Measurement of single cycle and sub-cycle pulse duration

Zhenglie Gong (龚正烈)[1], Wenzhuo Ge (葛文卓)[1],
Guizhong Zhang (张贵忠)[2], and Wanghua Xiang (向望华)[2]

[1]*Department of Physics, Tianjin University of Thchenology, Tianjin 300191*
[2]*School of Precision Instrument and Optoelectronics, Tianjin University, Tianjin 300072*

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This paper suggests that the linear interferometric correlation (LFC) can be used to measure pulse duration of a few cycles, single cycle or even sub-cycle light pulse. The relations between pulsewidth and LFC curve are derived for Gaussian- and hyperbolic secant-shaped pules. This new method abandons focusing, frequency doubling and filtering in the traditional second order correlation method, meanwhile the signal-to-noise ratio (SNR) is improved.

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The terms "single cycle" and "sub-cycle" used in present paper mean that the light pulse duration is equal to, or shorter than one optical cycle of a signal carrier.

So far, the pulsewidth obtained directly from mode-locking of visible light reaches 5 fs[1]. 4 fs[2] may be achieved by means of band broadening and pulse compression, which is close to an optical cycle. The pulsewidth of one optical cycle at 800-nm wavelength light is about 2.7 fs. Usually the duration of ultrashort pulse is measured using second harmonic intensity correlation (HIC). The development of HIC technique was largely due to the fact that there was no detector or display device at that time fast enough to respond to ps order pulses. (Streak camera with a temporal resolution up to ps was available at later time). The problem that the HIC is still valid for pulses as short as a cycle should be dressed.

For this purpose, we have studied the structure of the second order correlation function and the physical process in the correlation measurement. We have found that the HIC signal, the 3:1 curve, taken by a slow-response recorder in type-I co-linear configuration is an averaged result of the second harmonic interferometric correlation (HFC), the 8 : 1 curve. The 8 : 1 curve can be obtained using a fast-response recorder.

We can expect that it will be more difficult for a detector to respond to a single cycle or shorter pulses using HIC. The time resolution is, therefore, greatly reduced even if a correlation curve could be measured somehow.

In 1986, we obtained the 8 : 1 curve for the first time at home[3], and put forth a means of employing the equal-value width of the upper envelope of the 8 : 1 curve[4], and a method of computer-averaging the 8 : 1 curve to 3 : 1 curve for achieving higher precision[5,6]. All these means turn out to be useless for pulses of a single cycle or shorter, as there are few points in the correlation curves to process.

In fact, the shortest pulses so far were diagnosed by the 8 : 1 curve method. People measured the datum-points of 8 : 1 curve and then used curve-fitting to get the pulsewidth. This method is tedious and may miss some important points.

We put forward an idea to measure pulsewidth with linear interferometric correlation (LFC) from the original pulse. Computer simulation shows that, similar to the 8 : 1 curve, LFC curves of a cycle or shorter pulse can be dependent upon the peak number of the first order correlation curves (named as 2 : 1 curves), and upon the lobe heights. In the sub-cycle situation, the curve width is only related to the main peak, since there is no lobe.

This paper derives the relation between LFC function of light pulse and its duration, and presents simplified formulas for two specific cases.

Let the normalized electric field of optical pulses take the Gaussian or hyperbolic secant shapes

$$E(t) = \exp(-\frac{t^2}{T^2})\cos(\omega t), \quad \text{(Gaussian)} \qquad (1)$$

$$E(t) = \mathrm{sech}\frac{t}{T}\cos(\omega t), \quad \text{(hyperbolic secant)} \qquad (2)$$

where ω is the carrier frequency. The intensity of the pulse is

$$I(t) = (\mathrm{e}^{-\frac{t^2}{T^2}})^2 = \mathrm{e}^{-\frac{2t^2}{T^2}}, \quad \text{(Gaussian)} \qquad (3)$$

$$I(t) = \mathrm{sech}^2\frac{t}{T}, \quad \text{(hyperbolic secant)} \qquad (4)$$

where T is a parameter relating to the pulsewidth

$$\Delta t = T\sqrt{2\ln 2}, \quad \text{(Gaussian)} \qquad (5)$$

$$\Delta t = T\ln(3+2\sqrt{2}). \quad \text{(hyperbolic secant)} \qquad (6)$$

After being split in the optical correlator, the ultrashort pulses are co-linearly detected by a detector. When the time delay is changed slowly enough, the LFC function of the pulses can be recorded in a normalized way[7]

$$I(\tau) = 1 + \mathrm{e}^{-\frac{\tau^2}{2T^2}}\cos(\omega\tau), \quad \text{(Gaussian)} \qquad (7)$$

$$I(\tau) = 1 + \frac{\tau}{T}\mathrm{csch}\frac{\tau}{T}\cos\omega\tau. \quad \text{(hyperbolic secant)} \qquad (8)$$

The experiment and theory proved that LFC data represented by Eqs. (7) or (8) are exactly corresponding to

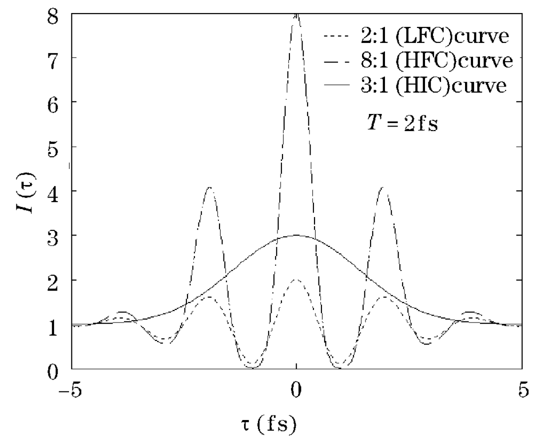

Fig. 1. The comparison of 3 correlation curves.

the HFC data and their trends are completely consistent to each other[4]. It should be pointed out that the scanning speed in HIC is much more fast than those in HFC and LFC in real experiments. For comparison, Fig. 1 gives three kinds of correlation curves with $T = 2$ fs at the same scan speed. It is known that HIC curve (3 : 1 curve) from which the pulsewidth is obtained is an average over HFC curve (8 : 1 curve). Therefore, it is reasonable to say that the pulsewidth should also be obtainable from the LFC curve. The only difference between the 3 : 1 and 2 : 1 curves is that one can not average over the 2 : 1 curve to obtain the pulsewidth since this average would be a straight line of value 1.

We are interested in deriving the simplified expressions of pulsewidth from Eqs. (7) and (8) under two specific conditions:

1) If side lobes exist in LFC curve, one measures the amplitude of the first lobe in the right wing $I(\tau_0)$ and its corresponding τ_0. It can be known from $\omega\tau_0 = 2\pi$ that τ_0 is the carrier cycle, denoted as P. Substituting it into Eqs. (7) and (8) and solving for T, one gets

$$\begin{cases} \Delta t = 0.833\{-\ln[I(\tau_0) - 1]\}^{-\frac{1}{2}}P, \\ \qquad \text{(Gaussian)} \\ \Delta t = 1.763\{\sqrt{\frac{120}{I(\tau_0)-1} - 20} - 10\}^{-\frac{1}{2}}P. \\ \qquad \text{(hyperbolic secant)} \end{cases} \tag{9}$$

For hyperbolic secant shaped pulses, one can take the first six terms in the expansion

$$e^{\pm x} = 1 \pm x + \frac{x^2}{2!} \pm \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots,$$

where $x = \tau/T$, and get

$$e^{x} - e^{-x} = 2(x + \frac{x^3}{3!} + \frac{x^5}{5!}). \tag{10}$$

2) When there is only one main peak without side lobe, the delay τ_0 in the right wing is corresponding to the half maximum of the peak. Insert the measured τ_0 into Eqs. (7) and (8), and the pulsewidth is

$$\begin{cases} \Delta t = 0.833(\ln 2 \cos\omega\tau_0)^{-\frac{1}{2}}\tau, \\ \qquad \text{(Gaussian)} \\ \Delta t = 1.763(\sqrt{240\cos\omega\tau_0 - 20} - 10)^{-\frac{1}{2}}\tau_0. \\ \qquad \text{(hyperbolic secant)} \end{cases} \tag{11}$$

The expressions (9) and (11) indicate that as long as LFC curve is obtained experimentally, the pulsewidth can be extracted from one datum point.

More generally, measuring any experimental value $I(\tau)$ of LFC curve and its corresponding τ will give the pulsewidth through

$$\Delta t = \sqrt{\ln 2}(\ln \frac{\cos\omega\tau}{I(\tau) - 1})^{-\frac{1}{2}}\tau, \quad \text{(Gaussian)} \tag{12}$$

$$\Delta t = \frac{\ln(3 + 2\sqrt{2})}{\sqrt{\sqrt{\frac{120\cos\omega\tau}{I(\tau)-1} - 20} - 10}}\tau. \quad \text{(hyperbolic secant)} \tag{13}$$

It is obvious that expressions (9) and (11) as two special results are contained in Eqs. (12) and (13) above.

In conclusion, we got the following results:

1) To experimentally realize Eqs. (7) and (8), the pulse and its replica must be completely co-linear. The criterion is that they form only one interferometric fringe. In order to achieve the 2 : 1 curve, the temporal delay should be varied slowly enough.

2) The solution for Gaussian pulse is exactly analytical. That is to say, there is no error in the mathematical model. The solution is quasi-analytical for hyperbolic secant pulse. For example, the error is only 0.017 fs for pulse width of 2 fs.

3) The benefit of using linear correlation is obvious. Focusing, frequency doubling and filtering in the traditional HIC are no longer required. And the signal-to-noise ratio (SNR) gets improved. It is worthy pointing out that the pulsewidth measurement for high harmonics such as 10^{-18}s-pulses with this technique may become much easier.

Z. Gong's e-mail address is gongzl@public.tpt.tj.cn.

References

1. A. Baltuška, Z. Y. Wei, M. S. Pshenichkov, and D. A. Wiersma, Opt. Lett. **22,** 102 (1997).
2. X. Hou, Henan University (in Chinese) **32,** 1 (2002) (and its Ref. [13]).
3. Z. L. Gong, G. Q. Ji, Q. Y. Wang, and G. X. Xiao, Acta Opt. Sin. (in Chinese) **7,** 935 (1987).
4. Z. L. Gong, S. R. Qian, C. Zhang, and Q. Y. Wang, Acta Opt. Sin. (in Chinese) **9,** 467 (1989).
5. Z. L. Gong, Z. Y. Huang, Z. G. Xu, X. M. Cheng, S. R. Qian, W. H. Xaing, and Y. Q. Zhan, Opto-Electronic Engineering (in Chinese) **17** (2), 8 (1990).
6. Z. L. Gong, Z. Y. Huang, Z. G. Xu, S. R. Qian, W. H. Xiang, and Q. Y. Wang, *Ultrafast Phenomena* **VI,** (Springer, Berlin, 1988) p. 156.
7. Z. L. Gong, Z. Y. Huang, X. M. Cheng, X. L. Xia, Z. P. Jiang, and Z. F. Liu, Proc. SPIE **2889,** 341 (1996).