensional space; $u_{ij}\in\{0,1\}$; $|0\rangle_j$ and $|1\rangle_j$ stand for two orthogonal states of the jth message particle. The initial state of the system composing of message particles $(1,2,\cdots,N)$ and $(N-1)$-EPR pairs $(2',2'')$, $(3',3'')$, $\cdots$, (N',N'') is read as

$$|\psi\rangle = |\psi\rangle_{1,2,\cdots,N}\otimes\prod_{j=2}^{N}1/\sqrt{2}(|00\rangle+|11\rangle)_{j',j''}. \tag{3}$$

We will present in the following implementation scheme 1 which allows N distant users $U_1, U_2, \cdots, U_N$ to perform the non-local quantum operation $U_{\rm CNOT}(1;2,3,\cdots,N)$ on the state (2), i.e. transform it to

$$U_{\rm CNOT}(1;2,3,\cdots,N)\,|\psi\rangle_{1,2,\cdots,N} = \sum_{i=1}^{2^{N-1}} \lambda_i\,|0\rangle_1\,|u_{i2}\rangle_2\,|u_{i3}\rangle_3\cdots|u_{iN}\rangle_N + \sum_{i=2^{N-1}+1}^{2^N} \lambda_i\,|1\rangle_1\,|\bar{u}_{i2}\rangle_2\,|\bar{u}_{i3}\rangle_3\cdots|\bar{u}_{iN}\rangle_N\,, \quad (4)$$

where the $U_{\rm CNOT}(1;2,3,\cdots,N)$ has $|\psi\rangle_1$ as its control, and $|\psi\rangle_2$, $|\psi\rangle_3$, $\cdots$, $|\psi\rangle_N$ as its targets; $\bar{u}_{ij}=1-u_{ij}$.

Step 1: U_1 performs $(N-1)$ local CNOT gates $U_{\rm CNOT}(1;2')$, $U_{\rm CNOT}(1;3')$, $\cdots$, $U_{\rm CNOT}(1;N')$ on particles $(1,2')$, $(1,3')$, $\cdots$, $(1,N')$, and then performs the computation basis measurements on qubits $2'$, $3'$, $\cdots$, N', respectively. The measurement results corresponding to particles $2'$, $3'$, $\cdots$, N' are sent to U_2, U_3, $\cdots$, and U_N through $(N-1)$-way classical channels. If the result is $|0\rangle_{j'}$ $(j'=2',3',\cdots,N')$, U_j $(j=2,3,\cdots,N)$ does nothing; if the result is $|1\rangle_{j'}$, U_j performs the $X_{j''}$ (here, and in what follows, we will write $X$and Z instead of σ_x and σ_z) operation on qubit j'' $(j''=2'',3'',\cdots,N'')$. After this, the state (3) will be

$$|\psi\rangle_1 = X_{N''}\cdots X_{3''}X_{2''}\,\langle u_{N'}|\cdots\langle u_{3'}|\,\langle u_{2'}| \times U_{\rm CNOT}(1;N')\cdots U_{\rm CNOT}(1;3')U_{\rm CNOT}(1;2')\,|\psi\rangle = \prod_{j''=2''}^{N''} X_{j''}\prod_{j'=2'}^{N'}\langle u_{j'}|\prod_{j'=2'}^{N'} U_{\rm CNOT}(1;j')\,|\psi\rangle\,, \quad (5)$$

where $|u'_j\rangle = |0\rangle_{j'},|1\rangle_{j'}$ is the computation basis in which the particle j' is measured.

Step 2—Step N: The last $(N-1)$ steps are essentially analogous. In the jth step, the user U_j $(j=2,3,\cdots,N)$ performs a local CNOT gate $U_{\rm CNOT}(j'';j)$ on qubits j'' and j, then a Hadamard gate $H_{j''}(H=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix})$ on qubit j''. The qubit j'' is measured in the computation basis, and the result of the measurement is transmitted to U_1. If the outcome is $|1\rangle_{j''}$, then a Z_1 operation is applied to the qubit 1; if the outcome is $|0\rangle_{j''}$, then no action is taken on qubit 1. After this, the state (5) collapses to

$$|\psi\rangle_2 = Z_1^{(N-1)}\,\langle u_{N''}|\cdots\langle u_{3''}|\,\langle u_{2''}|\,H_{N''}\cdots H_{3''}H_{2''} \times U_{\rm CNOT}(N'';N)\cdots U_{\rm CNOT}(3'';3)U_{\rm CNOT}(2'';2)\,|\psi\rangle_1 = Z_1^{(N-1)}\prod_{j''=2''}^{N''}\langle u_{j''}|\prod_{j''=2''}^{N''} H_{j''}\prod_{j=2}^{N} U_{\rm CNOT}(j'';j')\,|\psi\rangle_1 = (1/\sqrt{2})^{2(N-1)}U_{\rm CNOT}(1;2,3,\cdots,N)\,|\psi\rangle_{1,2,\cdots,N}\,. \quad (6)$$

Synthesizing all conditions ($2^{2(N-1)}$ kinds), we obtain the total probability of successful implementation $P=(1/\sqrt{2})^{4(N-1)}\times 2^{2(N-1)}=1$. We have thus shown how to implement the non-local quantum operation $U_{\rm CNOT}(1;2,3,\cdots,N)$ with unit fidelity and unit probability, and seen that the total required resources are $(N-1)$ initially shared e-bits (maximally entangled pairs of qubits) and $2(N-1)$ c-bits (bits of classical communication). It has been proven[13] that the upper bound of the resources required to carry out an arbitrary N-qubit non-local unitary operation efficiently is $2(N-1)$ e-bits and $4(N-1)$ c-bits. So our scheme is economical in terms of physical resources. In realistic conditions, instead of maximally entangled states, U_1 and U_j $(j=2,3,\cdots,N)$ may share partially entangled states. This means that the quantum channels between U_1 and U_j will be imperfect. There are two strategies to implement the non-local operation $U_{\rm CNOT}(1;2,3,\cdots,N)$ via partially entangled channels: 1) purifying the partially entangled channels to a maximally entangled channels before using them by entangled concentration[17], and then implementing the operation; 2) implementing the operation through the partially entangled channels directly, and then rectifying the distorted operation. In the following, we will use the second strategy to investigate how the non-local operation $U_{\rm CNOT}(1;2,3,\cdots,N)$ can be implemented with unit fidelity but less than unit probability by using partially entangled states as quantum channels. We present two explicit schemes for this purpose.

Suppose the state of N message particles, which the $U_{\rm CNOT}(1;2,3,\cdots,N)$ will perform on, is still in state (2), but the entangled particles used as the quantum channels are partially entangled. Without loss of generality, the quantum channels between U_1 and U_2, U_3, $\cdots$, U_N are $(N-1)$ independent entangled pairs with the state $\prod_{j=2}^{N}(\alpha_j\,|00\rangle+\beta_j\,|11\rangle)_{j',j''}$ (where $|\alpha_j|^2+|\beta_j^2|=1$, $|\alpha_j|>|\beta_j|$). The initial state of the whole system is

$$|\varphi\rangle = |\psi\rangle_{1,2,\cdots,N}\otimes\prod_{j=2}^{N}(\alpha_j\,|00\rangle+\beta_j\,|11\rangle)_{j',j''}\,. \quad (7)$$

Implementation scheme 2.1 is that user U_1 introduces an auxiliary qubit and performs a collective unitary transformation on the control and auxiliary qubits. The first N steps of the scheme 2.1 are analogous to those in scheme 1. After this, the state $|\varphi\rangle$ collapses to

$$|\varphi_1\rangle^a = \left(1/\sqrt{2}\right)^{N-1}\left[\left(\prod_{j=2}^{N}\alpha_j\right)\times\sum_{i=1}^{2^{N-1}}\lambda_i\,|0\rangle_1\,|u_{i2}\rangle_2\,|u_{i3}\rangle_3\cdots|u_{iN}\rangle_N + \left(\prod_{j=2}^{N}\beta_j\right)\sum_{i=2^{N-1}+1}^{2^N}\lambda_i\,|1\rangle_1\,|\bar{u}_{i2}\rangle_2\,|\bar{u}_{i3}\rangle_3\cdots|\bar{u}_{iN}\rangle_N\right] \quad (8a)$$

(if U_1's measurement outcomes are all in $|0\rangle_{j'}$ $(j'=2',3',\cdots,N')$); or

$$|\varphi_1\rangle^a = \left(1/\sqrt{2}\right)^{N-1}\left[\left(\prod_{j=2}^{N}\beta_j\right)\times\sum_{i=1}^{2^{N-1}}\lambda_i\,|0\rangle_1\,|u_{i2}\rangle_2\,|u_{i3}\rangle_3\cdots|u_{iN}\rangle_N + \left(\prod_{j=2}^{N}\alpha_j\right)\sum_{i=2^{N-1}+1}^{2^N}\lambda_i\,|1\rangle_1\,|\bar{u}_{i2}\rangle_2\,|\bar{u}_{i3}\rangle_3\cdots|\bar{u}_{iN}\rangle_N\right] \quad (8b)$$

(if U_1's measurement outcomes are all in $|1\rangle_{j'}$ ($j' = 2', 3', \cdots, N'$)).

But the users U_1, U_2, $\cdots$, U_N find they cannot evolve the state shown in Eq. (7) to the state (4) which they need, because the state given in Eq. (8a) or (8b) includes the parameters of the imperfect quantum channels, α_j and β_j ($j = 2, 3, \cdots, N$). In order to distill the state (4) from the state (8a) or (8b), an auxiliary particle A with the initial state $|0\rangle_A$ is introduced by user U_1, and a collective unitary transformation based on the basis $\{|00\rangle_{1A}, |10\rangle_{1A}, |01\rangle_{1A}, |11\rangle_{1A}\}$ is constructed

$$T^a = \begin{pmatrix} \prod_{j=2}^{N}\beta_j \Big/ \prod_{j=2}^{N}\alpha_j & 0 & \sqrt{1-\left(\frac{\prod_{j=2}^{N}\beta_j}{\prod_{j=2}^{N}\alpha_j}\right)^2} & 0 \\ 0 & 1 & 0 & 0 \\ \sqrt{1-\left(\frac{\prod_{j=2}^{N}\beta_j}{\prod_{j=2}^{N}\alpha_j}\right)^2} & 0 & -\prod_{j=2}^{N}\beta_j \Big/ \prod_{j=2}^{N}\alpha_j & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{9a}$$

or

$$T^b = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \prod_{j=2}^{N}\beta_j \Big/ \prod_{j=2}^{N}\alpha_j & 0 & \sqrt{1-\left(\frac{\prod_{j=2}^{N}\beta_j}{\prod_{j=2}^{N}\alpha_j}\right)^2} \\ 0 & 0 & -1 & 0 \\ 0 & \sqrt{1-\left(\frac{\prod_{j=2}^{N}\beta_j}{\prod_{j=2}^{N}\alpha_j}\right)^2} & 0 & -\prod_{j=2}^{N}\beta_j \Big/ \prod_{j=2}^{N}\alpha_j \end{pmatrix}. \tag{9b}$$

The user U_1 applies T^a (for Eq. (8a)) or T^b (for Eq. (8b)) on the control qubits 1 and auxiliary qubit A, and the non-normalized product state $|\varphi_1\rangle \otimes |0\rangle_A$ is transformed to the result

$$\left(1/\sqrt{2}\right)^{N-1} \prod_{j=2}^{N}\beta_j \Big(U_{\text{CNOT}}(1;2,3,\cdots,N) \times |\psi\rangle_{1,2,\cdots,N}|0\rangle_A + |W\rangle_{1,2,\cdots,N}|1\rangle_A\Big), \tag{10}$$

where $U_{\text{CNOT}}(1;2,3,\cdots,N)|\psi\rangle_{1,2,\cdots,N}$ is the desired state and $|W\rangle_{1,2,\cdots,N}$ is the wrong state.

Then a measurement on the auxiliary qubit follows. If the result is $|1\rangle_A$, the implementation fails. Otherwise the result is $|0\rangle_A$, and the state of particles $1, 2, \cdots$, and N collapses to

$$\left(1/\sqrt{2}\right)^{N-1} \prod_{j=2}^{N}\beta_j \left(U_{\text{CNOT}}(1;2,3,\cdots,N)|\psi\rangle_{1,2,\cdots,N}\right), \tag{11}$$

so the N users have $(1/\sqrt{2})^{2(N-1)} \prod_{j=2}^{N}|\beta_j|^2$probability to obtain the desired state. Synthesizing all conditions (2^N kinds), we obtain that total probability of successful implementation is $P = 2\left|\prod_{j=2}^{N}\beta_j\right|^2$.

We thus see that, by using an auxiliary qubit and performing a collective unitary transformation on the control qubit and auxiliary qubit, U_1, $U_2, \cdots$, and U_N are able to complete the non-local quantum operation $U_{\text{CNOT}}(1;2,3,\cdots,N)$ via partially entangled quantum channels with unit fidelity and certain successful probability. The scheme can be implemented with linear optics, because the collective operations introduced in the scheme are unitary and any unitary operation can be realized with a sequence of beamsplitters and phaseshifters[18].

Implementation scheme 2.2 is almost the same as the scheme 1, but the user U_j performs a rotation operation about the Y-axis on qubit j'' ($j'' = 2'', 3'', \cdots, N''$) instead of a Hadamard gate.

The first step of the scheme 2.2 is exactly the same as the one shown in scheme 1. After this, the state (7) will be

$$\begin{aligned} |\varphi_2\rangle_1 &= X_{N''}\cdots X_{3''}X_{2''}\langle u_{N'}|\cdots\langle u_{3'}|\langle u_{2'}| \\ &\times U_{\text{CNOT}}(1;N')\cdots U_{\text{CNOT}}(1;3')U_{\text{CNOT}}(1;2')|\varphi\rangle \\ &= \prod_{j''=2''}^{N''}X_{j''}\prod_{j'=2'}^{N'}\langle u_{j'}|\prod_{j'=2'}^{N'}U_{\text{CNOT}}(1;j')|\varphi\rangle. \end{aligned} \tag{12}$$

If U_1's measurement outcomes are all in $|0\rangle'_j$, Eq. (12) is

$$\begin{aligned} |\varphi_2\rangle_1^a &= (\prod_{j=2}^{N}\alpha_j)\sum_{i=1}^{2^{N-1}}\lambda_i|0\rangle_1|u_{i2}\rangle_2 \\ &\times|u_{i3}\rangle_3\cdots|u_{1N}\rangle_N\prod_{j''=2''}^{N''}|0\rangle_{j''} \\ &+(\prod_{j=2}^{N}\beta_j)\sum_{i=2^{N-1}+1}^{2^N}\lambda_i|1\rangle_1|u_{i2}\rangle_2 \\ &\times|u_{i3}\rangle_3\cdots|u_{1N}\rangle_N\prod_{j''=2''}^{N''}|1\rangle_{j''}; \end{aligned} \tag{13a}$$

and if U_1's measurement outcomes are all in $|1\rangle_{j'}$, Eq. (12) is

$$\begin{aligned} |\varphi_2\rangle_1^b &= (\prod_{j=2}^{N}\beta_j)\sum_{i=1}^{2^{N-1}}\lambda_i|0\rangle_1|u_{i2}\rangle_2 \\ &\times|u_{i3}\rangle_3\cdots|u_{1N}\rangle_N\prod_{j''=2''}^{N''}|0\rangle_{j''} \\ &+(\prod_{j=2}^{N}\alpha_j)\sum_{i=2^{N-1}+1}^{2^N}\lambda_i|1\rangle_1|u_{i2}\rangle_2 \\ &\times|u_{i3}\rangle_3\cdots|u_{1N}\rangle_N\prod_{j''=2''}^{N''}|1\rangle_{j''}. \end{aligned} \tag{13b}$$

Subsequently, the users $U_2, U_3, \cdots, U_N$ perform $(N-1)$ manipulations in the same manner. In the jth step, the user U_j performs a local CNOT $U_{\rm CNOT}(j''; j)$ on qubits "j''" and "j", then sends the qubit j'' through a $R_{Yj''}$ gate

$$R_{Yj''} = \begin{pmatrix} \beta_j & -\alpha_j \\ \alpha_j & \beta_j \end{pmatrix}, \qquad (14)$$

where $R_{Yj''}$ is the rotation operation about Y-axis. This is then followed by computation basis measurements on qubit j''. If all the outcomes are $|0\rangle_j''$ (for Eq. (13a)) or $|1\rangle_j''$ (for Eq. (13b)) , the implementation succeeds, otherwise the implementation fails. After the (N-1)-time manipulations, the final state of the N particles $(1, 2, \cdots, N,)$ belonging N distant users is given by

$$\begin{aligned} |\varphi_2\rangle_2 &= \langle u_{N''}| \cdots \langle u_{3''}| \langle u_{2''}| R_{YN''} \cdots R_{Y3''} R_{Y2''} \\ &\quad \times U_{\rm CNOT}(N''; N) \cdots U_{\rm CNOT}(3''; 3) U_{\rm CNOT}(2''; 2) |\varphi_2\rangle_1 \\ &= \prod_{j''=2''}^{N''} \langle u_{j''}| \prod_{j''=2''}^{N''} R_{Yj''} \prod_{j=2}^{N} U_{\rm CNOT}(j''; j) |\varphi_2\rangle_1 \\ &= \prod_{j=2}^{N} \alpha_j \beta_j \left(U_{\rm CNOT}(1; 2, 3, \cdots, N) |\psi\rangle_{1,2,\cdots,N} \right). \qquad (15) \end{aligned}$$

Synthesizing all conditions (2 kinds), we obtain that total probability of successful implementation is $P = 2\left|\prod_{j=2}^{N} \alpha_j \beta_j\right|^2$.

In summary, we have presented explicit quantum construction for implementing a non-local CNOT gate with $(N-1)$-target $U_{\rm CNOT}(1; 2, 3, \cdots, N)$ with unit fidelity and unit probability, which uses only local single-qubit operations, local two-qubits CNOT gates, EPR pairs, and projective measurements in the computation basis—all of which are within the reach of current technology. A special feature of the scheme is that it consumes less overall resources than the "double teleportation" scheme presented in Ref. [13].

We have also put forward two schemes for probabilistic implementing the operation via partially entangled quantum channels with unit fidelity. These will be useful in some practical cases of distributed quantum information processing where the quantum channels are partially entangled. Comparing the two schemes, we have found the total physical resources required for accomplishing these schemes are different, and the successful implementation probabilities are also different. In the first one, we have found that by using a auxiliary qubit and performing a collective unitary transformation on the control qubit and auxiliary qubit, the N distant users are able to complete the non-local operation with successful probability $P = 2\left|\prod_{j=2}^{N} \beta_j\right|^2$. The probability of success is determined by the smaller Schmidt coefficients of the entangled states that are used as quantum channels. The second scheme gets an advantage over the first one in the simplicity of manipulation, and consumes fewer overall resources at the expense of the much smaller successful probability $P = 2\prod_{j=2}^{N} |\alpha_j \beta_j|^2$, which is determined by both Schmidt coefficients of the entangled pairs. This is of no great advantage to the situation where the N is large. The probabilistic implementation of quantum gate may be not so useful in the context of quantum computation, as probabilistic operations may change the complexity class of the problem and may thus destroy the (exponential) speed up of the quantum algorithm. However, probabilistic gates are useful for processes such as entanglement distillation[8], which itself is already a probabilistic process. For example, this may help in the implementation of quantum repeaters[19,20] using photons only. Due to the fact that photons are ideal candidates for quantum information processing, it is highly desirable to manipulate them directly rather than map their states on the states of another physical system, e.g. of an ion or an atom, and *vice versa*.

This work was supported by the Natural Science Foundation of Guangdong Province, China (No. 020127) and the Natural Science Research Foundation of Education Department of Guangdong Province (No. Z02069).
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