Lasing with and without inversion due to spontaneously generated coherence in an open V-type system

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Received June 22, 2004

It is shown that in an open V-type atomic system without incoherent pumping, the spontaneously generated coherence (SGC) can not only lead to lasing with inversion but also lead to lasing without inversion (LWI) including at resonance of the driving and probe fields. This conclusion is much different from that obtained in the corresponding closed V-type system.

OCIS codes: 270.1670, 270.3430.

Quantum coherence and interference in atomic systems have led to a number of important consequences, such as lasing without inversion (LWI)[1], coherent population trapping (CPT)[2], potentiality for sensitive measurements of magnetic fields[3], electromagnetically induced transparency (EIT)[4], and so on. There are many ways to generate atomic coherence and quantum interference. Generally, they are produced by a coherent driving field or by initial coherence. The spontaneous emission may give rise to a coherent superposition. This kind of coherence can be created by interference of spontaneous emission (usually called as spontaneously generated coherence (SGC)) of either of two close lying atomic levels to a common atomic level (V-type atom)[5,6] or by a single excited level to two close lying atomic levels (Λ-type atom)[7]. In a ladder type system, it can also be created in the nearly spaced atomic levels case[8,9]. The effects of SGC on absorption, dispersion, population inversion and so on have been extensively investigated recently. For example, Menon and Agarwal[10,11] have studied effects of SGC on the pump-probe response of a Λ-type atomic system and found that SGC can result in new gain features instead of the usual absorption features and a large refractive index with low absorption. Paspalakis *et al.*[12] have shown that SGC can lead to large modification of the dispersion and absorption properties in a V-type atomic system. Paspalakis *et al.*[13] have also studied the consequences of SGC in the propagation dynamics of a short laser pulse in a V-type medium and found that under specific conditions, the (otherwise opaque) medium becomes transparent to the laser pulse. Gong *et al.*[7,14] have shown that in the Λ- and V-type atomic systems the unexpected population inversion can be achieved due to SGC. Xu *et al.*[15] have investigated the effects of the coherence on the transient process in the Λ- and V-type systems, respectively, and found that the transient gain (absorption) properties can be dramatically affected by such coherence. Liu *et al.*[16] have shown that in a Λ-type system, in the case of small dephasing, instead of EIT at resonance, electromagnetically induced absorption (EIA) can occur due to this coherence. For a four-level double-V laser without inversion, Hu and Peng[17] have revealed that SGC can lead to intensity noise squeezing. However, all these studies are for the closed atomic systems. In this paper, we analyze the effect of SGC on the gain and populations in an open V-type atomic system without incoherent pumping.

The system considered here is shown in Fig. 1. The excited state $|2\rangle$ is coupled to the ground state $|1\rangle$ through a strong driving field with frequency ω_a and Rabi frequency $\Omega = \vec{\mu}_{12} \cdot \vec{\varepsilon}_a/\hbar$. The excited state $|3\rangle$ is coupled to the ground state$|1\rangle$ through a weak probe field with frequency ω_b and Rabi frequency $E = |\mu_{13} \cdot |\varepsilon_b/\hbar$. Both Rabi frequencies are considered as real in this paper. The two closely spaced upper levels $|2\rangle$ and $|3\rangle$ decay spontaneously to the ground state $|1\rangle$ with decay rates $2\gamma_2$ and $2\gamma_3$, respectively. The atomic injection rates for levels $|1\rangle$ and $|2\rangle$ are J_1 and J_2, respectively. The atomic exit rate from the cavity is r_0. An incoherent pump with a pumping rate $2R$ is applied between levels $|1\rangle$ and $|3\rangle$. In the rotating wave frame the density matrix equations of motion for the system can be written as

$$\dot{\rho}_{11} = 2\gamma_2\rho_{22} + 2\gamma_3\rho_{33} - r_0\rho_{11} - i\Omega(\rho_{12} - \rho_{21})$$
$$-iE(\rho_{13} - \rho_{31}) + 2R(\rho_{33} - \rho_{11})$$
$$+2p\sqrt{\gamma_2\gamma_3}(\rho_{23} + \rho_{32}) + J_1, \qquad (1)$$

$$\dot{\rho}_{22} = -(2\gamma_2 + r_0)\rho_{22} - p\sqrt{\gamma_2\gamma_3}(\rho_{23} + \rho_{32})$$
$$+i\Omega(\rho_{12} - \rho_{21}) + J_2, \qquad (2)$$

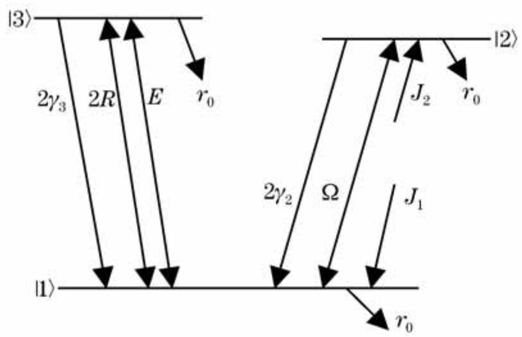

Fig. 1. An open V-type atomic system.

$$\dot{\rho}_{33} = 2R\rho_{11} - (2\gamma_3 + 2R + r_0)\rho_{33}$$

$$-p\sqrt{\gamma_2\gamma_3}(\rho_{23} + \rho_{32}) + iE(\rho_{13} - \rho_{31}), \quad (3)$$

$$\dot{\rho}_{23} = -[\gamma_{23} + i(\Delta_3 - \Delta_2)]\rho_{23}$$

$$-p\sqrt{\gamma_2\gamma_3}(\rho_{22} + \rho_{33}) + i\Omega\rho_{13} - iE\rho_{21}, \quad (4)$$

$$\dot{\rho}_{21} = -(\gamma_{21} - i\Delta_2)\rho_{21} - p\sqrt{\gamma_2\gamma_3}\rho_{31}$$

$$+i\Omega(\rho_{11} - \rho_{22}) - iE\rho_{23}, \quad (5)$$

$$\dot{\rho}_{31} = -(\gamma_{31} - i\Delta_3)\rho_{31} - p\sqrt{\gamma_2\gamma_3}\rho_{21}$$

$$+iE(\rho_{11} - \rho_{33}) - i\Omega\rho_{31}, \quad (6)$$

constrained by $\rho^*_{mn} = \rho_{nm}$. In the following discussion, we always make $J_1 + J_2 = r_0$ for keeping the total number of the atoms as a constant. γ_{ij} stands for the coherence decay rate on the corresponding transition $|i\rangle \to |j\rangle$: $\gamma_{23} = \gamma_2 + \gamma_3 + R$, $\gamma_{21} = \gamma_2 + R$, $\gamma_{31} = \gamma_3 + R$. $\Delta_3(=\omega_{31} - \omega_b)$ and $\Delta_2(=\omega_{21} - \omega_a)$ are the detunings of the probe and driving fields from their relevant atomic transitions, respectively. $p\sqrt{\gamma_2\gamma_3}$ represents the effect of spontaneous emission of one photon from the transition $|2\rangle \to |1\rangle$ and absorption of the same photon in the transition $|1\rangle \to |3\rangle$ or *vice versa*. The physical explanation is that this term is the result of quantum interference of spontaneous emission from the two close upper levels. The parameter p denotes the alignment of the two dipole matrix elements ($\vec{\mu}_{21}$ and $\vec{\mu}_{13}$) and is defined as $p \equiv \vec{\mu}_{21} \cdot \vec{\mu}_{13}/(|\vec{\mu}_{21}|\,|\vec{\mu}_{13}|) = \cos\theta$. The parameter p plays a very important role in the creation of coherence. If we consider linearly polarized electric fields with the restriction that $\vec{\mu}_{12} \cdot \vec{\varepsilon}_b = 0$ and $\vec{\mu}_{13} \cdot \vec{\varepsilon}_a = 0$, then the Rabi frequencies are connected to the p parameter by the relation $\Omega = \Omega_0\sqrt{1-p^2} = \Omega_0 \sin\theta$ and $E = E_0\sqrt{1-p^2} = E_0 \sin\theta$, with $\Omega_0 = |\vec{\mu}_{12}|\,|\vec{\varepsilon}_a|/\hbar$ and $E_0 = |\vec{\mu}_{13}|\,|\vec{\varepsilon}_b|/\hbar$[10]. If $\vec{\mu}_{12}$ and $\vec{\mu}_{13}$ are orthogonal, then p=0 and Eqs. (1)−(6) reduce to the equations for an open V-type three level atomic system without SGC[18−22]; when $J_1 = J_2 = \gamma_0$=0, Eqs. (1)−(6) reduce to the equations for a closed V-type three level atomic system with SGC[12].

For the steady state case $\mathrm{d}\rho_{nm}/\mathrm{d}t = 0$, Eqs. (1)—(6) reduce to a set of coupled algebraic equations. After splitting them into real and imaginary parts, we obtain a system of 9 algebraic equations. These equations can be treated in all orders using the symbolic computation package *Mathemetica* or *Maple*. The polarization ρ_{31} can be expanded in powers of the probe field Rabi frequency E and approximated as[23]

$$\rho_{31} = A(\Omega, \gamma_2, \gamma_3, p, R, \Delta_2, \Delta_3, J_1, J_2, \gamma_0)$$

$$+B(\Omega, \gamma_2, \gamma_3, p, R, \Delta_2, \Delta_3, J_1, J_2, \gamma_0)E, \quad (7)$$

where A is the lowest order nonlinear susceptibility. The dispersion and absorption (gain) of the medium correspond to the real and imaginary parts of $\sigma_{31} \equiv \rho_{31} - A$, respectively. If $\mathrm{Im}(\sigma_{31}) < 0$, the system exhibits gain for the probe field; if $\mathrm{Im}(\sigma_{31}) > 0$, the probe laser field is attenuated. When $\mathrm{Im}(\sigma_{31})$ and the population difference, $\rho_{33} - \rho_{11}$, between levels coupled by the probe field are simultaneously negative, LWI can be realized; if $\mathrm{Im}(\sigma_{31}) < 0$ and $\rho_{33} - \rho_{11} > 0$, the lasing with inversion occurs. The parameter p designates the effect of SGC. The gain (absorption) of the probe field and population difference will have different behaviors for different values of p.

Our numerical calculation result of the steady state shows that the effect of SGC on the gain and populations in the open system has some obvious difference from that in the corresponding closed system. In the following, we will discuss in detail the case.

First we study the case without the incoherent pumping ($R = 0$).

Figure 2 illustrates gain (absorption) $\mathrm{Im}(\sigma_{31})/E$ and the population difference $\rho_{33} - \rho_{11}$ as functions of the probe detuning Δ_3 for different values of p in the open and closed V-type atomic systems. The parameter values used in Fig. 2 are $\gamma_3 = 1$, $\gamma_2 = 6$, $\Delta_2 = 0$, $\Omega_0 = 20$, $E_0 = 1$ and $J_1 = J_2 = r_0 = 0$ for the closed V-type system, $J_1 = 0.1$, $J_2 = 0.05$, $r_0 = 0.15$ for the open V-type system. It should be noted that in Fig. 2 the values of $\rho_{33} - \rho_{11}$ were divided by 10.

When $p = 0$ (i.e., SGC is absent), Fig. 2(a) shows that $\rho_{33} - \rho_{11} < 0$ and $\mathrm{Im}(\sigma_{31})/E > 0$ for any value of the probe detuning Δ_3 in both the open and closed systems, so both LWI and lasing with inversion cannot take place.

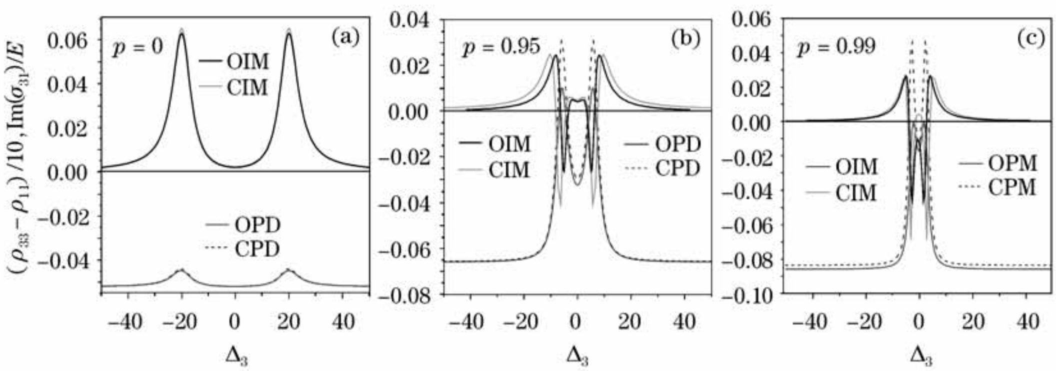

Fig. 2. $\mathrm{Im}(\sigma_{31})/E$ and $\rho_{33} - \rho_{11}$ as functions of Δ_3 for different values of p in the open and closed V-type atomic systems without incoherent pumping. The parameter values are $\gamma_3 = 1$, $\gamma_2 = 6$, $\Delta_2 = 0$, $\Omega_0 = 20$, $E_0 = 1$, $R = 0$, and $J_1 = J_2 = r_0 = 0$ for the closed system, $J_1 = 0.1$, $J_2 = 0.05$, $r_0 = 0.15$ for the open system. In the figure, OIM (CIM) and OPD (CPD) denote $\mathrm{Im}(\sigma_{31})/E$ and population difference $\rho_{33} - \rho_{11}$ for the open (closed) system, respectively.

The values of $\mathrm{Im}(\sigma_{31})/E$ and $\rho_{33} - \rho_{11}$ in the open system are nearly same as those in the closed system, except that the maximum values of $\mathrm{Im}(\sigma_{31})/E$ and $\rho_{33} - \rho_{11}$ in the open system are little smaller than the corresponding values in the closed system.

When $p \neq 0$ (i.e., SGC is present), with p increasing, different phenomena appear.

When $p = 0.95$ (Fig. 2(b)), for both the open and closed systems, lasing with inversion appears in two symmetric regions about Δ_3=0; but at Δ_3=0 and in the region near Δ_3=0, lasing with inversion does not exist. The maximum values of the gain of lasing with inversion and the unexpected population inversion in the open system are obviously smaller than those in the closed system.

When $p = 0.99$ (Fig. 2(c)), for the closed system, similar to the case for $p = 0.95$, only lasing with inversion can be realized and amplification of the probe field is still impossible at Δ_3=0 and in the region near Δ_3=0, however, the maximum values of the lasing gain and unexpected population increase obviously against as those when $p = 0.95$; for the open system, the case is much different, the open system can exhibit LWI, moreover, LWI also can occur at Δ_3=0 and in the region near Δ_3=0.

In addition, our numerical calculation result shows that when the incoherent pumping is present ($R \neq 0$), similar to the case in the closed V-type system[12], SGC can result in LWI in the open V-type system, and the LWI gain in the open system is always smaller than that in the corresponding closed system as shown in Fig. 3, where the parameter values are same as those used in Fig. 2 but $\Delta_3 = 0$ and $R = 2$.

The quantum coherence and interference (QCI) in atomic system lead to LWI. Comparing with the closed system, the present of the injection and exit rates in the open system will obviously affect QCI, thereby will change the behaviors of LWI of the system, so that the result obtained from the open V-type system in this paper has some remarkable difference from that of the corresponding closed V-type system studied by Paspalakis *et al.*.

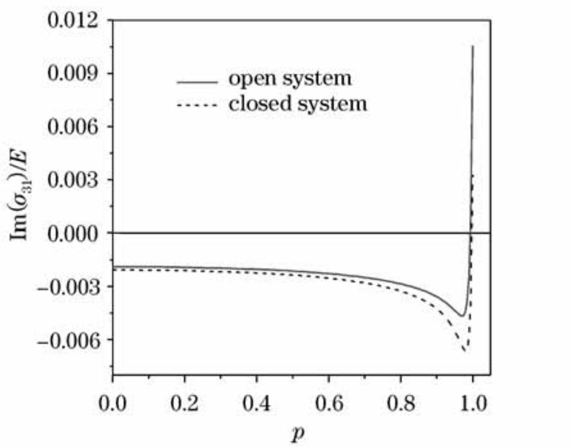

Fig. 3. $\mathrm{Im}(\sigma_{31})/E$ as a function of p in the open and closed systems with an incoherent pumping. The parameters values are same as those used in Fig. 2 but $\Delta_3 = 0$ and $R = 2$.

In conclusion, we have investigated the effect of SGC on gain (absorption) and populations in an open V-type atomic system without incoherent pumping and given a comparison with the corresponding closed system. We found that SGC can lead to lasing with inversion and also to LWI including at resonance of the driving and probe fields in the open system. And this conclusion is much different from that obtained in the corresponding closed system, in which only the lasing with inversion can be realized, moreover any lasing cannot occur at resonance of the driving and probe fields.

This work was supported by the Key Laboratory for High Intensity Optics, Shanghai Institute of Optics and Fine Mechanics, and by the National Natural Science Foundation of China under Grant No. 69978011. X. Fan is the author to whom the correspondence should be addressed, his e-mail address is fanxj108@beelink.com.

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