Theoretical analysis of a collimated hollow-laser-beam generated by a single axicon using diffraction integral

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A novel method to generate a collimated hollow-laser-beam (HLB) by only a single axicon is proposed. With some reasonable assumptions, the radial light intensity distribution is calculated in detail by diffraction integral theory. The result of numerical simulation shows that this method is valid. Compared with other methods of generating HLB, this scheme is extraordinarily simple in principle and can be utilized experimentally to construct a light trap in atomic fountain for convenience.

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The methods for generating hollow-laser-beam (HLB) can be summarized as follows: ring pattern of TEM_{01}^{*} mode directly from laser[1], HLB from the output beam of the LP_{01} mode selectively excited in a mini-sized hollow-core optical fiber[2], non-diffracting beams using computer-generated holograms[3], and beams generated by using axicon[4]. Due to the simple implementation and good quality of HLB, axicon is the most widely used optical element. This optical element with a conical surface was introduced and characterized by Mcleod[5]. It has an interesting feature of the possibility of focusing a Gaussian laser beam into a HLB with ring-shaped profile. Since then axicons are mainly utilized in laser machining[6], i.e. generating ring-shaped foci associated with CO_2 lasers as required for cutting holes with great precision on the surface of various materials[7]. Durnin, Indebetow, Herman *et al.*[8−10] also suggested the use of axicon to produce "diffraction-free Bessel beams"[11]. In the field of laser cooling and trapping of neutral atoms, HLB has attracted increasing interests[12,13] because the repulsive action of the optical dipole force in a blue-detuned laser field allows to confine the motion of atoms in the dark inner region of HLB. This light trap can minimize unexpected effects such as "heating" by photon scattering and level perturbation by light shifts[14]. The motivation of the paper is to use HLB generated by axicon in our atomic fountain experiment.

Rioux *et al.*[6] described the deflection property of axicon to incident collimated beam and gave the deflection angle's expression on the premise of very small base angle of axicon. The deflection behavior of axicon is shown in Fig. 1. From the simple geometrical relations given by

$$\sin\alpha = n\sin\alpha_1,\quad \sin\beta = n\sin\beta_1,\\ \varphi = \alpha_1+\beta_1,\quad \delta = \alpha+\beta-\varphi, \tag{1}$$

the deflection angle δ can be expressed as a function of α, φ, n as

$$\delta = \alpha + \arcsin(\sin\varphi\sqrt{n^2-\sin^2\alpha} - \sin\alpha\cos\varphi) - \varphi. \tag{2}$$

When α, $\varphi \to 0$, $\delta \approx (n-1)\varphi$, which is the most usual expression in many literatures. The condition to form "diffraction-free Bessel beams" is given as $\mathrm{tg}\varphi > \frac{\sin\alpha}{\sqrt{n^2-\sin^2\alpha}-1}$ which can be satisfied easily for well-collimated laser (α is small) and a usual axicon.

Our principal scheme is shown in Fig. 2 using only one axicon to generate collimated HLB. A well-collimated laser beam is deflected by an axicon and forms convergent HLB. Then after being reflected by a reflector and passing through the same axicon, a collimated HLB is generated. The scheme is equivalent to the case that beams pass through two face-to-face axicons with same shape, which is shown in Fig. 3. For simplicity, we assume that beams propagate in 2-dimensional plane y-z. The discussion below can be generalized to the case of 3 dimensions without essential difference and the similar result will be obtained. The Gaussian

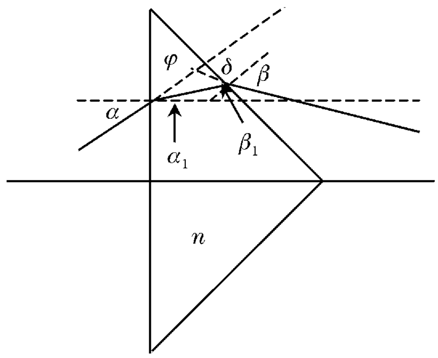

Fig. 1. Ray path through single axicon. α is the incident angle, φ is the base angle of axicon, n is the refractive index of axicon, and δ is the deflection angle.

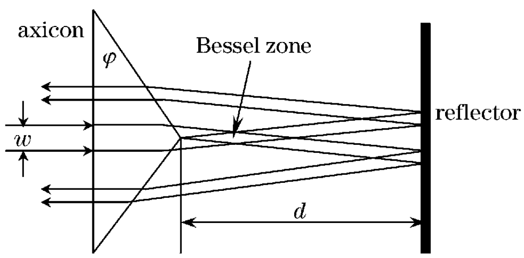

Fig. 2. Schematic plan. w is the waist of the well-collimated laser and d is the distance between the axicon's apex and reflector. The Bessel zone is denoted.

amplitude distribution of incident TEM_{00} beam is $E_1 \propto \exp(-\frac{y_1^2}{w_1^2})$. Due to the fact that the calibre of axicon's flat surface is greatly larger than incident beam's diameter, the electric amplitude distribution after the first axicon can be estimated from the usual Huygens-Fresnel's diffraction integral

$$\begin{aligned} E_2 &= \frac{e^{ikz_{21}}}{i\lambda z_{21}} \int_{-\infty}^{\infty} dy_1 E_1 e^{-ik(n-1)\tan\varphi|y_1|} e^{i\frac{k}{2z_{21}}(y_1-y_2)^2} \\ &= \frac{e^{ikz_{21}}}{i\lambda z_{21}} \left[e^{i\frac{k}{2z_{21}}(y_{20}^2+2y_{20}y_{2-})} \int_{-\infty}^{0} dy_1 E_1 e^{i\frac{k}{2z_{21}}(y_1-y_{2-})^2} \right. \\ &\quad \left. + e^{i\frac{k}{2z_{21}}(y_{20}^2-2y_{20}y_{2+})} \int_{0}^{\infty} dy_1 E_1 e^{i\frac{k}{2z_{21}}(y_1-y_{2+})^2} \right], \quad (3) \end{aligned}$$

where $z_{21} = z_2 - z_1$, $y_{20} = (n-1)z_{21}\tan\varphi$, $y_{2\pm} = y_2 \pm y_{20}$, and k, λ are the wave vector and wavelength of laser, respectively. When $y_{2\pm} \to 0$ (i.e. the light intensity mainly distributes at the ring with radius y_{20}), the terms in the integral can be approximately taken as even function of y_1. So we can obtain

$$E_2 = C_{2+}E_{2+} + C_{2-}E_{2-}, \quad (4)$$

where $C_{2\pm} = \frac{e^{ikz_{21}}}{2i\lambda z_{21}} e^{i\frac{k}{2z_{21}}(y_{20}^2 \mp 2y_{20}y_{2\pm})}$ and $E_{2\pm} = \int_{-\infty}^{\infty} dy_1 E_1 e^{i\frac{k}{2z_{21}}(y_1-y_{2\pm})^2}$ ($C_{2\pm}$ is irrelevant to $y_{2\pm}$ and $C_{2+} = C_{2-}$ when $y_{2\pm} \to 0$). $E_{2\pm}$ can be further simplified by standard integral formula to

$$E_{2\pm} \propto \exp\left[-\frac{y_{2\pm}^2}{w_1^2(\lambda^2 z_{21}^2/\pi^2 w_1^4 + 1)}\right]. \quad (5)$$

In the case of our experiment, the magnitudes of parameters are $\lambda = 780$ nm, w_1 and z_{21} are of the order of millimeter and centimeter, respectively, thus $\lambda z_{21}/\pi w_1^2 \ll 1$. $w_1 \approx w_2$ is satisfied as long as the incident beam is collimated well enough, so expression (5) can be further approximated as

$$E_{2\pm} \propto \exp\left(-\frac{y_{2\pm}^2}{w_2^2}\right). \quad (6)$$

So the intensity of beam through the first axicon can be expressed as

$$\begin{aligned} I_2 = |E_2|^2 &\propto \exp\left(-\frac{2y_{2+}^2}{w_2^2}\right) + \exp\left(-\frac{2y_{2-}^2}{w_2^2}\right) \\ &\quad + \exp\left(-\frac{y_{2+}^2 + y_{2-}^2}{w_2^2}\right). \quad (7) \end{aligned}$$

By the same procedure, the intensity of beam after the second axicon has the similar form as

$$\begin{aligned} I &\propto \exp\left(-\frac{2y_{+}^2}{w^2}\right) + \exp\left(-\frac{2y_{-}^2}{w^2}\right) \\ &\quad + \exp\left(-\frac{y_{+}^2 + y_{-}^2}{w^2}\right), \quad (8) \end{aligned}$$

where w is the waist radius, $y_\pm = y \pm y_0 = y \pm y_{20}$ [for $\varphi_1 = \varphi_2$, $y_0 = (z-z_2)(n-1)(\tan\varphi_1 - \tan\varphi_2) + y_{20} = y_{20}$]. The light intensity distribution is shown in Fig. 4, where $w = 1.2$ mm and $y_0 = 4$ mm. Generalizing expression (8) to the case of x-y-z plane, the radial light intensity can be obtained and shown in Fig. 5. As can be seen from Figs. 4 and 5, the inner region is dark enough compared with the intensity of ring. From expression (8), we can see that the intensity I is irrelevant to axial distance z so long as w is slowly varying in the range of several hundred millimeters, which is enough to confine the cold atoms in our experiment. This is satisfied for well-collimated laser whose far-field divergent angle is small enough, so the output HLB is nearly collimated.

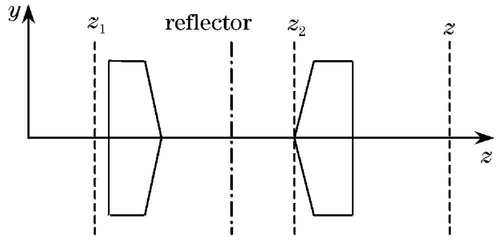

Fig. 3. Equivalent diagrammatical sketch of Fig. 2 in 2-dimensional plane y-z.

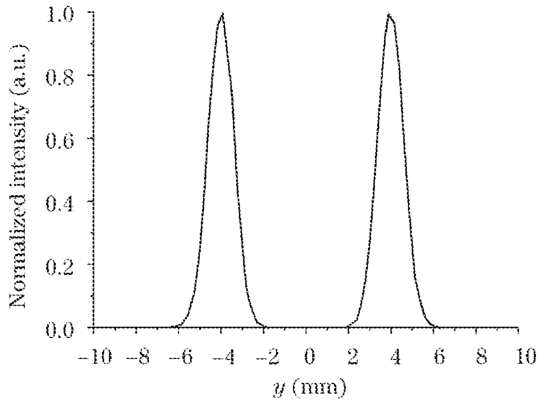

Fig. 4. Normalized intensity of HLB as a function of y.

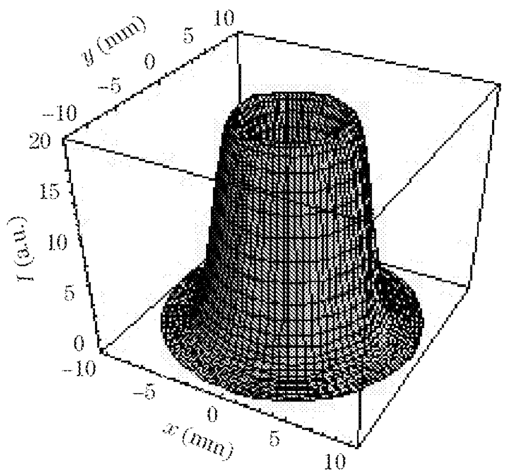

Fig. 5. Radial intensity of HLB as a function of x and y.

In conclusion, it is possible to generate a well-collimated HLB using only a single axicon. An input Gaussian laser beam can be transformed into double-Gaussian HLB whose inner region is dark enough to serve as a proper light trap confining the transverse motion of cold atomic sample when being ejected up and down in atomic fountain.

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