Entangled squeezed states: Bell state measurement and teleportation

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We construct orthogonal Bell states with entangled squeezed vacuum states and show that these states can be discriminated with arbitrary precision when the amplitude of the squeezed states becomes sufficiently large. A scheme of teleporting a superposition state of the squeezed vacuum states based on the Bell state measurement is presented.

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Quantum entanglement is one of the most striking features of quantum mechanics. In the process of processing and transmitting quantum information, such as teleportation[1−7], cryptography[8], and quantum computation[9−10], entanglement lies in extremely important position.

There are different types of entanglement for light fields. For instance, in the teleportation experiments from the Innsbruck group[2], it is the polarization directions of single photons that are entangled. In the Caltech teleportation experiments[3], two em field modes are entangled with respect to photon numbers and the state used for teleportation is a two-mode squeezed state. Recently, van Enk *et al.* have used nonorthogonal entangled coherent states called quasi-Bell states as quantum channel to teleport one qubit encoded in Schrödinger cat states and have proved that the successful probability is 1/2[11]. The method has been generalized to teleport entangled coherent states[12] and superposition states of squeezed vacuum states[13], but for the latter, the successful probability is only 1/4.

In the standard process of teleportation, the sender Alice and the receiver Bob share the two members of an entangled pair. Alice is given an unknown quantum state $|\psi\rangle$ which she wants to teleport. She makes a Bell state measurement, a projection to an orthonormal base, on her Einstein-Podolsky-Rosen (EPR) member and the state, and sends the result through a classical information channel to Bob. After receiving the classical information, Bob carries out the appropriate unitary transformation, and obtains the state $|\psi\rangle$.

In above process of teleportation, an important problem is discrimination of the Bell states. It has been shown that Bell state measurement on a product Hilbert space of two two-level systems is not possibly completed by using linear elements[14].

Recently, Jeong *et al.*[15] have constructed an orthogonal Bell basis set from nonorthogonal coherent states. They have suggested an experimental setup (a beam splitter and two photodetectors) to discriminate these Bell states constructed from entangled coherent states and have shown that arbitrary precision can be achieved when the amplitude of the coherent states becomes sufficiently large. The results can be used to teleport a superposition state of coherent states.

The squeezed vacuum state is another important state. It is possible to construct an orthonormal basis by superposing both nonorthogonal and linearly independent squeezed vacuum states $|\zeta\rangle$ and $|-\zeta\rangle$. In this letter, we generalize Jeong's method[15] to entangled squeezed vacuum states. We show how to discriminate the Bell states constructed from entangled squeezed vacuum states and how to realize teleportation of a superposition state of squeezed vacuum states.

The squeezed vacuum state can be obtained by acting squeezing operator on a vacuum state. In its number-state representation, it takes the form

$$|\zeta\rangle = \hat{S}(\zeta)|0\rangle = \cosh^{-\frac{1}{2}} r \sum_{n=0}^{\infty} \frac{[(2n)!]^{\frac{1}{2}}}{n!} \times \left(-\frac{1}{2}\mathrm{e}^{i\theta}\tanh r\right)^{n} |2n\rangle, \tag{1}$$

where

$$\hat{S}(\zeta) = \exp\left(\frac{\zeta^*}{2}\hat{a}^2 - \frac{\zeta}{2}\hat{a}^{+2}\right), \tag{2}$$

here $\hat{a}$ is annihilation operator, $\zeta = r\mathrm{e}^{i\theta}$ is complex squeezed parameter.

Following Jeong[15], we construct an orthonormal basis by superposing both nonorthogonal and linearly independent squeezed vacuum states $|\zeta\rangle$ and $|-\zeta\rangle$,

$$|\Psi_+\rangle = \frac{1}{\sqrt{N_\varphi}}(\cos\varphi|\zeta\rangle - \sin\varphi|-\zeta\rangle), \tag{3}$$

$$|\Psi_-\rangle = \frac{1}{\sqrt{N_\varphi}}(-\sin\varphi|\zeta\rangle + \cos\varphi|-\zeta\rangle), \tag{4}$$

where $N_\varphi = \cos^2 2\varphi$ is a normalization factor and the parameter φ is defined as

$$\sin 2\varphi = \langle\zeta|-\zeta\rangle = \cosh^{-1/2} 2r. \tag{5}$$

For simplicity, we suppose that ζ is real. From Eqs. (3) and (4), four Bell states can be defined as

$$|B_{1,2}\rangle = \frac{1}{\sqrt{2}}(|\Psi_+\rangle|\Psi_+\rangle \pm |\Psi_-\rangle|\Psi_-\rangle), \tag{6}$$

$$|B_{3,4}\rangle = \frac{1}{\sqrt{2}}(|\Psi_+\rangle|\Psi_-\rangle \pm |\Psi_-\rangle|\Psi_+\rangle). \tag{7}$$

which may be represented by $|\zeta\rangle$ and $|-\zeta\rangle$ as

$$|B_1\rangle = \frac{1}{\sqrt{2N_\varphi}}\{|\zeta\rangle|\zeta\rangle + |-\zeta\rangle|-\zeta\rangle - \sin 2\varphi\,(|\zeta\rangle|-\zeta\rangle + |-\zeta\rangle|\zeta\rangle)\}, \quad (8)$$

$$|B_2\rangle = \frac{1}{\sqrt{2N_\varphi}}(|\zeta\rangle|\zeta\rangle - |-\zeta\rangle|-\zeta\rangle), \quad (9)$$

$$|B_3\rangle = \frac{1}{\sqrt{2N_\varphi}}\{|\zeta\rangle|-\zeta\rangle + |-\zeta\rangle|\zeta\rangle - \sin 2\varphi\,(|\zeta\rangle|\zeta\rangle + |-\zeta\rangle|-\zeta\rangle)\}, \quad (10)$$

$$|B_4\rangle = \frac{1}{\sqrt{2N_\varphi}}(|\zeta\rangle|-\zeta\rangle - |-\zeta\rangle|\zeta\rangle). \quad (11)$$

If the four different Bell states that can be discriminated, we can teleport a superposition state of two squeezed vacuum states by the normal method.

It is well known that a lossless 50/50 beam splitter can transform the coherent state $|\alpha\rangle_1|\beta\rangle_2$ as $\left|(\alpha+i\beta)/\sqrt{2}\right\rangle_1\left|(\beta+i\alpha)/\sqrt{2}\right\rangle_2$[16]. Let $\hat{a}_1$ and $\hat{a}_2$ denote the annihilation operators for the two light beams entering the two input ports of the beam splitter. Let $\hat{A}_1$ and $\hat{A}_2$ denote the annihilation operators for the light beams leaving the two output ports of the 50/50 beam-splitter. The boundary conditions at the surface of the beam splitter lead to the well-known input-output relation

$$\hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 + i\hat{a}_2), \quad \hat{A}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 + i\hat{a}_1). \quad (12)$$

We assume that the input light beams of the beam splitter are in squeezed vacuum states

$$|\Psi\rangle_{\rm in} = S_1(\zeta_1)S_2(\zeta_2)|0,0\rangle, \quad (13)$$

the operators $S_i(\zeta_i)$ are defined by Eq. (2). Cai *et al.*[13] proved that if the two input light beams have the same squeezing amplitudes and phases, i.e., $r_2 = r_1 = r$ and $\theta_2 = \theta_1 = \theta$, the output light field is in a two-mode squeezed vacuum state[17]

$$|\Psi\rangle_{\rm out} = \exp\left[-re^{i(\theta+\pi/2)}\hat{A}_1^+\hat{A}_2^+ + re^{-i(\theta+\pi/2)}\hat{A}_1\hat{A}_2\right]|0,0\rangle. \quad (14)$$

If the two input light beams have the same squeezing amplitudes but a phase difference of π, i.e., $r_2 = r_1 = r$ and $\theta_2 - \theta_1 = \pi$, the output light field is in a direct product of two single-mode squeezed vacuum states

$$|\Psi\rangle_{\rm out} = \exp\left[-\frac{1}{2}re^{i\theta}\hat{A}_1^{+2} + \frac{1}{2}re^{-i\theta}\hat{A}_1^2\right] \times \exp\left[-\frac{1}{2}re^{i(\theta+\pi)}\hat{A}_2^{+2} + \frac{1}{2}re^{-i(\theta+\pi)}\hat{A}_2^2\right]|0,0\rangle. \quad (15)$$

Obviously, in Eq. (14), the state is an entangled state, but in Eq. (15), the state is a non-entangled state.

Suppose each mode of the Bell state is incident on the beam splitter. After passing the beam splitter, the Bell states become (assume ζ is a real number)

$$|B_1\rangle \overset{\rm BS}{\to} |B_1\rangle' = \frac{1}{\sqrt{2N_\varphi}}\{[\exp(-ir\hat{A}_1^+\hat{A}_2^+ - ir\hat{A}_1\hat{A}_2) + \exp(ir\hat{A}_1^+\hat{A}_2^+ + ir\hat{A}_1\hat{A}_2)]|0,0\rangle - \sin 2\varphi\left[\exp\left(-\frac{r}{2}\hat{A}_1^{+2} + \frac{r}{2}\hat{A}_1^2\right)\exp\left(\frac{r}{2}\hat{A}_2^{+2} - \frac{r}{2}\hat{A}_2^2\right) + \exp\left(\frac{r}{2}\hat{A}_1^{+2} - \frac{r}{2}\hat{A}_1^2\right)\exp\left(-\frac{r}{2}A_2^{+2} + \frac{r}{2}A_2^2\right)\right]|0,0\rangle\}$$

$$= \frac{1}{\sqrt{2N_\varphi}\cosh r}\{\sum_{n=0}^{\infty}[(-1)^n + 1](i\tanh r)^n|n,n\rangle - \sin 2\varphi\sum_{n_1,n_2=0}^{\infty}\{\frac{\sqrt{(2n_1)!\,(2n_2)!}}{n_1!n_2!}\left(\frac{1}{2}\tanh r\right)^{n_1+n_2} \times [(-1)^{n_1} + (-1)^{n_2}]|2n_1,2n_2\rangle\}\}, \quad (16)$$

$$|B_2\rangle \overset{\rm BS}{\to} |B_2\rangle' = \frac{1}{\sqrt{2N_\varphi}}[\exp(-ir\hat{A}_1^+\hat{A}_2^+ - ir\hat{A}_1\hat{A}_2) - \exp(ir\hat{A}_1^+\hat{A}_2^+ + ir\hat{A}_1\hat{A}_2)]|0,0\rangle$$

$$= \frac{1}{\sqrt{2N_\varphi}\cosh r}\sum_{n=0}^{\infty}\{[(-1)^n - 1](i\tanh r)^n|n,n\rangle\}, \quad (17)$$

$$|B_3\rangle \overset{\rm BS}{\to} |B_3\rangle' = \frac{1}{\sqrt{2N_\varphi}} \times\{\left[\exp\left(-\frac{r}{2}\hat{A}_1^{+2} + \frac{r}{2}\hat{A}_1^2\right)\exp\left(\frac{r}{2}\hat{A}_2^{+2} - \frac{r}{2}\hat{A}_2^2\right) + \exp\left(\frac{r}{2}\hat{A}_1^{+2} - \frac{r}{2}\hat{A}_1^2\right)\exp\left(-\frac{r}{2}\hat{A}_2^{+2} + \frac{r}{2}\hat{A}_2^2\right)\right]|0,0\rangle - \sin 2\varphi[\exp(-ir\hat{A}_1^+\hat{A}_2^+ - ir\hat{A}_1\hat{A}_2) + \exp(ir\hat{A}_1^+\hat{A}_2^+ + ir\hat{A}_1\hat{A}_2)]|0,0\rangle\}$$

$$= \frac{1}{\sqrt{2N_\varphi}\cosh r}\left\{\sum_{n_1,n_2=0}^{\infty}\{\frac{\sqrt{(2n_1)!\,(2n_2)!}}{n_1!n_2!} \times \left(\frac{1}{2}\tanh r\right)^{n_1+n_2}[(-1)^{n_1} + (-1)^{n_2}]|2n_1,2n_2\rangle\} - \sin 2\varphi\sum_{n=0}^{\infty}\{[(-1)^n + 1](i\tanh r)^n|n,n\rangle\}\right\}, \quad (18)$$

$$|B_4\rangle \overset{\rm BS}{\to} |B_4\rangle' = \frac{1}{\sqrt{2N_\varphi}} \times \left[\exp\left(-\frac{r}{2}\hat{A}_1^{+2} + \frac{r}{2}\hat{A}_1^2\right)\exp\left(\frac{r}{2}\hat{A}_2^{+2} - \frac{r}{2}\hat{A}_2^2\right)\right.$$

$$-\exp\left(\frac{r}{2}\hat{A}_1^{+2}-\frac{r}{2}\hat{A}_1^{2}\right)\exp\left(-\frac{r}{2}\hat{A}_2^{+2}+\frac{r}{2}\hat{A}_2^{2}\right)\Big]|0,0\rangle$$

$$=\frac{1}{\sqrt{2}N_\varphi\cosh r}\sum_{n_1,n_2=0}^{\infty}\{\frac{\sqrt{(2n_1)!\,(2n_2)!}}{n_1!n_2!}$$

$$\left(\frac{1}{2}\tanh r\right)^{n_1+n_2}[(-1)^{n_1}-(-1)^{n_2}]\,|2n_1,2n_2\rangle\}. \quad (19)$$

Next, we detect the photon number in two output ports using the photodetectors. According to the measurement results and Eqs. (16)−(19), we can discriminate that the incident light field is in which Bell state.

If two identical odd numbers of photons are detected at two detectors, the incident state on the measurement setup is $|B_2\rangle$. If two different numbers of photons are detected at two detectors and n_1 is an odd number (this implies that, in mode 1, the photon number is $2n_1$) but n_2 is an even number (or n_1 is an even number but n_2 is an odd number), the incident state is $|B_4\rangle$.

When $r\gg 1$, $\sin 2\varphi\to 0$. If two different numbers of photons are detected at two detectors and n_1 and n_2 are both odd numbers (or even numbers), $(-1)^{n_2}=(-1)^{n_1}$, the incident state is $|B_3\rangle$. If two identical even numbers of photons are detected at two detectors, the incident state on the measurement setup is, most probably, $|B_1\rangle$.

We have noticed that, if two identical even numbers of photons are detected at two detectors, the input light field is likely to be in state $|B_3\rangle$ rather than $|B_1\rangle$. But for $r\gg 1$, the relatively wrong probability is

$$P_{\mathrm{W}}=\frac{\sum_{m=0}^{\infty}|\langle 2m|_1\langle 2m|_2 B_3\rangle'|^2}{\sum_{m=0}^{\infty}|\langle 2m|_1\langle 2m|_2 B_1\rangle'|^2}, \quad (20)$$

which is negligible (see Table 1).

So, for $r\gg 1$, all the Bell states may be discriminated with arbitrarily high precision.

Following, we describe how to teleport a superposition of squeezed vacuum state. Suppose the state that Alice need to teleport is

$$|\psi\rangle_0=A|\zeta\rangle_0+B|-\zeta\rangle_0, \quad (21)$$

which can be written as

$$|\psi\rangle_0=A'|\Psi_+\rangle_0+B'|\Psi_-\rangle_0. \quad (22)$$

Suppose the quantum channel shared by Alice and Bob is

$$|B_2\rangle_{ab}=\frac{1}{\sqrt{2}}(|\Psi_+\rangle_a|\Psi_+\rangle_b-|\Psi_-\rangle_a|\Psi_-\rangle_b). \quad (23)$$

Table 1. Relatively Wrong Probability P_{W} Versus Varions r Values

r	$\sin 2\varphi$	P_{W}
2.0	0.1914	0.2112
4.0	0.0259	0.0074
6.0	0.0035	0.0002
8.0	4.7×10^{-4}	4.8×10^{-6}

More recently, Zhou *et al.* have proposed a method to generate such states[18]. We suppose the systems 0 and a are located at Alice's side and system b is at Bob's side. Alice let light beams 0 and a through the beam splitter. She then measures the photon number in the two output modes, respectively, and determines that the input light is located in which Bell state. Suppose the state is located in $|B_1\rangle_{0a}$. In this case, the state on Bob's side collapses into

$${}_{0a}\langle B_1|\psi\rangle_0|B_2\rangle_{ab}=\frac{A}{2}|\zeta\rangle_b-\frac{B}{2}|-\zeta\rangle_b. \quad (24)$$

Bob then performs a unitary transformation $(|\zeta\rangle_b\langle\zeta|_b-|-\zeta\rangle_b\langle-\zeta|_b)/N_\varphi$, and obtains the state of Eq. (21).

In conclusion, we have constructed an orthogonal Bell basis with entangled squeezed vacuum states and have shown that these states can be discriminated with arbitrary precision when the amplitude of the squeezed states becomes sufficiently large. A scheme of teleporting a superposition state of squeezed vacuum states based on the Bell state measurement is presented. However, this does not imply that the scheme is near to realization with current experimental expertise. In fact, as in Refs. [11−13] and [15], in our scheme an ideal photon counter is also not available at present.

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