Study on digital holography with single phase-shifting operation

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The digital holography with single phase-shifting operation has been studied. Experimental results and computer-simulated work are presented. The phase-shifting error makes the intensity of primary image decrease and the conjugate image appear in reconstruction. The explicit equation for explaining these effects is given. The calculation of the normalized intercorrelation peak between the input and the reconstruction for different algorithms shows that, when the phase-shifting operation is the main error source, the quality of the image reconstructed from the digital holography with single phase-shifting operation is favorable.

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Digital holography has been widely applied in various areas with the development of charge coupled device (CCD) and computer. The in-line optical arrangement is mostly adopted in digital holography for the poor resolving power of CCD device. To remove the unwanted zero-order and conjugate images of the in-line digital holography, I. Yamaguchi *et al.* have found an excellent solution called phase-shifting digital holography (PSDH)[1] using the phase-shifting interferometry. The PSDH technique has been successfully applied to diffusely reflecting objects, three-dimensional microscopy[2], encryption[3], pattern recognition[4] as well as color image recording and reconstruction[5,6].

Different methods for deriving the complex amplitude and suppressing the unwanted reconstruction terms have been proposed[7−9]. Among them there are two often adopted methods[5,6,10], the four-step algorithm (FSA) and the three-step algorithm (TSA). For FSA, three phase-shifting operations $(\frac{\pi}{2}, \pi, \frac{3\pi}{2})$ should be taken, and for TSA two phase-shifting operations $(\frac{\pi}{2}, \pi)$ should be taken. The stepped phase shifts are carried out by using same phase modulator. The deviation of the actual shift values ϕ'_j from the designed values ϕ_j is not avoidable because of the limitation of the instrument accuracy and operation errors. These deviations cannot be easily compensated because they always change in different operations and different experiments. In the color digital holography, the phase shift value changes with wavelength for the same operation. Therefore, the phase-shifting error could be one of the main error sources for PSDH. To our knowledge, the effect of the phase-shifting error on the reconstruction has not been sufficiently understood yet.

In this paper, our attention is mainly concentrated on the single phase-shifting algorithm (SPSA) of digital holography. In this algorithm only one phase-shifting operation is carried out. Therefore, the better quality of the reconstruction image from SPSA than others is expected when the phase-shifting operation is the main error source. Theoretical study shows that the effects of phase-shifting error are the intensity decrease of the primary image and the appearance of the conjugate image in reconstruction. The explicit equation for explanation of these effects is obtained. Some experimental results as well as computer-simulated work are presented to support the theoretical results.

For generality we consider the off-axis setup of PSDH, as shown in Fig. 1. A collimated laser beam is divided by the beam splitter (BS) into two beams. One, as the reference wave, passes through the phase shifter (PS). The other produces the object wave after reflecting from the object. Then the two beams are combined by the half-mirror (HM), which can adjust the angle between the reference wave and the object wave. Shutters K_1 and K_2 are used to control optical paths. When both of K_1 and K_2 are open, the intensity in CCD plane can be expressed as the usual holographic equation

$$\begin{aligned} I(x,y) &= |O(x,y) + R(x,y)|^2 \\ &= |O(x,y)|^2 + |R(x,y)|^2 \\ &\quad + O(x,y)R*(x,y) + O*(x,y)R(x,y), \quad (1) \end{aligned}$$

where (x, y) denotes the coordinates of the CCD plane, $O(x,y)$ and $R(x,y)$ represent the complex amplitudes of object wave and reference wave, respectively. $I(x,y)$ contains the amplitude and phase information of the object wave. Taking this point of view in mind, we only need two equations to uniquely solve the object wave

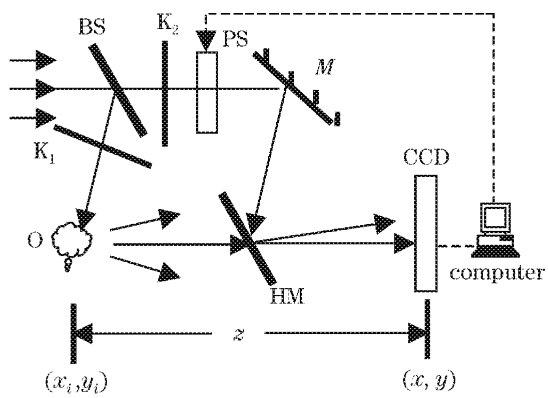


Fig. 1. The scheme for the discussion of SPSA principle. BS: beam spliter; K_1, K_2: shutters; PS: phase shifter; M: mirror; O: object; HM: half mirror; CCD: camera.

function $O(x,y)$ from the intensity records. For this purpose, a phase-shift ϕ independent of (x,y) is given to the reference beam. Then the holographic equation can be written as

$$I(x,y;\phi) = \left|O(x,y) + R(x,y)\,\mathrm{e}^{i\phi}\right|^2$$
$$= |O(x,y)|^2 + |R(x,y)|^2 + O(x,y)\,R*(x,y)\,\mathrm{e}^{-i\phi}$$
$$+O*(x,y)\,R(x,y)\,\mathrm{e}^{i\phi}. \quad (2)$$

Equations (1) and (2) can be reformed as

$$I(x,y) - I_{\mathrm{O}} - I_{\mathrm{R}}$$
$$= O(x,y)\,R*(x,y) + O*(x,y)\,R(x,y), \quad (3)$$
$$I(x,y;\phi) - I_{\mathrm{O}} - I_{\mathrm{R}}$$
$$= O(x,y)\,R*(x,y)\,\mathrm{e}^{-i\phi} + O*(x,y)\,R(x,y)\,\mathrm{e}^{i\phi}, \quad (4)$$

where

$$I_{\mathrm{O}} = |O(x,y)|^2, \quad (5)$$

$$I_{\mathrm{R}} = |R(x,y)|^2. \quad (6)$$

I_{O} and I_{R} can be recorded by controlling the shutters K_1 and K_2. From Eqs. (3) and (4), $O(x,y)$ can be solved as

$$O(x,y)$$
$$= \frac{[I(x,y) - I_{\mathrm{O}} - I_{\mathrm{R}}]\,\mathrm{e}^{i\phi} - [I(x,y;\phi) - I_{\mathrm{O}} - I_{\mathrm{R}}]}{2iR*(x,y)\sin\phi}. \quad (7)$$

Equation (7) shows that registering the intensities $I(x,y)$, I_{O}, I_{R} and $I(x,y;\phi)$ can rebuild the object wave function $O(x,y)$. For reconstruction of the object image, the Fresnel transformation of the object wave function $O(x,y)$ can be calculated as

$$O(x_i,y_i) = \frac{\exp(ikZ)}{i\lambda Z}$$
$$\int\int O(x,y)\exp\left\{i\frac{\pi}{\lambda Z}\left[(x_i-x)^2 + (y_i-y)^2\right]\right\}\mathrm{d}x\mathrm{d}y, \quad (8)$$

where (x_i, y_i) are the coordinates of output plane which coincides with the object plane if Z is chosen as the distance between object plane and CCD plane.

Equation (7) holds also for the in-line optical arrangement because there is not any assumption on the reference wave. The phase shift ϕ can be any given value. When $\phi = \frac{\pi}{2}$, the formula becomes

$$O(x,y)$$
$$= \frac{i\,[I(x,y) - I_{\mathrm{O}} - I_{\mathrm{R}}] - \left[I\left(x,y;\frac{\pi}{2}\right) - I_{\mathrm{O}} - I_{\mathrm{R}}\right]}{2iR*(x,y)}. \quad (9)$$

For convenience to compare, the formulae of FSA and TSA are quoted from Ref. [10] as

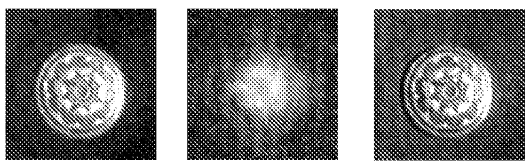
$$O(x,y) =$$
$$\frac{I(x,y) - I(x,y;\pi) + i\left[I\left(x,y;\frac{\pi}{2}\right) - I\left(x,y;\frac{3\pi}{2}\right)\right]}{4R*(x,y)}, \quad (10)$$

$$O(x,y) =$$
$$\frac{(1-i)\left\{I(x,y) - I\left(x,y;\frac{\pi}{2}\right) + i\left[I\left(x,y;\frac{\pi}{2}\right) - I(x,y;\pi)\right]\right\}}{4R*(x,y)}. \quad (11)$$

To demonstrate whether the SPSA works well or not, a very simple experimental setup is built as shown in Fig. 1. Some typical experimental results are shown in Fig. 2. Figure 2(a) shows the input object, which is a cloth button with a diameter of 20 mm at a distance of 30 cm from the CCD plane. The in-line optical arrangement is adopted in this experiment. A DALSA 1M30 CCD camera (objective lens removed) is used to capture the interference patterns. A plane parallel plate, which is specially designed for $\frac{\pi}{2}$ phase delay, can be inserted or removed in the path of reference beam. A He-Ne laser with 10 mW power and 633-nm wavelength is used as the light source. By registering $I(x,y)$, I_{O}, I_{R} and $I\left(x,y;\frac{\pi}{2}\right)$, the computer first calculates $O(x,y)$ according to Eq. (9). Then the Fresnel transformation is performed according to Eq. (8). The reconstruction image is shown in Fig. 2(c). This result shows that the zero-order and conjugate images are really removed after one phase-shifting operation. For comparison, Fig. 2(b) shows the reconstruction result for the same input with only one exposure. Because of the existence of the zero-order and conjugate images, we can hardly recognize the reconstructed object.

Theoretically the exact reconstruction of the object wave function $O(x,y)$ can be obtained from each one of Eqs. (9), (10) and (11). In practice, the actual phase shift value is not exactly equal to the value as the equations require. For SPSA, if the true value of phase shift contained in the recorded intensity is ϕ', rather than assumed value ϕ, according to Eq. (7) the calculation will not give the exact $O(x,y)$, but

$$O'(x,y) =$$
$$\frac{[I(x,y) - I_{\mathrm{O}} - I_{\mathrm{R}}]\exp(i\phi) - [I(x,y;\phi') - I_{\mathrm{O}} - I_{\mathrm{R}}]}{2iR*(x,y)\sin\phi}. \quad (12)$$



(a) (b) (c)

Fig. 2. Experimental results of the in-line holography. (a) object; (b) the reconstruction image from traditional in-line digital holography; (c) the reconstruction image from SPSA.

To find the effects of the phase-shifting error, $O'(x,y)$ should be resolved from Eqs. (1) and (2) as

$$O'(x,y) = O(x,y)\frac{\exp(i\phi) - \exp(-i\phi')}{2i\sin\phi} + O*(x,y)R^2\frac{\exp(i\phi) - \exp(i\phi')}{2iI_{\mathrm{R}}\sin\phi}. \quad (13)$$

The explicit equation (13) shows that the effects of the phase error are the decrease of the amplitude $O(x,y)$ and the appearance of the conjugate image $O*(x,y)$. Conjugate image may seriously damage the quality of the desired reconstruction for the in-line holography or decrease the effective capability of the system for off-axis holography because the conjugate image will occupy some space. At moment our experimental system is specially designed for SPSA with a fixed phase operation. The appearance of conjugate image for different algorithms is confirmed by the computer simulated experimental results shown in Fig. 3. In this example the input is an English character "F". For easier observation of the error effect, an off-axis arrangement is adopted. The angle between object beam and reference beam is set to 1.5°. During registering intensity we assume each phase-shifting operation has a 0.3-rad error. Figures 3(a), (b) and (c) show the reconstructed results based on Eqs. (9), (10) and (11), respectively. One can see that the conjugate image appears in the reconstructions for each algorithm. For SPSA the effects of phase error can easily eliminated by adjusting the parameter ϕ around the theoretical value until the conjugate image disappears. From Eq. (13) one can recognize that when ϕ reaches ϕ', the second term will be zero and Eq. (13) will reach the exact $O(x,y)$. This search procedure is possible even the approximation value of phase shift is not known. For FSA and TSA, this will be a complex task because there are two or three parameters to be adjusted.

For estimating and comparing the phase error effect of different algorithms, we calculate the intercorrelation

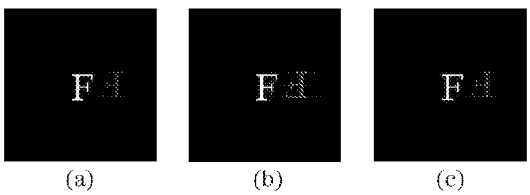


Fig. 3. Computer-simulated experimental results of the off-axis holography using SPSA, FSA and TSA. (a) The reconstruction from SPSA; (b) the reconstruction from TSA; (c) the reconstruction from FSA.

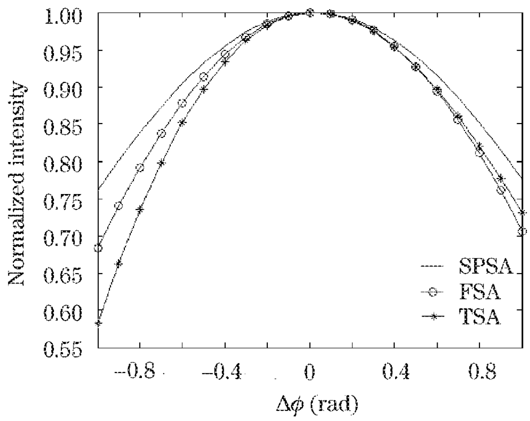


Fig. 4. Normalized intercorrelation peak between input and reconstruction as a function dependent of phase-shifting error.

between the input $U(x_i,y_i)$ in object plane and the output $O'(x_i,y_i)$. The results are given in Fig. 4, where the intercorrelation peak normalized by the autocorrelation of $U(x_i,y_i)$ shows a function dependence with the phase error $\Delta\phi = \phi'_j - \phi_j$ (ϕ'_j and ϕ_j are previously defined). The normalized intercorrelation peak decreases with the increase of the phase error in each case. The curve for SPSA is always over the curves for FSA and TSA. If the normalized intercorrelation peak can be an adopted standard, one may conclude that the quality of the reconstruction based on SPSA is the best one among the three algorithms under the same experimental condition.

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