Effects of residual second- to fourth-order dispersion in ultra high-speed optical time division multiplexing transmission

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We present an expression of maximum fiber-link length, at which the output pulses can return to its original rms time width, in an optical fiber link with up to fourth-order dispersion. The fourth order dispersion is compensated by combination of the effects of proper source chirping and negative residual second-order dispersion. The interesting fact is that the optical pulses can restore itself at a longest distance even in case of chirp parameter being positive, as well as being negative traditionally. The validity of the analytical formulas is also confirmed by split-step Fourier numerical stimulation.

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Optical time division multiplexing (OTDM) system can be an alternative of future high-capacity optical communication system for its excellent properties by comparison with wavelength division multiplexing (WDM) system, such as easy to implement high speed routing and packet switching, single wavelength channel, etc. But, there are still some difficulties, which have kept OTDM off commercial application. To realize high-speed OTDM transmission system with channel capacities of several hundreds of gigabytes per second, the crucial factor is the ability to control the chromatic dispersion. Second- and third-order dispersion (TOD) are traditional limiting factors in high-speed OTDM system. Their effects have been widely studied, and several methods for their simultaneous compensation by means of a suitable combination of different kinds of fibers have been reported[1,2]. In ultra high-speed OTDM system, the channel capacities can be close to 1 Tb/s, so the pulse width must be limited under 1 ps, and fourth-order dispersion (FOD) must be considered. Because FOD of the existing fibers have the same sign (negative), it cannot be compensated by fiber combination method. However, FOD was compensated by employing cosine phase modulation and residual second-order dispersion (SOD) in two related experiments[3,4]. The combination of FOD and proper excess negative SOD of the whole transmission line can be approximated by a cosine function near ω_0[3]

$$1+\frac{\beta_2}{2}(\varpi-\varpi_0)^2+\frac{\beta_4}{24}(\varpi-\varpi_0)^4 \cong \beta_4\cos(\varpi-\varpi_0), \quad (1)$$

in case of $\beta_2=-\beta_4$, where ω_0 is the center frequency. Second- and fourth-order dispersion are represented in the terms including β_2 and β_4, respectively.

Recently, a closed-form expression for computation of the output pulse rms time width in an optical fiber link with up to FOD by use of an optical source with arbitrary linewidth and chirp parameters has been reported[5]. When SOD and TOD were thoroughly compensated by certain fiber combination[5], FOD can be compensated for to an upper link-length limit, above which other techniques must be employed, by suitable source chirping.

In this paper, we demonstrate that the FOD can be compensated for by proper source chirping and excess SOD to an upper link-length, which is longer by comparison with the fiber-link with zero averaged SOD. The expression for the output pulse's rms time width σ[5] is

$$\frac{\sigma}{\sigma_0}=[1+4CD+(4D^2+12FC)(1+C^2+V^2)+(9B^2+24DF)(1+C^2+V^2)^2+60F^2(1+C^2+V^2)^3]^{1/2}, \quad (2)$$

where $\sigma_0=T_0/\sqrt{2}$ is the input Gaussian pulse's rms time width. D, B and F represent second-, third- and fourth-order dispersion length parameters, respectively,

$$D=\frac{\beta_2' L}{2T_0^2},\quad B=\frac{\beta_3 L}{6T_0^3},\quad F=\frac{\beta_4 L}{24T_0^4}. \quad (3)$$

Parameters V and C account for the source's normalized linewidth and chirp parameter, respectively. Here we take the assumption that the SOD β_2 can be substituted by averaged SOD $\beta_2'=(\beta_{21}L_1+\beta_{22}L_1)/(L_1+L_2)$. $L_{1,2}$, $\beta_{21,22}$ are the length and SOD of each section of fiber-link, respectively. TOD is compensated by combination of different kinds of fibers, so we have TOD parameter $B=0$. Moreover, we can choose low-linewidth sources where $V\ll 1$. Under these conditions, Eq. (2) can be reduced to

$$\frac{\sigma}{\sigma_0}=[1+4CD+(4D^2+12FC)(1+C^2)+24DF(1+C^2)^2+60F^2(1+C^2)^3]^{1/2}. \quad (4)$$

In order to compensate FOD, we have $\sigma/\sigma_0=1$, thus Eq. (4) can be written as

$$L=\frac{-24T_0^4(\beta_4 C+\beta_4 C^2+4\beta_2 C T_0^2)}{5\beta_4^2(1+C^2)^3+24\beta_2\beta_4 T_0^2(1+C^2)^2+48\beta_2^2 T_0^4(1+C^2)}. \quad (5)$$

The whole fiber-link length L is plotted against chirp parameter and averaged SOD for the case of $T_0 = 380$ fs, $\beta_4 = 0.00086$ ps^2/km. According to Fig. 1, there are two vertexes of L, one is at the point of $C = 0.58$ and $\beta_2' = -0.0036$ ps^2/km, the other is at the point of $C = -0.58$ and $\beta_2' = -0.00036$ ps^2/km. Because of the instability in the transmission system due to the uneven dispersion in optical fiber and random jitter of source chirping, we can operate the system at one of these two vertexes so as to decrease the negative effects of the perturbation of SOD or source chirping to the extremely extent. (As can be seen from Eq. (2), there is only one quadratic term of B, so the optimum β_3 is zero.) In order to get precise value of each parameter, we can also derive the results from solving the equations below,

$$\frac{\partial L(\beta_2, C)}{\partial \beta_2} = 0, \quad \frac{\partial L(\beta_2, C)}{\partial C} = 0. \tag{6}$$

Substituting the solutions of Eq. (6)

$$\beta_2 = -(3 \pm \sqrt{6})\frac{\beta_4}{9T_0^2} \quad \text{and} \quad C = \mp\frac{1}{\sqrt{3}} \tag{7}$$

into Eq. (5), we derive the expression of maximum fiber-link length,

$$L_{\max} = 1.5910\frac{T_0^4}{\beta_4}. \tag{8}$$

$L_{\max}$ is a function of initial pulse time width T_0 and FOD β_4, and increases with T_0 significantly. When FOD $\beta_4 = 0.00086$ ps^4/km and $T_0 = 1$ ps, L' is 1850 km. Thus, under these conditions, the effects of FOD can be compensated very well by proper source chirping and excess SOD. However, the optimum chirp parameter is a constant, and has nothing to do with β_4 and T_0. When $\beta_4 = 0.00086$ ps^4/km and $T_0 = 380$ fs, the optimum averaged SOD and chirp parameter are $\beta_2' = -0.00036$ ps^2/km, $C = -1/\sqrt{3}$, or $\beta_2' = -0.00361$ ps^2/km, $C = 1/\sqrt{3}$. The maximum fiber-link length, at which the optical pulses can return to its initial rms time width, is 38.575 km. The surprising facts are that the same maximum fiber-link length L' exists with different pair of optimum parameters β_2' and C,

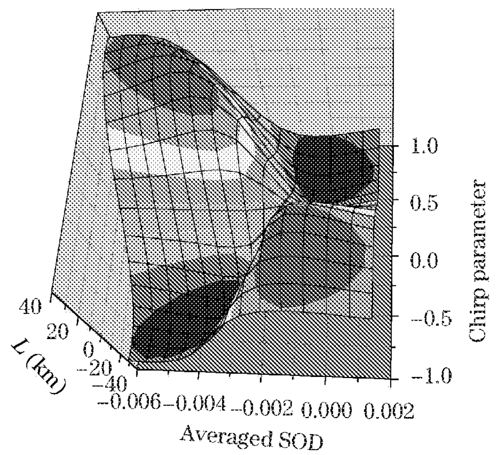

Fig. 1. L' as a function of chirp parameter and averaged SOD obtained by direct application of Eq. (5).

and the optical pulse can also restore itself at a maximum length when C is positive. Comparing with the case, in which average SOD is zero[5], the pulse can only restore itself to an upper distance when C is negative.

We also simulate the ratio of the output-to-input rms time width as functions of source chirp C and averaged SOD respectively by split-step Fourier integration in the dispersion compensation fiber-link. The nonlinear Shrödinger equation is given in the form of

$$i\frac{\partial A}{\partial z} + \frac{i\alpha}{2}A - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - i\frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} + \frac{\beta_4}{24}\frac{\partial^4 A}{\partial T^4} + \gamma |A^2| A = 0. \tag{9}$$

The fiber link we considered is consisted of a 35-km single mode fiber and 3.5-km dispersion compensation fiber. The SOD and TOD values are 16.5 ps^2/km and 0.02 ps^3/km, respectively, for the single mode fiber. The SOD of dispersion compensation fiber is ranging around 165 ps^2/km, and TOD value is 0.2 ps^3/km. By adjusting the pulse state of polarization at various points along the fiber link to manipulate the faster/slower polarization axis, the total PMD of the respective fiber sections cancel[4]. The input optical power is 1 mW, and nonlinear constant γ is 1.27 m^{-1}W^{-1}. The other higher order nonlinear terms are neglected.

In Figs. 2(a) and (b), the ratio of output-to-input rms time width is plotted against averaged SOD with chirp

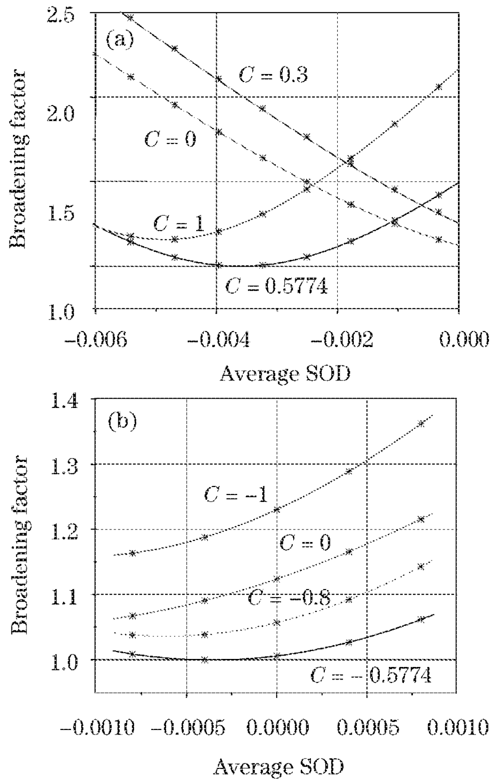

Fig. 2. σ/σ_0 versus averaged SOD obtained by direct application of Eq. (4) (curves) and split-step Fourier integration (asterisks). (a) Optimum chirp parameter $C = 0.5774$; (b) optimum chirp parameter $C = -0.5774$.

as a parameter, obtained by direct application of Eq. (4) (curves) and by solution of nonlinear Shrödinger equation (asterisks). After 38.5-km transmission, the output optical pulse can return to its original rms time width at certain value of averaged SOD only when chirp parameter $C = 0.5774$ (Fig. 2(a)) or $C = -0.5774$ (Fig. 2(b)). In Figs. 3(a) and (b), σ/σ_0 is plotted against source chirping with averaged SOD as a parameter, obtained both by

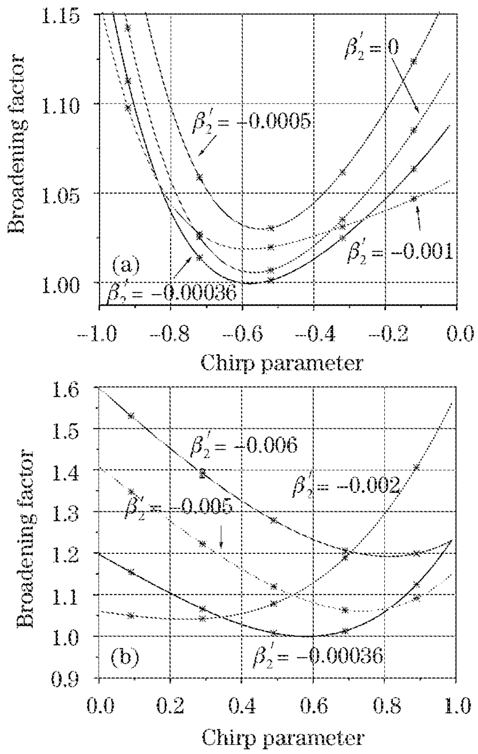

Fig. 3. σ/σ_0 versus chirp parameter obtained by direct application of Eq. (4) (curves) and split-step Fourier integration (asterisks). (a) Optimum averaged SOD $\beta_2' = -0.00036$ ps^2/km; (b) optimum averaged SOD $\beta_2' = -0.0036$ ps^2/km.

Eq. (4) (curves) and nonlinear Shrödinger equation (asterisks). In the same way, after 38.5-km transmission, the output optical pulse can return to its original rms time width at certain value of chirp parameter only when averaged SOD $\beta_2' = -0.00036$ (Fig. 3(a)) or $\beta_2' = -0.0036$ (Fig. 3(b)). Because of the optical pulses are very short, the broadening effects of the optical pulses in the fiber line is very serious, so the effect of self phase modulation is very weak and can be neglected[4]. Therefore, as can be seen from Figs. 2 and 3, the agreement between the results given by Eq. (4) and the nonlinear Shrödinger equation is so well.

In conclusion, we have shown FOD can be compensated for to an upper link-length limit via proper source chirping and excess SOD, instead of zero-averaged SOD fiber link. And the system can operate relatively stable in these conditions under small nonlinear effects. The chirp parameter of the output pulse is not considered. We also present the numerical solution of nonlinear Schrödinger equation, which exhibit a well agreement with the approximate analytic resolution.

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