Collision-induced timing jitter in dispersion-managed WDM soliton system with filtering

Jianxin Du (杜建新)[1,2] and Qihong Lou (楼祺洪)[1]

[1]*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800*

[2]*Graduate School of the Chinese Academy of Sciences, Beijing 100039*

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Taking into account the randomicity of collision positions and the arbitrary encoding of data in channel, the influences of different dispersion management on collision-induced timing jitter in a filtered wavelength division multiplexing (WDM) soliton system are obtained statistically and numerically by applying a set of coupled ordinary differential equations which are derived through variational procedure. The optimal dispersion managements which can greatly reduce the collision-induced timing jitter are found. The multi-channel collision-induced timing jitters in a filtered WDM soliton system are given with an optimal dispersion management and constant dispersion.

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In wavelength division multiplexing (WDM) soliton systems there can be serious timing displacement effects due to nonuniform soliton collisions in the presence of optical amplifiers, which induce residual frequency and velocity shifts of the soliton. The permanent frequency shifts then induce undesirable timing jitter of the signal. In recent analyses, the dispersion management technique has been shown to be effective for reducing the timing jitter in WDM systems[1−3]. However, to our knowledge no statistical analysis taking into account the randomicity of collision positions and the arbitrary encoding of data in a channel simultaneously in a filtered dispersion-managed WDM soliton system has been reported. In this letter, on the base of this statistical analysis, we give the optimal dispersion managements which can greatly reduce the collision-induced timing jitter.

The analysis is based on the coupled nonlinear Schrödinger equation (NLS) that governs a two-channel WDM system

$$i\frac{\partial u_j}{\partial \zeta}+\frac{\sigma(\zeta)}{2}\frac{\partial^2 u_j}{\partial \tau^2}+a^2(\zeta)\left(|u_j|^2+2|u_{3-j}|^2\right)u_j=0, \tag{1}$$

where $u_j(\zeta,\tau)$ are the normalized electric-field envelopes for each channel (with $j=1$, 2). Coordinates ζ and τ are the nondimensional space and time, normalized to the dispersion length Z_0 and the characteristic time T_0: $Z_0=2\pi c T_0^2/(\lambda^2 D_{\rm av})$ and $T_0=T_{\rm FWHM}/1.665$, respectively, where $\lambda=1.550$ μm is the central wavelength, $D_{\rm av}$ is the average dispersion parameter, $T_{\rm FWHM}$ is the full-width at half-maximum of the pulse intensity, and c is the speed of light in vacuum. The periodic gain-loss cycle given by $\sigma(\zeta)$ is represented by $a(\zeta)$ and the local dispersion, normalized to its average value $D_{\rm av}$.

Equation (1) can be solved by the variational approach[4], and we give its Lagrangian

$$L=\int_{-\infty}^{+\infty}\left\{\sum_{j=1}^{2}\left[\left(u_j\frac{\partial u_j^*}{\partial\zeta}-u_j^*\frac{\partial u_j}{\partial\zeta}\right)\right.\right.$$
$$\left.\left.+\sigma(\zeta)\left|\frac{\partial u_j}{\partial\tau}\right|-a^2(\zeta)|u_j|^4\right]-4a^2(\zeta)|u_1|^2|u_2|^2\right\}\mathrm{d}\tau. \tag{2}$$

We assume that a trial solution of Eq. (1) takes the following form,

$$u_j=Ab\sqrt{\eta_j}\exp\left\{-\left[m\eta_j(\tau-q_j)\right]^2\right\}$$
$$\times\exp\left[-ik_j(\tau-q_j)+iC_j(\tau-q_j)^2\right], \tag{3}$$

where η_j is the normalized inverse width, C_j is the quadratic phase chirp, q_j is the temporal position, and k_j the central frequency of pulse u_j. The constants $m=0.669$ and $b\approx1.0324$. The constant A satisfies[5,6]

$$A^2=1+0.7S^2, \tag{4}$$

$$S=\left|\frac{\lambda_0^2}{2\pi c}\frac{(D_{\rm an}-D_{\rm av})L_1-(D_{\rm n}-D_{\rm av})L_2}{T_0^2}\right|, \tag{5}$$

where $D_{\rm an}$ and $D_{\rm n}$ are the anomalous and normalous dispersion, L_1 and L_2 are the anomalous and normalous dispersion lengths in the unit cell of the dispersion map. S is defined as dispersion map strength.

Substituting Eq. (3) into Eq. (2) and evaluating the Lagrangian, we then obtain

$$L=\sum_{j=1}^{2}\left\{\frac{-i\sqrt{2\pi}A^2b^2}{m}\left(k_j\frac{\mathrm{d}q_j}{\mathrm{d}\zeta}+\frac{1}{4m^2\eta_j^2}\frac{\mathrm{d}C_j}{\mathrm{d}\zeta}\right)\right.$$
$$\left.+\frac{\sqrt{2\pi}}{2}A^2b^2\left(\frac{m^4\eta_j^4+C_j^2}{m^3\eta_j^2}+\frac{k_j^2}{2m}\right)-\frac{\sqrt{\pi}a^2A^4b^4\eta_j}{2m}\right\}$$
$$-\frac{2\sqrt{2\pi}a^2A^4b^4\eta_1\eta_2}{m\sqrt{\eta_1^2+\eta_2^2}}$$
$$\times\exp\left[2m^2\left(\eta_1^2q_1^2+\eta_2^2q_2^2\right)-\frac{2m^2\left(\eta_1^2q_1+\eta_2^2q_2\right)}{\eta_1^2+\eta_2^2}\right]. \tag{6}$$

The coupled ordinary differential equations (ODEs) by variations with respect to the free parameters η_j, C_j, q_j, and k_j are obtained. The simplified ODEs are then given by

$$\frac{\mathrm{d}\eta_j}{\mathrm{d}\zeta} = -2\sigma(\zeta)\,\eta_j C_j, \tag{7}$$

$$\frac{\mathrm{d}C_j}{\mathrm{d}\zeta} = 2\sigma(\zeta)\left(m^4\eta_j^4 - C_j^2\right) - 2\frac{a^2(\zeta)\,m^3\eta_j^3}{\sqrt{\pi}}$$

$$\left\{A^2 + \sqrt{2}A^2\left[1 - 2m^2(q_j - q_{3-j})^2\eta_j^2\right]\times F\right\}, \tag{8}$$

$$\frac{\mathrm{d}q_j}{\mathrm{d}\zeta} = -\sigma(\zeta)\,k_j, \tag{9}$$

$$\frac{\mathrm{d}k_j}{\mathrm{d}\zeta} = \frac{8}{\sqrt{\pi}}a^2(\zeta)\,A^2m^3\eta_j^3(q_j - q_{3-j})\times F, \tag{10}$$

$$F = \exp\left[-m^2(q_j - q_{3-j})^2\eta_j^2\right].$$

The filters placed at every amplifier location are assumed to have a Gaussian transfer function

$$H(\xi) = \sqrt{G_\mathrm{e}}\exp\left[-2\frac{(\xi - \xi_\mathrm{f})^2}{B^2}\right], \tag{11}$$

$$G_\mathrm{e} = \exp(2\delta\zeta_\mathrm{a}), \tag{12}$$

$$B = \sqrt{\frac{2}{\beta\zeta_\mathrm{a}}}, \tag{13}$$

where G_e is the extra power gain necessary to compensate the loss introduced by the filter, B is the normalized units and ξ_f is the central frequency. ζ_a is the normalized amplifier separation. δ and β are the distributed filter parameters. When a pulse passes through a filter, its parameters are changed according to the equations[7]

$$A_\mathrm{out} = \frac{A_\mathrm{in}\sqrt{B}\sqrt{\kappa\eta_\mathrm{in}}}{\sqrt[4]{8(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + B^2\kappa^4\eta_\mathrm{in}^2}}$$

$$\times\exp\left\{-\frac{\kappa^2\eta_\mathrm{in}^2(\xi_\mathrm{in} - \xi_\mathrm{f})^2}{\sqrt[4]{8(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + B^2\kappa^4\eta_\mathrm{in}^2}}\right\}\sqrt{G_\mathrm{e}}, \tag{14}$$

$$\eta_\mathrm{out} = \frac{B}{\kappa}\sqrt{\frac{8(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + B^2\kappa^4\eta_\mathrm{in}^2}{64(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + 16B^2\kappa^4\eta_\mathrm{in}^2 + B^4}}, \tag{15}$$

$$C_\mathrm{out} = \frac{C_\mathrm{in}B^4}{64(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + 16B^2\kappa^4\eta_\mathrm{in}^2 + B^4}, \tag{16}$$

$$q_\mathrm{out} = q_\mathrm{in} + \frac{4C_\mathrm{in}(\xi_\mathrm{in} - \xi_\mathrm{f})}{8(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + B^2\kappa^4\eta_\mathrm{in}^2}, \tag{17}$$

$$\xi_\mathrm{out} = \frac{8\xi_\mathrm{f}(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + \xi_\mathrm{in}B^2\kappa^4\eta_\mathrm{in}^2}{8(C_\mathrm{in}^2 + \kappa^4\eta_\mathrm{in}^2) + B^2\kappa^4\eta_\mathrm{in}^2}. \tag{18}$$

We have considered two-step dispersion maps with constant average dispersion $D_\mathrm{av} = 0.1$ ps/(km·nm) and period equal to the amplifier spacing $L_\mathrm{a} = 50$ km; fiber loss is 0.2 dB/km; $T_\mathrm{FWHM} = 10$ ps; initial inverse width $\eta_j = 1$; chirp $C_j = 0$; channel separation is 996 GHz (in normalized units $\Delta\xi = \xi_1 - \xi_2 = 35.5$); $\beta = 0.3$, $\delta = 0.1$.

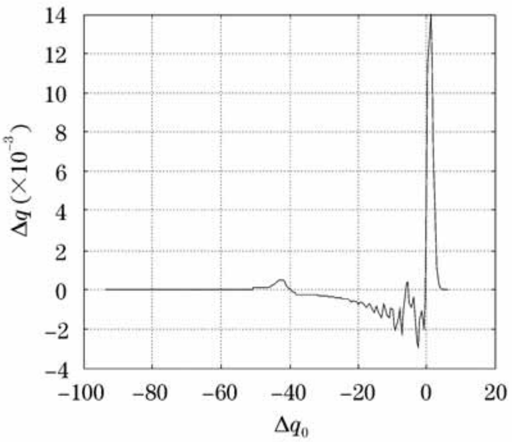

Fig. 1. Residual time displacement Δq (transmission distance of 3000 km) as a function of initial separation Δq_0 corresponding to $\Delta\sigma = 25$ and $L_1 = 25$ km in a filtered WDM system.

The initial pulse separation, $q_0 = (q_1 - q_2)$ $(\zeta = 0)$, will greatly affect the residual time displacement Δq. Figure 1 shows Δq as a function of q_0 when transmission distance is 3000 km corresponding to $L_1 = 25$ km, $\Delta\sigma = \sigma_\mathrm{an} - \sigma_\mathrm{n} = 25$ in a filtered dispersion-managed WDM soliton system by solving Eqs. (7)−(10) and Eqs. (14)−(18) numerically.

The relative timing shifts between adjacent solitons are important. $\delta q(z_\mathrm{c})$ is defined as timing shift corresponding to the given collision center position z_c in the fiber. The total relative timing shift is expressed by[1,2]

$$\Delta q_\mathrm{sum} = \sum_{c=1}^{N}(b_{n+1} - b_n)\,\delta q(z_\mathrm{c}), \tag{19}$$

where b_{n+1} and b_n represent the $(n+1)$th and nth solitons in channel #B which can be 0 or 1 depending on the arbitrary encoding data, N is the total number of collisions in the system length L. Considering the randomicity of the encoded binary data, the average of squares of relative timing shifts is given by

$$\left\langle(\Delta q_\mathrm{sum})^2\right\rangle = \frac{1}{2}\sum_{c=1}^{N}\left\langle(\delta q(z_\mathrm{c}))^2\right\rangle. \tag{20}$$

The normalized minimum pulse separation which corresponds to the reciprocal of the bit rate per channel is assumed to be 5.

The initial pulse separation, $q_0 = (q_1 - q_2)$ $(\zeta = 0)$, which determines collision position, is random. We assume it follows an average distribution. Taking into account the randomicity and applying Eqs. (7)−(10) and Eqs. (14)−(20) numerically, we give the contour plot Fig. 2 for the various possible dispersion maps when the transmission distance is 3 Mm. In Fig. 2, the contour plotted values are root-mean-square (rms) timing jitters.

Figure 2 shows two minima that yield optimal dispersion maps which can greatly reduce collision-induced timing jitter in a WDM soliton system with filters:

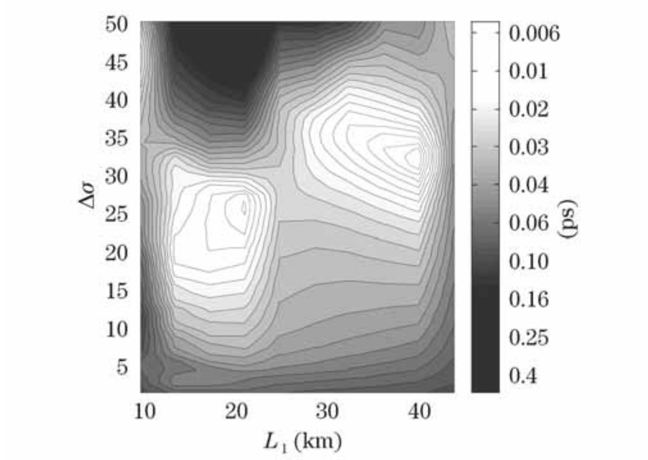

Fig. 2. Contour plot of rms timing jitters after 3000-km propagation versus dispersion differences $\Delta\sigma$ and lengths of first step L_1 in a filtered WDM soliton system taking into account the randomicity of collision positions and the arbitrary encoding of data in channel.

$L_1 = 21$ km, $\Delta\sigma = 25$, corresponding to dispersion map strength of 2.06 and the timing jitter (rms) is 0.007 ps; $L_1 = 40$ km, $\Delta\sigma = 32$, corresponding to dispersion map strength of 1.85 and the timing jitter (rms) is 0.005 ps.

The total variance in the relative arrival times of adjacent solitons in the jth channel is the sum of the variances resulting from the interactions with the other $J-1$ channels, J being the total number of channels. That is, the total mean square timing jitter for the each channel j is

$$\left\langle (\Delta q_{\text{tota}})^2 \right\rangle_j = \sum_{\substack{k=1 \\ k \neq j}}^{J} \left\langle (\Delta q_{\text{sum}})^2 \right\rangle_{jk}, \tag{21}$$

with $\left\langle (\Delta q_{\text{sum}})^2 \right\rangle_{jk}$ given by Eq. (20).

The dimensionless minimum frequency separation $\Delta\xi_{\min} = 11.2$ for $T_{\text{FWHM}} = 10$ ps. The multi-channel collisions are exemplified in Table 1, where, for a 9-channel filtered WDM soliton system, we report the total timing jitter experienced by solitons in a given channel as a result of collisions with the solitons in all the other channels, as obtained by using Eqs. (7)−(10) and Eqs. (14)−(21) numerically. As shown, optimal dispersion map can greatly reduce the total timing jitter caused by inter-channel nonlinear interactions compared with constant dispersion.

Table 1

	Channel λ (nm)	Without DM Δt_{rms} (ps)	With DM Δt_{rms} (ps)
1	1544.96	8.97	0.0740
2	1546.22	8.55	0.0737
3	1547.48	8.52	0.0724
4	1548.74	8.25	0.0710
5	1550.00	8.15	0.0699
6	1551.26	8.25	0.0710
7	1552.52	8.52	0.0724
8	1553.78	8.55	0.0737
9	1555.04	8.97	0.0740

The total timing jitter experienced by solitons in a given channel as a result of collisions with the solitons in all the other channels in a filtered WDM soliton system. "Without DM" refers to constant dispersion with $D_{\text{av}} = 1$ ps/(km·nm); "With DM" refers to an optimal dispersion management with $L_1 = 21$ km, $\Delta\sigma = 25$, $D_{\text{av}} = 0.1$ ps/(km·nm).

In conclusion, on the base of the coupled ordinary differential equations for a two-channel WDM system, which are obtained through variational procedure, the optimal dispersion managements for reducing collision-induced timing jitters are derived when the randomicity of collision positions and the arbitrary encoding of data in channels are considered simultaneously in a filtered WDM soliton system. The multi-channel collision-induced timing jitters in a filtered WDM soliton system are given with an optimal dispersion management and constant dispersion as a comparison.

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