Optimized design of parallel beam-splitting prism

Peitao Zhao (赵培涛) and Guohua Li (李国华)

Laser Research Institute, Qufu Normal University, Qufu 273165

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A large lateral shearing distance of parallel beam-splitting prism is often needed in laser modulation and polarization interference. In this letter, we present an optimized design of parallel beam-splitting prism and list some different cases in detail. The optimized design widens the use range of parallel beam-splitting prism. At the wavelength of 632.8 nm, the law that the enlargement ratio changes with the refractive index and the apex angle is verified.

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The polarizing prism can be simply divided into two categories, one is laser-polarizing prism whose type is LGP-Series; the other is polarization beam-splitting prism whose type is LSP-Series. They are applied widely in polarization techniques. Parallel beam-splitting prism belongs to the latter and can be applied in some special conditions such as laser modulation, transformation of polarization state and polarization interference, for its two output beams propagate in a parallel way. It is often made of natural crystal calcite with special cut and polish. Calcite has such advantage as large maximum birefringence ($n_o - n_e$), no deliquescence, and steady chemical composition. Its geometry structure is such a type that the angle between optical axis orientation and beam terminal face is 45°. One beam can be divided into two polarized beams through the prism, whose vibration planes are perpendicular to each other, but the output beams propagate in a parallel way. One beam propagates along the incidence direction, the other has an offset in comparison with the former. The offset was named lateral shearing distance. It is difficult to get a large lateral shearing distance with a single prism due to the properties of calcite. In fact, if the length of the prism is 10 mm, the value of lateral shearing distance is only about 1.1 mm. Therefore, in order to get large lateral shearing distance, we must increase the length of prism but it is not economical. One can attempt to modify the structure angle of Wollaston prism to get the same result, but this method is too troublesome to practice. In this paper, we present a new design that two right-angle prisms are inserted, forming a compounded type to satisfy the purpose.

The design is illustrated in Fig. 1, where M_1 and M_2 stand for two equal right-angle prisms with apex angle α. From

$$n_0 \sin\theta = n \sin\theta', \tag{1}$$

we have

$$d_1 = AB\cos\theta, \quad d_2 = AB\cos\alpha;$$

$$d_2 = CD\cos\theta, \quad d_3 = CD\cos\alpha, \tag{2}$$

therefore we can get

$$\frac{d_3}{d_1} = \frac{\cos^2\alpha}{\cos^2\theta} = \frac{1-\sin^2\alpha}{1-n^2\sin^2\alpha}. \tag{3}$$

In the above equations, θ is the angle of incidence and θ' is the angle of emergence. It is easy to find that the value of d_3/d_1 is larger than 1, and changes with the refractive index n and apex angle α. So by inserting two right-angle prisms into the optical path and adjusting the orientation of prism, we can extend the lateral shearing distance.

Now let us represent the relationship of the value of d_3/d_1, α, n, and the transmittance of the beams in detail. Our calculations will be based on the following mathematical formulas for the transmittance of the two monochromatic beams. We use the parameters R_{o1}, R_{o2}, R_{e1}, R_{e2} to denote the reflectivities of o-beam and e-beam on two right-angle prisms, respectively. From Fresnel equation they can be written as

$$R_{o1} = \left(\frac{\cos\theta - n\cos\theta'}{\cos\theta + n\cos\theta'}\right)^2$$

$$= \left(\frac{\sqrt{1-n^2\sin^2\alpha} - n\sqrt{1-\sin^2\alpha}}{\sqrt{1-n^2\sin^2\alpha} + n\sqrt{1-\sin^2\alpha}}\right)^2,$$

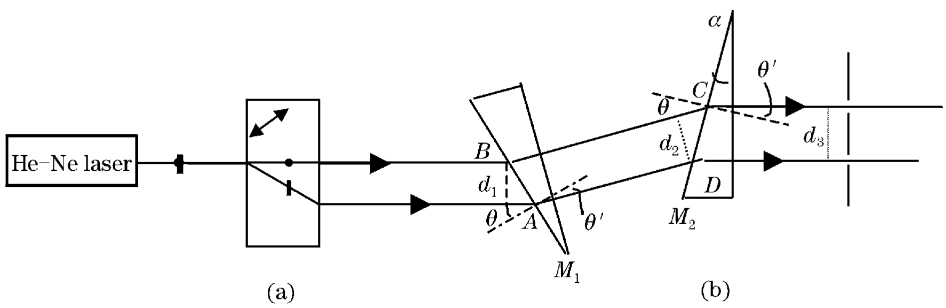

Fig. 1. The optimization design of parallel beam-splitting prism.

$$R_{o2} = \left(\frac{n-1}{n+1}\right)^2,$$

$$R_{e1} = \left(\frac{n\cos\theta - \cos\theta'}{n\cos\theta + \cos\theta'}\right)^2 = \left(\frac{n\sqrt{1-n^2\sin^2\alpha} - \sqrt{1-\sin^2\alpha}}{n\sqrt{1-n^2\sin^2\alpha} + \sqrt{1-\sin^2\alpha}}\right)^2,$$

$$R_{e2} = \left(\frac{n-1}{n+1}\right)^2.$$

The total transmittance of o-beam is

$$T = (1-R_{o1})^2(1-R_{o2})^2 = \left[1-\left(\frac{n-1}{n+1}\right)^2\right]^2 \times \left[1-\left(\frac{\sqrt{1-n^2\sin^2\alpha} - n\sqrt{1-\sin^2\alpha}}{\sqrt{1-n^2\sin^2\alpha} + n\sqrt{1-\sin^2\alpha}}\right)^2\right]^2,$$

and the total transmittance of e-beam is

$$T = (1-R_{e1})^2(1-R_{e2})^2 = \left[1-\left(\frac{n-1}{n+1}\right)^2\right]^2 \times \left[1-\left(\frac{n\sqrt{1-n^2\sin^2\alpha} - \sqrt{1-\sin^2\alpha}}{n\sqrt{1-n^2\sin^2\alpha} + \sqrt{1-\sin^2\alpha}}\right)^2\right]^2.$$

Assuming that the value of refractive index n is 1.5147, we can get the changes of d_1/d_3, and T with α, which is illustrated in Fig. 2. Figure 3 is similar to Fig. 2 but for the refractive index n of 1.6.

a) When the refractive index n is 1.5147, the value of d_1/d_3 is decreased with an increasing apex α. At the same time, the transmittance of e-beam shows a slow increase before it reaches the top point, and then it decreases rapidly. Meanwhile, the transmittance of o-beam shows a slow decrease at first but drops rapidly at the latter of the process.

b) Selecting the appropriate apex α, we can design a double-output type prism. In this case, it is needed that the transmittances of two beams are both high and the difference between them is small. We design the model in such a type, the refractive index of glass n is 1.5147, the value of d_3/d_1 is 2, the apex angle α is 31.8°. Thus

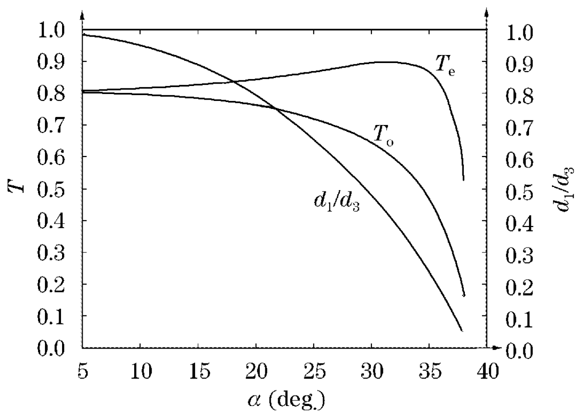

Fig. 2. The changes of d_1/d_3 and T with α (n =1.5147).

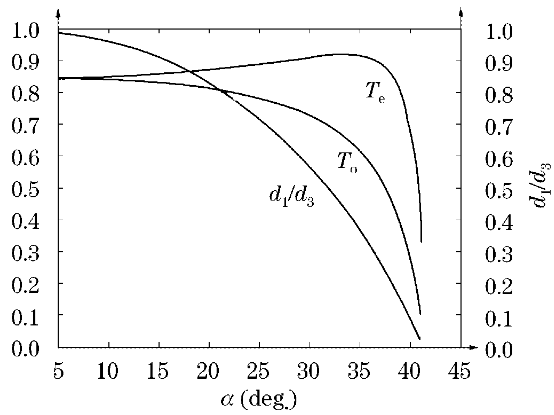

Fig. 3. The changes of d_1/d_3 and T with α (n=1.6).

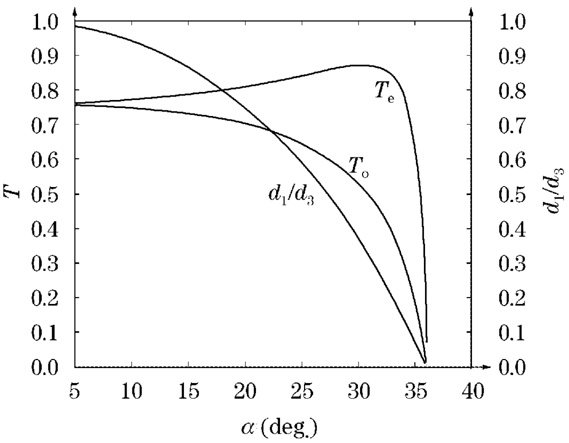

Fig. 4. The changes of d_1/d_3 and T with α (n=1.7).

the transmittance of e-beam and o-beam are T_o=69.3%, T_e=86.3%, the value of lateral shearing distance is 2.4 mm.

c) When designing the type of single output, let the value of d_3/d_1 be 3 and the value of lateral shearing distance be 3.6 mm. In this case, assuming n = 1.5147, the apex α is 35.6°, the transmittances of two beams are T_o=54.3%, T_e=91%, so it is easy to eliminate the o-beam and preserve the e-beam.

d) Figure 4 illustrates the changes of d_1/d_3 and transmittance T with α.

We concluded that as the value of refractive index n increases, the transmittance has the trend of decrease. As the value of n is 1.6 and d_3/d_1 is 3, the result of the apex α is 33.2°, the transmittances of two beams are T_o=89.3%, T_e=54.3%. As d_3/d_1 is 2, the apex α is 29.5°, the transmittances of two beams are T_o=65% and T_e=89%.

Assuming that the value of n is 1.7 and d_3/d_1 is 3, we should let the apex α be 30.7°, the transmittances of two beams are T_o=54% and T_e=87%. When the value of d_3/d_1 is 2, we should let the apex α be 27.2°, the transmittances of two beams are T_o = 60% and T_e = 85.5%.

e) In the process of designing a double output type, we can conglutinate the two right-angle prisms with glycerin in order to get the high transmittance. Figure 5 illustrates the operation. Based on the analysis of the former

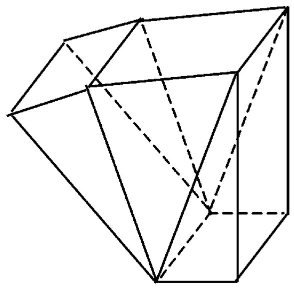

Fig. 5. The sectional drawing of two rectangular prisms bonding.

case, we get from Eq. (3) that

$$d_3/d_1 = \cos^2\alpha/(\cos\theta_1\cos\theta_2), \tag{4}$$

where θ_1 and θ_2 are the angles of incidence on the first and the second prism's surface. Assuming that the value of refractive index of glycerin n is 1.4311, the result of Eq. (4) is 2, so the value of α is 32.45°. The values of θ_1 and θ_2 can be calculated from the former equation, the results are 54.36° and 34.6°, respectively. The above data should be taken into the Fresnel equation to get

$$\begin{aligned} R_{e1} &= 5.035\times10^{-4},\\ R_{e2} &= 8.054\times10^{-4},\\ R_{e3} &= 2.545\times10^{-4},\\ R_{e4} &= 4.189\times10^{-2},\\ R_{o1} &= 1.397\times10^{-1},\\ R_{o2} &= 8.054\times10^{-4},\\ R_{o3} &= 1.576\times10^{-3},\\ R_{o4} &= 4.189\times10^{-2}. \end{aligned}$$

The total transmittance of e-beam is

$$\begin{aligned} T_e = &(1-R_{e1})\times(1-R_{e2})\\ &\times(1-R_{e3})\times(1-R_{e4}) = 95.6\%; \end{aligned}$$

the total transmittance of o-beam is

$$\begin{aligned} T_o = &(1-R_{o1})\times(1-R_{o2})\\ &\times(1-R_{o3})\times(1-R_{o4}) = 85.7\%. \end{aligned}$$

It is easy to find that the transmittances of two beams in this case is higher than the case b).

We can represent the Eq. (4) by α and β as

$$\beta = \frac{d_3}{d_1} = \frac{\cos^2\alpha}{(1-n^2\sin^2\alpha)^{\frac{1}{2}}(1-n'^2\sin^2\alpha)^{\frac{1}{2}}}, \tag{5}$$

n' stands for n/n_1, n and n_1 are the refractive indices of prism and glycerin, respectively. By differentiating both sides of Eq. (4) with respect to α, we get

$$\begin{aligned} \frac{d\beta}{d\alpha} = &\frac{-\sin 2\alpha}{(1-n^2\sin^2\alpha)^{\frac{1}{2}}(1-n'^2\sin^2\alpha)^{\frac{1}{2}}}\\ &+\left[\frac{n^2\sin 2\alpha\times\cos^2\alpha}{2(1-n^2\sin^2\alpha)^{\frac{3}{2}}(1-n'^2\sin^2\alpha)^{\frac{1}{2}}}\right]\\ &\times\left[\frac{n'^2\sin 2\alpha}{2(1-n^2\sin^2\alpha)^{\frac{1}{2}}(1-n'^2\sin^2\alpha)^{\frac{3}{2}}}\right]. \end{aligned}$$

We can get the value of $\frac{d\beta}{d\alpha} = 3.76$ after taking α=32.45° into the above equation. So when the error of α is 0.05°, the influence error caused by $\Delta\alpha$ is only 0.19, this value is small. We can also use the same approach to calculate the other case b), the influence error caused by the machining error is much bigger than the case e), the value of $\frac{d\beta}{d\alpha}$ can reach 8.8. So the case b) is with low precision.

The parameters of two prisms, such as α and n, should be same, so we take the approach of adhering two prisms to integral one to machining, the apex angles of two prisms are equal to each other. It is interesting to note that the discussion and analysis only aim at monochromatic light, whose wavelength is 632.8 nm.

According to the different request, we can design different type. The best one among the above designs is case e). This case can extend the lateral shearing distance, but it also causes the two beams to have a small offset, so we should pay attention to it in real applications. We can also evaporate the antireflecting film on the surfaces of two right-angle prisms and the calcite to improve the transmittance.

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