

Effects of phase fluctuation in an open four-level inversionless lasing system

Xijun Fan (樊锡君)¹, Mengzheng Zhu (朱孟正)¹, Zhengping Hong (洪正平)¹,
Shangqing Gong (龚尚庆)², and Zhizhan Xu (徐至展)²

¹Department of Physics, Shandong Normal University, Ji'nan 250014

²Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

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A steady analytical solution of an open four-level inversionless lasing system with a driving field having the phase fluctuation has been given, and the effects of the finite width due to the phase fluctuation on the gain, dispersion, and population difference have been analyzed by using the numerical simulation from the steady analytical solution. It is found that: with the linewidth increasing, the gain decreases and the absolute value of population difference between levels coupled by the probe field increases, but the variation of the linewidth cannot change the properties of the inversionless lasing and refractive index increase of the system; when the linewidth does not equal to zero, the system can still get a high refractive index with zero absorption, and these conclusions have very obvious difference from those obtained in other inversionless lasing systems.

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Quantum coherence and interference in atomic systems have a number of important consequences, including lasing without inversion (LWI), subrecoil cooling of atoms, potentiality for sensitive measurements of magnetic fields, electromagnetically induced transparency, and so on. In particular, LWI has attracted much more attention (for example, see Refs. [1–11]) due to its important science sense and potential wide application. For mathematical simplicity, in the study on LWI systems the phase of the driving field is usually assumed to be fixed, but the real phase is fluctuant. Recently, Zhu *et al.*^[12] have put forward a theoretical model of an open four-level LWI system and studied its gain and dispersion properties, however, the phase of the driving field is still assumed as fixed. In this paper, we investigate the effects of the phase fluctuation of the driving field on the gain, dispersion, and population difference from different respects.

Consider an open four-level system with the ground state $|1\rangle$ and excited states $|2\rangle$, $|3\rangle$, and $|4\rangle$ as illustrated in Fig. 1. The transition $|1\rangle \leftrightarrow |2\rangle$ of frequency ω_{21} is driven by a strong coherent field with frequency ω_l and Rabi frequency 2Ω . The transition $|1\rangle \leftrightarrow |3\rangle$ of frequency

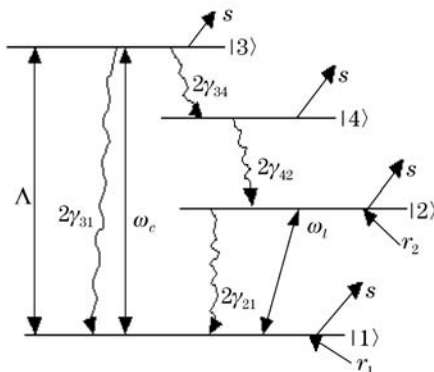


Fig. 1. An open four-level system.

ω_{31} is incoherently pumped with a rate Λ . A weak probe field with frequency ω_c and Rabi frequency $2g$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. $2\gamma_{ij}$ is the spontaneous decay rate from state $|i\rangle$ to state $|j\rangle$. The transitions $|2\rangle \leftrightarrow |3\rangle$ and $|4\rangle \leftrightarrow |1\rangle$ are forbidden. If the probe field is amplified, lasing can be established on the transition $|1\rangle \leftrightarrow |3\rangle$. In the rotating wave, slowly varying envelope and mean field approximations, the density matrix motion equations of the system can be written as^[12]

$$\dot{\rho}_{11} = -\Lambda\rho_{11} + (\Lambda + 2\gamma_{31})\rho_{33} + 2\gamma_{21}\rho_{22} + i(\Omega\rho_{21} - \Omega^*\rho_{12}) + ig(\rho_{31} - \rho_{13}) - s\rho_{11} + r_1, \quad (1a)$$

$$\dot{\rho}_{22} = -2\gamma_{21}\rho_{22} + 2\gamma_{42}\rho_{44} + i(\Omega^*\rho_{12} - \Omega\rho_{21}) - s\rho_{22} + r_2, \quad (1b)$$

$$\dot{\rho}_{33} = \Lambda\rho_{11} - (\Lambda + 2\gamma_{31} + 2\gamma_{34})\rho_{33} + ig(\rho_{13} - \rho_{31}) - s\rho_{33}, \quad (1c)$$

$$\dot{\rho}_{44} = 2\gamma_{34}\rho_{33} - 2\gamma_{42}\rho_{44} - s\rho_{44}, \quad (1d)$$

$$\dot{\rho}_{12} = i\Omega(\rho_{22} - \rho_{11}) - (\Lambda/2 + \gamma_{21} + i\Delta_1)\rho_{12} + ig\rho_{32}, \quad (1e)$$

$$\dot{\rho}_{13} = ig(\rho_{33} - \rho_{11}) - (\Lambda + \gamma_{31} + \gamma_{34} + i\Delta_2)\rho_{13} + i\Omega\rho_{32}, \quad (1f)$$

$$\dot{\rho}_{32} = -i\Omega\rho_{31} + ig\rho_{12} - (\Lambda/2 + \gamma_{31} + \gamma_{34} + \gamma_{21} - i(\Delta_2 - \Delta_1))\rho_{32}. \quad (1g)$$

In Eqs. (1), r_1 (r_2) is the atomic injection rate for level $|1\rangle$ ($|2\rangle$), and s is the atomic exit rate from the cavity. Δ_1 ($=\omega_{31} - \omega_l$) and Δ_2 ($=\omega_{31} - \omega_c$) express the detunings

of the driving and probe field from their relevant atomic transition, respectively. The gain coefficient of the probe field is proportional to $\text{Im}\rho_{13}$. If $\text{Im}\rho_{13} > 0$, the system exhibits gain for the probe field; if $\text{Im}\rho_{13} < 0$, the probe field is attenuated. If $\text{Im}\rho_{13} > 0$ and $\rho_{33} - \rho_{11} < 0$, then LWI can be realized in the system. The dispersion is determined by $\text{Re}\rho_{13}$, $\text{Re}\rho_{13} > 0$ corresponds to the red shift of the frequency of the probe field; $\text{Re}\rho_{13} < 0$ shows the blue shift^[13]. The refractive index of medium is proportional to $\text{Re}\rho_{13}$.

Let $\phi(t)$ represent the phase fluctuation of the driving field

$$\Omega = \Omega_0 \exp[i\phi(t)].$$

The phase is characterized by the following random equation of motion^[14],

$$\dot{\phi}(t) = u(t),$$

with zero average, i.e., $\langle u(t) \rangle = 0$. Here $u(t)$ is a δ -correlated Langevin-noise term, whose diffusion coefficient gives the linewidth $2R_L$ of the driving field^[15], i.e.,

$$\langle u(t)u(t') \rangle = 2R_L \delta(t - t').$$

In order to clarify the effect of the finite linewidth, we redefine the variables ρ_{12} and ρ_{32} as

$$\rho_{12} = \rho'_{12} \exp[i\phi(t)], \quad \rho_{32} = \rho'_{32} \exp[i\phi(t)].$$

For the new variables, we have

$$\begin{aligned} \dot{\rho}'_{12} = & i\Omega_0(\rho_{22} - \rho_{11}) - (\Lambda/2 + \gamma_{21} + i\Delta_1)\rho'_{12} \\ & - iu(t)\rho'_{12} + ig\rho'_{32}, \end{aligned} \quad (2a)$$

$$\begin{aligned} \dot{\rho}'_{32} = & -i\Omega_0\rho_{31} + ig\rho'_{12} - (\Lambda/2 + \gamma_{31} + \gamma_{34} + \gamma_{21} \\ & - i(\Delta_2 - \Delta_1))\rho'_{32} - iu(t)\rho'_{32}. \end{aligned} \quad (2b)$$

In this case, the density matrix equations (1a)–(1d), (1f), (2a), and (2b) should be averaged over the randomly fluctuating phase. Dropping both the primes of the quantities ρ'_{12} and ρ'_{32} , assuming a small probe field strength, from the above equations, we derive the semiclassical set of equations for the stochastic averaged values of the population and polarization, $\langle \rho_{ii} \rangle$ ($i = 1-4$) and $\langle \rho_{12} \rangle$, in the zeroth order of the probe field, for the stochastic averaged values of the polarizations, $\langle \rho_{13} \rangle$ and $\langle \rho_{23} \rangle$, in the first order of the probe field,

$$\begin{aligned} \langle \dot{\rho}_{11} \rangle = & -\Lambda\langle \rho_{11} \rangle + (\Lambda + 2\gamma_{31})\langle \rho_{33} \rangle + 2\gamma_{21}\langle \rho_{22} \rangle \\ & + i\bar{\Omega}(\langle \rho_{21} \rangle - \langle \rho_{12} \rangle) + i\bar{g}(\langle \rho_{31} \rangle - \langle \rho_{13} \rangle) - s\langle \rho_{11} \rangle + r_1, \end{aligned} \quad (3a)$$

$$\begin{aligned} \langle \dot{\rho}_{22} \rangle = & -2\gamma_{21}\langle \rho_{22} \rangle - 2\gamma_{42}\langle \rho_{44} \rangle \\ & + i\bar{\Omega}(\langle \rho_{12} \rangle - \langle \rho_{21} \rangle) - s\langle \rho_{22} \rangle + r_2, \end{aligned} \quad (3b)$$

$$\begin{aligned} \langle \dot{\rho}_{33} \rangle = & \Lambda\langle \rho_{11} \rangle - (\Lambda + 2\gamma_{31} + 2\gamma_{34})\langle \rho_{33} \rangle \\ & + i\bar{g}(\langle \rho_{13} \rangle - \langle \rho_{31} \rangle) - s\langle \rho_{33} \rangle, \end{aligned} \quad (3c)$$

$$\langle \dot{\rho}_{44} \rangle = 2\gamma_{34}\langle \rho_{33} \rangle - 2\gamma_{42}\langle \rho_{44} \rangle - s\langle \rho_{44} \rangle, \quad (3d)$$

$$\begin{aligned} \langle \dot{\rho}_{12} \rangle = & i\bar{\Omega}(\langle \rho_{22} \rangle - \langle \rho_{11} \rangle) \\ & - (\Lambda/2 + \gamma_{21} + R_L + i\Delta_1)\langle \rho_{12} \rangle, \end{aligned} \quad (3e)$$

$$\begin{aligned} \langle \dot{\rho}_{13} \rangle = & i\bar{g}(\langle \rho_{33} \rangle - \langle \rho_{11} \rangle) \\ & - (\Lambda + \gamma_{31} + \gamma_{34} + i\Delta_2)\langle \rho_{13} \rangle + i\bar{\Omega}\langle \rho_{32} \rangle, \end{aligned} \quad (3f)$$

$$\begin{aligned} \langle \dot{\rho}_{32} \rangle = & -(\Lambda/2 + \gamma_{31} + \gamma_{34} + \gamma_{21} + R_L \\ & - i(\Delta_2 - \Delta_1))\langle \rho_{32} \rangle - i\bar{\Omega}\langle \rho_{31} \rangle + i\bar{g}\langle \rho_{12} \rangle, \end{aligned} \quad (3g)$$

where $\bar{\Omega}$ and \bar{g} are the semiclassical steady-state values of Ω_0 and g , respectively. For the convenience of expression, $\bar{\Omega}$ and \bar{g} are still shown as Ω and g , respectively. For obtaining Eqs. (3) from Eqs. (1), we used the relation^[15]

$$i\langle u(t)\rho_{lj}(t) \rangle = R_L\langle \rho_{lj} \rangle,$$

where the value of lj is 12 or 32. Comparing Eqs. (1) and (3), we find that the phase fluctuation in the driving field modifies the off-diagonal damping rates $\Lambda/2 + r_{21}$ and $\Lambda/2 + r_{31} + r_{34} + r_{21}$ to $\Lambda/2 + r_{21} + R_L$ and $\Lambda/2 + r_{31} + r_{34} + r_{21} + R_L$, respectively. For the steady state, from Eqs. (3) we obtain the following general exact linear analytical solutions for the polarization $\langle \rho_{13} \rangle$ and population difference $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$

$$\text{Im}\langle \rho_{13} \rangle = gb_0/h_0, \quad (4a)$$

$$\text{Re}\langle \rho_{13} \rangle = gc_0/h_0, \quad (4b)$$

$$\langle \rho_{33} \rangle - \langle \rho_{11} \rangle = -d_0/h_0. \quad (4c)$$

The detail expressions of b_0 , c_0 , d_0 , and h_0 can be found in the appendix of Ref. [12], just k_2 and k_3 are replaced by $k_2 + R_L$ and $k_3 + R_L$, respectively.

Now we discuss the effect of the linewidth on the gain $\text{Im}\langle \rho_{13} \rangle$, dispersion $\text{Re}\langle \rho_{13} \rangle$, and population difference $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$ from different respects by using the numerical calculation results from Eqs. 4. In the calculation we make always $r_1 + r_2 = s$ to keep $\rho_{33} + \rho_{22} + \rho_{11} = 1$.

Figure 2 illustrates $\text{Im}\langle \rho_{13} \rangle/g$, $\text{Re}\langle \rho_{13} \rangle/g$, and $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$ versus R_L , where the parameters values are $\Lambda = 8$, $\Omega = 17$, $\gamma_{21} = 6$, $\gamma_{34} = 0.25$, $\gamma_{31} = 1$, $\gamma_{42} = 5$, $s = 0.1$, $r_1 = 0.08$, $r_2 = 0.02$, $\Delta_1 = 1$, $\Delta_2 = 4$. We can see from Fig. 2 that: 1) With the linewidth R_L increasing, $\text{Im}\langle \rho_{13} \rangle$, $\text{Re}\langle \rho_{13} \rangle$, and $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$ decrease ($|\langle \rho_{33} \rangle - \langle \rho_{11} \rangle|$ increases) monotonously. 2) $\text{Im}\langle \rho_{13} \rangle$ changes from positive to negative, this means that the probe field changes from gain to absorption. 3) $\text{Re}\langle \rho_{13} \rangle$ changes from positive to negative, and this represents that the frequency of the probe field changes from blue shift to red shift. 4) $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$ is always negative, i.e., population inversion cannot arise, and this means that variation of the linewidth cannot change the properties of the inversionless lasing of the system. When choosing different parameters values from those in Fig. 2, the variation rules of $\text{Im}\langle \rho_{13} \rangle$, $\text{Re}\langle \rho_{13} \rangle$, and $\langle \rho_{33} \rangle - \langle \rho_{11} \rangle$ with R_L increasing may be different, as shown in Fig. 3. In Fig. 3, the following parameters values are used: $\Lambda = 6$, $\Omega = 12$, $\gamma_{21} = 6$, $\gamma_{34} = 0.25$, $\gamma_{31} = 1$, $\gamma_{42} = 5$, $s = 0.2$, $r_1 = 0.15$,

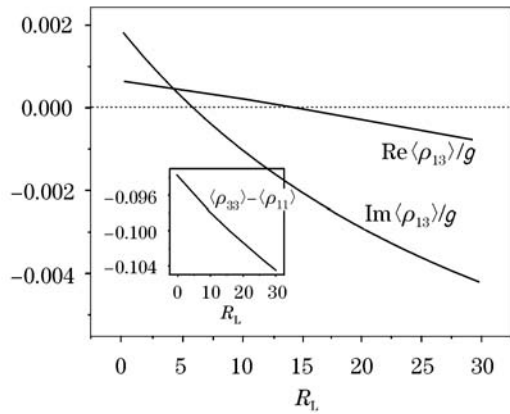


Fig. 2. Plots of $\text{Im}\langle\rho_{13}\rangle/g$, $\text{Re}\langle\rho_{13}\rangle/g$, and $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ versus R_L with $\Lambda = 8$, $\Omega = 17$, $\gamma_{21} = 6$, $\gamma_{34} = 0.25$, $\gamma_{31} = 1$, $\gamma_{42} = 5$, $s = 0.1$, $r_1 = 0.08$, $r_2 = 0.02$, $\Delta_1 = 1$, and $\Delta_2 = 4$.

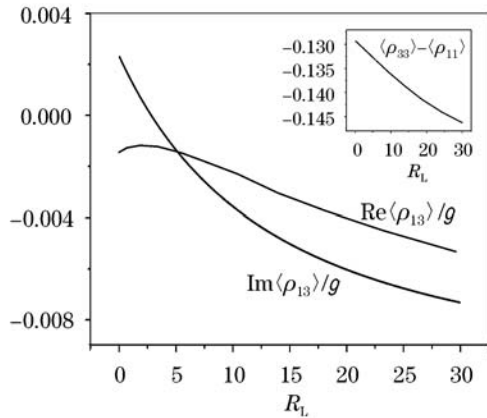


Fig. 3. Plots of $\text{Im}\langle\rho_{13}\rangle/g$, $\text{Re}\langle\rho_{13}\rangle/g$, and $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ versus R_L with $\Lambda = 6$, $\Omega = 12$, $\gamma_{21} = 6$, $\gamma_{34} = 0.25$, $\gamma_{31} = 1$, $\gamma_{42} = 5$, $s = 0.2$, $r_1 = 0.15$, $r_2 = 0.05$, $\Delta_1 = 5$, and $\Delta_2 = 8$.

$r_2 = 0.05$, $\Delta_1 = 5$, and $\Delta_2 = 8$. However, we can still obtain such conclusion that $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ decreases due to linewidth increasing, $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ is always negative and variation of the linewidth cannot change the properties of the inversionless lasing of the system, and this conclusion is much different from that obtained from a closed four level inversionless lasing system in which with R_L increasing, the lasing of the probe field changes from inversion to inversionless^[15].

Using the same parameters values as those in Fig. 2 except for $\Delta_1 = 0$, Fig. 4 illustrates curves of $\text{Im}\langle\rho_{13}\rangle/g$, $\text{Re}\langle\rho_{13}\rangle/g$, and $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ versus the probe field detuning Δ_2 when the driving field is resonant. Figure 4 shows that: 1) With R_L increasing, the gain and population difference decrease, the population difference $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ is always negative, and these are the same as the conclusions obtained from Figs. 2 and 3. 2) The variation of the driving field detuning Δ_2 has no effect on the population difference. 3) When $R_L \neq 0$, the system can still get a high dispersion (refractive index) without absorption. This conclusion is much different from the one given by Gong *et al.*^[16] for a closed V-type three level system, that it, due to the finite linewidth produced by the phase diffusion of the driving field, the system cannot generate a large refractive index along with zero absorption.

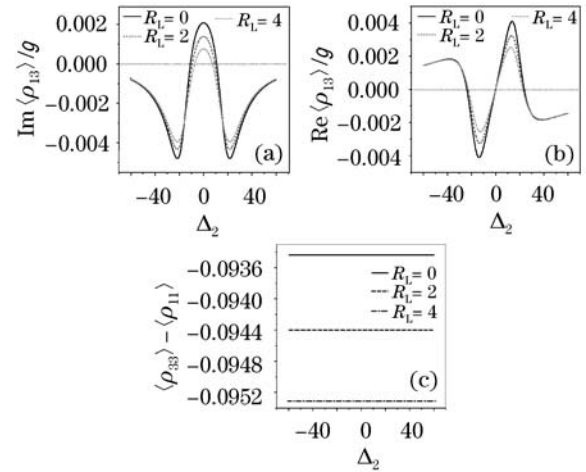


Fig. 4. Plots of $\text{Im}\langle\rho_{13}\rangle/g$, $\text{Re}\langle\rho_{13}\rangle/g$, and $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ versus Δ_2 with the same parameters values as those in Fig. 2 except for $\Delta_1 = 0$.

The quantum coherence and interference (QCC) in atomic system lead to LWI, and the dispersion is also closely relative to QCC. Comparing with the closed system, the presence of the injection and exit rates in the open system will obviously affect QCC, thereby will change the behaviors of LWI and dispersion of the system, so that the result obtained in this paper has some remarkable difference from that of the closed four-level and V-type system.

In conclusion, we have given an exact steady linear analytical solution of the open four-level LWI system with the driving field having the phase fluctuation, and analyzed the effects of the finite linewidth due to the phase fluctuation of driving field on the gain, dispersion, and population difference from different respects by using the numerical results from the steady analytical solution. We found that: 1) The phase fluctuation of the driving field results in a finite linewidth R_L . With R_L increasing, the gain $\text{Im}\langle\rho_{12}\rangle$ and population difference $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ decrease ($|\langle\rho_{33}\rangle - \langle\rho_{11}\rangle|$ increases), $\langle\rho_{33}\rangle - \langle\rho_{11}\rangle$ is always negative. 2) Variation of the linewidth R_L cannot change the property of the inversionless lasing of the system and this is much different from that obtained in a closed four level LWI system^[17]. 3) When $R_L \neq 0$, the system can still get a high refractive index without absorption, and this is obviously different from that obtained from a closed V-type three-level LWI system^[16].

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