The effect of conductor loss on half-wave voltage and modulation bandwidth of electro-optic modulators

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In this paper, we theoretically deduce the expressions of half-wave voltage and 3-dB modulation bandwidth in which conductor loss is taken into account. The results suggest that it will affect the theoretical values of half-wave voltage and bandwidth as well as the optimized electrode's dimension whether considering the conductor loss or not. As an example, we present a Mach-Zehnder (MZ) type polymer waveguide amplitude modulator. The half-wave voltage increases by 1 V and the 3-dB bandwidth decreases by 30% when the conductor loss is taken into account. Besides, the effects of impedance mismatching and velocity mismatching between microwave and light wave on the half-wave voltage, and 3-dB bandwidth are discussed.

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The potential applications for electro-optic (EO) modulators are numerous, including electrical-to-optical signal transduction for cable television (CATV), high speed switching at nodes in optical networks, pixel addressing in flat panel displays, voltage sensing for biomedical and electric-power applications, back-panel interconnection in high-speed computers, high-speed signal processing (e.g., greater than 50-Gb/s analogue-to-digital conversion) optically-detected radar, and so on. Indeed, EO modulators may well be critically important components of information processing devices, displays, and sensors in future information superhighways[1].

In the course of design of modulators, the traveling electrodes should satisfy two requirements[2]: one is how to utilize the microwave driving power efficiently, which means the microwave loss (mainly resulting from conductor loss) and half-wave voltage must be as little as possible; the other is how to get a higher modulation bandwidth as possible. The limitation of modulation bandwidth is caused by two factors: the mismatching between the microwave and the light wave and the conductor loss caused by the skin effect of microwave. It suggests that the conductor loss has an effect on half-wave voltage and modulation bandwidth of modulators. However, to our knowledge, this effect has seldom been taken into account in literatures[3]. The aim of this study is to quantitatively analyze this effect. Our results show that conductor loss will play an important role when the half-voltage and bandwidth are theoretically designed. So the results including the conductor loss will be more accurate when designing the traveling electrodes.

Firstly, we introduce two transmission coefficients of microwave voltage, T_Z and T_L, caused by impedance mismatching and conductor loss, respectively[2], and

$$T_Z = \frac{2Z_0}{Z_0 + 50}, \tag{1}$$

$$T_L = \frac{1 - \mathrm{e}^{-\alpha L}}{\alpha L}, \tag{2}$$

where

$$\alpha = \alpha_0 \sqrt{f} \tag{3}$$

is the conductor loss in dB/cm, α_0 is the conductor loss (dB·cm^{-1}·GHz$^{-1/2}$) at 1 GHz, which depends on the dimensions of electrodes and kind of dielectric, f is the operating frequency, Z_0 is characteristic impedance of electrodes, and L is the electrode length. From Eq. (1) the factor T_Z reaches its maximum of 1 when impedance matching is satisfied. Equation (2) shows the attenuation of modulation potential along the channel of waveguides due to conductor loss. Now we can define the effective electric fields E_{eff}[2] as

$$E_{\text{eff}} = T_Z T_L \frac{V}{D}, \tag{4}$$

where D is the electrode space and V is the modulation potential. For travelling electrode, V can be written as

$$V(z,t) = V_{\text{m}} \mathrm{e}^{j(\omega_{\text{m}} t - \beta_{\text{m}} z)}, \tag{5}$$

where V_{m} is the amplitude of modulation potential, $\beta_{\text{m}} = \omega_{\text{m}} \frac{n_{\text{m}}}{c}$ is the propagation constant of microwave and n_{m} is the effective refraction index of it, c is velocity of light in free space. We assume that photons enter into the modulation region at $t = t_0$ and $z = 0$. Then we can get

$$t - t_0 = \frac{N_{\text{eff}} z}{c} \tag{6}$$

at the arbitrary position z and time t, where N_{eff} is the effective refractive index of light wave in the channel. Substituting Eq. (6) into Eq. (5), we can get

$$V(z,t) = V_{\text{m}} \mathrm{e}^{j\omega_{\text{m}} t_0} \mathrm{e}^{j \frac{\omega_{\text{m}}}{c} (N_{\text{eff}} - n_{\text{m}}) z}. \tag{7}$$

The term of $(N_{\text{eff}} - n_{\text{m}})$ on right of Eq. (7) suggests the effect of velocity mismatching between microwave and light wave.

After modulation fields were applied, the change of N_{eff} caused by EO effect is given by

$$\Delta N_{\text{eff}} = -\frac{1}{2} \Gamma n^3 \gamma E_{\text{eff}}, \tag{8}$$

where γ is EO coefficient of materials, Γ is the modal overlap integral. Accordingly, the total phase-shift is

$$\Delta\Phi = \int_0^L \mathrm{d}\Phi = \int_0^L \Delta N_{\mathrm{eff}} k_0 \mathrm{d}z, \tag{9}$$

where k_0 is the wave number in vacuum. After integrating Eq. (9) and taking its absolute value, the result is

$$\Delta\Phi = \Delta\Phi_0 B \tag{10}$$

and

$$\Delta\Phi_0 = \pi \frac{V_{\mathrm{m}}}{\lambda_0 D / \Gamma n^3 \gamma L}, \tag{11}$$

where $\Delta\Phi_0$ is the ideal phase-shift when the conductor loss, impedance mismatching, and velocity mismatching are ignored. We can get the ideal half-wave voltage $V_{\pi(0)}$ when the ideal shift $\Delta\Phi_0$ is π,

$$V_{\pi(0)} = \frac{\lambda_0 D}{\Gamma n^3 \gamma L}. \tag{12}$$

In Eq. (10), B is a reduction factor whose value is less than 1 and can be written as

$$B = \left| T_Z T_L \frac{\sin\left[\frac{\pi}{c}\left(N_{\mathrm{eff}} - n_{\mathrm{m}}\right) f L\right]}{\frac{\pi}{c}\left(N_{\mathrm{eff}} - n_{\mathrm{m}}\right) f L} \right|. \tag{13}$$

It is clear that the conductor loss, impedance mismatching, and velocity mismatching are included in the factor B. From Eq. (10), the real half-wave voltage V_π is

$$V_\pi = \frac{V_\pi^0}{B}. \tag{14}$$

It is shown that the real half-wave voltage is larger than the ideal one due to the effect of conductor loss, impedance mismatching, and velocity mismatching.

For optical phase modulators, the 3-dB bandwidth can be defined as the microwave frequency when modulation index $\Delta\Phi$ drops to 1/2 of its maximum. Similarly, the microwave frequency when modulation index $\Delta\Phi$ drops to $1/\sqrt{2}$ of its maximum is the 3-dB optical intensity modulation bandwidth. Then, using Eqs. (10)−(13), the formulas of 3-dB bandwidth of optical intensity modulation and phase modulation f_{TW} can be written as

$$\frac{1 - \mathrm{e}^{-\alpha_0 \sqrt{L}\sqrt{f_{TW} L}}}{\alpha_0 \sqrt{L}\sqrt{f_{TW} L}} \left| \frac{\sin\left(\frac{\pi}{c}\left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right| f_{TW} L\right)}{\frac{\pi}{c}\left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right| f_{TW} L} \right| = \frac{\sqrt{2}}{2}, \tag{15a}$$

and

$$\frac{1 - \mathrm{e}^{-\alpha_0 \sqrt{L}\sqrt{f_{TW} L}}}{\alpha_0 \sqrt{L}\sqrt{f_{TW} L}} \left| \frac{\sin\left(\frac{\pi}{c}\left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right| f_{TW} L\right)}{\frac{\pi}{c}\left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right| f_{TW} L} \right| = \frac{1}{2}, \tag{15b}$$

respectively. The first and second terms on left of Eqs. (15a) and (15b) represent the effect of conductor loss and velocity mismatching on modulation bandwidth, respectively.

If we ignore the effect of conductor loss, Eqs. (15a) and (15b) are simplified as

$$f_{TW} L \approx \frac{1.4c}{\pi \left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right|}, \tag{16a}$$

and

$$f_{TW} L \approx \frac{2c}{\pi \left|N_{\mathrm{eff}} - n_{\mathrm{m}}\right|}, \tag{16b}$$

respectively. Under the velocity matching condition, modulation bandwidth is only affected by conductor loss. Thus Eqs. (15a) and (15b) can be written as[4]

$$f_{TW} \approx \frac{41}{(\alpha_0 L)^2} (\mathrm{GHz}), \tag{17a}$$

and

$$f_{TW} \approx \frac{191}{(\alpha_0 L)^2} (\mathrm{GHz}). \tag{17b}$$

Equations (16a) and (16b) are conventional forms of modulation bandwidth[5−7]. Of course, both of them ignore the effect of conductor loss on the bandwidth and thus large errors are caused. For accurate calculations, Eq. (15a) or (15b) should be adopted.

As an example, an EO polymer waveguide amplitude modulator with a structure of Mach-Zehnder interferometer (MZI) was considered (see Fig. 1). Half-wave voltage and 3-dB bandwidth of this modulator were calculated using Eqs. (14) and (15). Using genetic algorithm[8] we optimized the structure of electrodes in order to maximize the 3-dB bandwidth and half-wave voltage. The results are listed in Tables 1 and 2, respectively. According to Eq. (14), optimal half-wave voltage V_π is 7.86 V

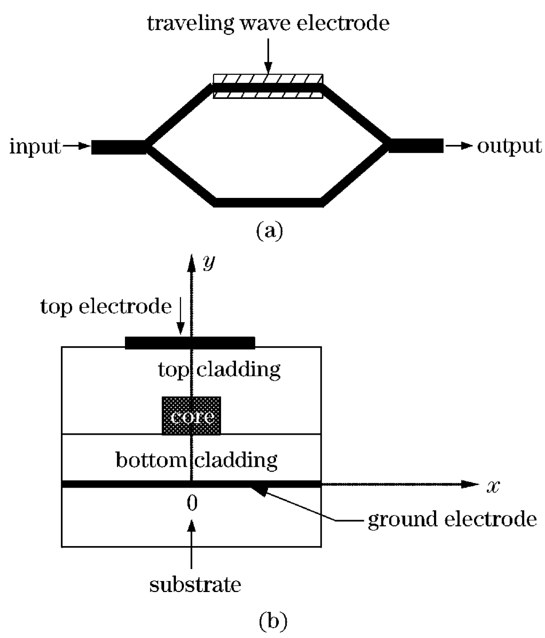

Fig. 1. EO waveguide polymer modulators of MZI type. (a): Top view; (b): cross sectional view.

Table 1. The Optimal Half-Wave Voltage

V_π (V)	6.58[a]	6.80[b]	7.61[c]	7.85[d]	7.86[e]
W (μm)	40.0	20.0	18.6	40.0	18.6
D (μm)	8.0	8.0	8.0	8.0	8.0
T (μm)	8.0	3.6	8.0	8.0	7.9
α_0 (dB·cm^{-1}·GHz$^{-1/2}$)	0.35	0.36	0.33	0.35	0.33
Z_0 (Ω)	31.1	50.0	50.0	31.1	50.0
Δn	0.134	0.166	0.166	0.134	0.166

[a]From Eq. (12). [b]Without considering the conductor loss. [c]Without considering the velocity mismatching. [d]Without considering the impedance mismatching. [e]From Eq. (14). W, D, and T are the electrode width, space, and thickness, respectively.

Table 2. Optimal 3-dB Bandwidth of Optical Intensity Modulation (IM) and Phase Modulation (PM). The Length of Electrode Is 1.0 cm

	f_{TW}	W	D	T	α_0	Z_0	Δn
IM	73.1 (94.8*)	40.0	9.3	8.0	0.30	34.7	0.141
PM	115.6 (142.5*)	40.0	8.0	8.0	0.35	31.1	0.134

*From Eqs. (16a) and (16b).

when the effects of conductor loss, impedance mismatching, and velocity mismatching are considered. However, the optimal V_π is 6.8 V when we do not take conductor loss into account in Eqs. (13) and (14), i.e. with the term T_L disappeared in factor B. Although the difference of effective refraction index between microwave and light wave, i.e. $\Delta n = |N_{\mathrm{eff}} - n_{\mathrm{m}}|$, which suggests the extent of velocity mismatching, and the characteristic impedance of electrodes are all identical for the two cases, the difference of V_π between them is 1 V. Also, the differences of dimensions of electrodes are large for the two cases, especially for electrode thickness. Thus it can be seen that the effect of conductor loss on the design of devices is very important. In addition, Table 1 gives the results without considering the impedance mismatching and velocity mismatching. Although the value of V_π is almost invariable when impedance mismatching was ignored, the designed electrode width is changed greatly. In this case, the extent of velocity mismatching dropped because Δn decreased from 0.166 to 0.134. That will increase the bandwidth designed. From the table, the design results are identical except for a small change of V_π whether velocity mismatching is considered or not.

From the above analysis, the most important factor that impacts on half-wave voltage is the conductor loss α_0. But what is the relation between α_0 and V_π? This is illuminated in Fig. 2. It is shown that V_π decreases as α_0 increases, thus an increasing conductor loss is required in order to reduce the half-wave voltage. Unfortunately, the increase of conductor loss will result in two disadvantages, one is the rise of propagation loss of microwave, the other is the deterioration of performance of modulation, especially for polymer modulators due to the disturbance of index of refraction caused by thermal effect. This requires an appropriate conductor loss and the value is 0.33 dB·cm^{-1}·GHz$^{-1/2}$ given by our calculations.

According to Eqs. (15a) and (15b), we also calculated the optimal 3-dB bandwidth and corresponding dimensions of electrodes. The results are shown in Table 2. The values in brackets are achieved using Eqs. (16a) and

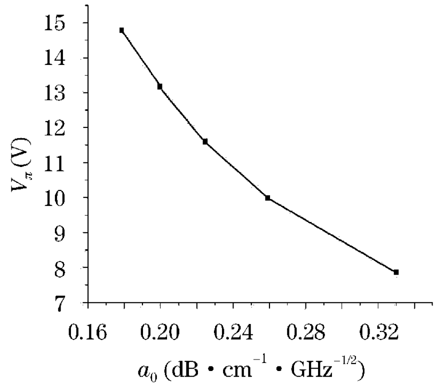

Fig. 2. Half-wave voltage (V_π) as a function of conductor loss (α_0).

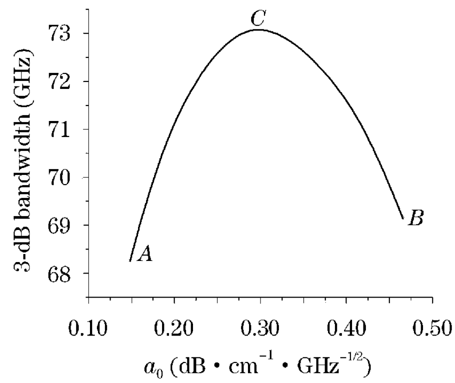

Fig. 3. The 3-dB bandwidth of an optical intensity modulator as a function of conductor loss (α_0).

(16b), respectively, that is to say, they were obtained when the conductor loss was ignored. For optical intensity modulation, the relative difference between the two bandwidths is 30%, and for optical phase modulation it is 23%. The conductor loss is also proved to be an important factor impacting on bandwidth besides velocity mismatching. Figure 3 shows the relation between the conductor loss and the bandwidth. At the two sides of the curve in this figure, the bandwidth is smaller than that at the middle part, i.e. the modulation bandwidth is not optimal when the conductor loss is small or large. This phenomenon can be explained as follows.

There are two factors that together determine the bandwidth. As conductor loss increases, the bandwidth decreases through Eqs. (17a) and (17b). But on the other hand, the extent of velocity mismatching (i.e. Δn) also drops (see Fig. 4), which reversely causes the bandwidth rising. At segment AC of the curve in Fig. 3, the latter is dominant and thus the bandwidth increases as the conductor loss increases. On the contrary, the former is dominant at segment CB and thus bandwidth decreases as the conductor loss increases. It is clear that the transition occurs at point C and then both of them are not dominant, accordingly the bandwidth reaches its maximum. We found that the bandwidth is maximal at the conductor loss of 0.30 dB·cm^{-1}·GHz$^{-1/2}$ by optimization calculation.

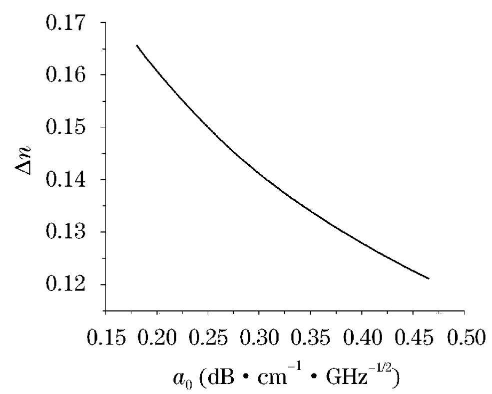

Fig. 4. The difference between the effective refractive indices of microwave and lightwave, Δn, as a function of conductor loss (α_0).

In conclusion, we theoretically derived the formulas of half-wave voltage and optical 3-dB bandwidth of EO waveguide modulators considering the conductor loss, impedance mismatching, and velocity mismatching. Numerical results show that the effect of conductor loss on design of modulators is very important. Increasing conductor loss can reduce the half-wave voltage. But it will also consume more powers of microwave and deteriorate the performance of modulation due to thermal effect. The effect of conductor loss on modulation bandwidth is relatively complex. The bandwidth firstly increases and then decreases with the increase of the conductor loss. Only at an appropriately large conductor loss, the bandwidth reaches its maximum. The optimal conductor loss can be found by optimization calculation.

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