Effect of Doppler broadening on gain in an open V-type inversionless lasing system

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Our study shows that for the copropagating probing and driving fields, the gain without inversion doesn't monotonously decrease or increase with the increasement of Doppler width. When the driving field is resonant, at a suitable Doppler width, we can get a maximum value of the gain without inversion, which is much larger than that obtained when Doppler broadening is absent.

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Since the seminal work of Kocharovskaya, Harris and Scully *et al.,* the study of lasing without inversion (LWI) has attracted tremendous attention[1−26], as this study has an important science significance and wide potential applications. LWI could provide a new tool in the pursuit of "tabletop" UV, X- and γ-ray lasers and has other interesting statistical properties. We have investigated the open V-type LWI system from different respects[16−21]. However, in these studies we have assumed homogeneously broadened atomic media, and not considered moving atoms. In this paper we explore the effect of Doppler broadening on the gain of the system.

Using the electric dipole, slowly varying envelope and rotating wave approximations, the semiclassical density matrix equations of motion for an open V-type atomic system (Fig. 1) can be written as follows

$$\dot{\rho}_{11} = -\gamma_{13}\rho_{11} + \Lambda(\rho_{33} - \rho_{11}) - i\alpha(\rho_{31} - \rho_{13}) - r_0\rho_{11}, \tag{1a}$$

$$\dot{\rho}_{22} = -\gamma_{23}\rho_{22} + i\Omega(\rho_{23} - \rho_{32}) + J_2 - r_0\rho_{22}, \tag{1b}$$

$$\begin{aligned}\dot{\rho}_{33} = {} & \gamma_{13}\rho_{11} + \gamma_{23}\rho_{22} + \Lambda(\rho_{11} - \rho_{33}) + i\alpha(\rho_{13} - \rho_{31}) \\ & + i\Omega(\rho_{23} - \rho_{32}) + J_3 - r_0\rho_{33},\end{aligned} \tag{1c}$$

$$\dot{\rho}_{23} = -\Gamma_{23}\rho_{23} + i[\Omega(\rho_{33} - \rho_{22}) - \alpha\rho_{21}], \tag{1d}$$

$$\dot{\rho}_{13} = -\Gamma_{13}\rho_{13} - i[\alpha(\rho_{33} - \rho_{11}) - \Omega\rho_{12}], \tag{1e}$$

$$\dot{\rho}_{12} = -\Gamma_{12}\rho_{12} + i(\alpha\rho_{32} - \Omega\rho_{13}), \tag{1f}$$

along with the equations of their complex conjugates, where α and Ω are the Rabi frequencies of the probe and driving fields, respectively. For the convenience of calculation, α and Ω are chosen to be real, but this does not affect generality of the following discussion. The atoms are injected into the lower levels $|2\rangle$ and $|3\rangle$ with rates J_2 and J_3, respectively, and exit out the system at rate r_0. We also assume that the number of interacting atoms is constant, which means that $J_2 + J_3 = r_0 \cdot \gamma_{ij}$. γ_{ij} denotes the radiant decay from level $|i\rangle$ to $|j\rangle$. Λ is incoherent pump rate. The Γ_{ij}'s are the complex coherence decay rates defined as

$$\begin{aligned}\Gamma_{13} &= (\gamma_{13} + 2\Lambda)/2 - i\Delta_p, \\ \Gamma_{23} &= (\gamma_{23} + \Lambda)/2 - i\Delta_d, \\ \Gamma_{12} &= (\gamma_{13} + \gamma_{23} + \Lambda)/2 + i\Delta_p - i\Delta_d,\end{aligned}$$

where Δ_p ($= \omega_p - \omega_{13}$) and Δ_d ($= \omega_d - \omega_{23}$) are the detunings of the probing field (frequency ω_p) and driving field (frequency ω_d), respectively. In our notation, if $\mathrm{Im}\rho_{13} < 0$ the system exhibits gain for the probing field, that is, lasing can be established on the transition $|3\rangle \leftrightarrow |1\rangle$; if $\mathrm{Im}\rho_{13} > 0$, the probing field is attenuated. When $\rho_{11} - \rho_{33} < 0$ and $\mathrm{Im}\rho_{13} < 0$, the probing field gets gain without inversion.

We solve Eqs. (1) in the steady state and obtain the exact nonlinear analytical solutions for $\mathrm{Im}\rho_{13}$ and $\rho_{11} - \rho_{33}$ as following

$$\mathrm{Im}\rho_{13} = -\alpha(b_2 I^2 + b_1 I + b_0)/\Phi, \tag{2a}$$

$$\rho_{11} - \rho_{33} = (d_2 I^2 + d_1 I + d_0)/\Phi. \tag{2b}$$

In Eqs. (2), $I = \alpha^2$, b_i, d_i ($i = 0, 1, 2, 3$) and Φ are functions of the Rabi frequency of the driving field and other parameters of the system, the detail expressions are given in the appendix of Ref. [18] (note the difference of the levels' titles). In the following discussion, $\rho_{11} - \rho_{33} < 0$ is always satisfied.

As already stated, the calculation of the gain outlined above is only valid for the static atoms. Now consider atomic motion and the resulting inversionless gain when the Doppler effect is included. For a single atom, moving with a velocity v along the z axis, the probing frequency $\omega_p(v)$, as seen by the atom, is given by

$$\omega_p(v) = \omega_p(1 \pm v/c), \tag{3a}$$

where the negative (positive) sign corresponds to copropagating (counterpropagating) of atom and probe. Similarly, the frequency of the driving field, as seen by the atom is

$$\omega_d(v) = \omega_d(1 \pm v/c). \tag{3b}$$

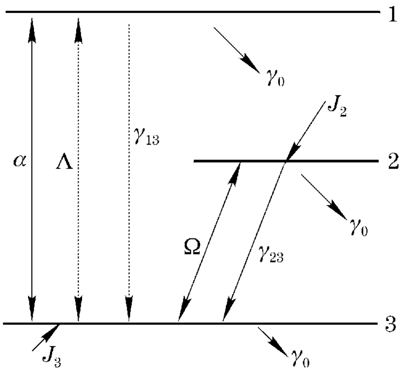

Fig. 1. Open V-type three- level atomic system.

We denote by $\delta_p(v)$ $(= \omega_p(v) - \omega_{13})$ and $\delta_d(v)$ $(= \omega_d(v) - \omega_{23})$, the detunings of the probing and driving fields from their respective transitions, as seen by the moving atom. In terms of $\delta_p(v)$ and stationary atom parameters, $\delta_d(v)$ can be written as

$$\delta_d(v) = \Delta_d \pm \omega_d v/c, \tag{4a}$$

$$\delta_p(v) = \Delta_p \pm \omega_p v/c. \tag{4b}$$

Assuming a Maxwell-Boltzmann distribution for the atomic velocities, we adopt a probability distribution function for $\delta_p(v)$, which is given by

$$\rho(\delta_p) = \frac{1}{\sqrt{2\pi D^2}} \exp[-(\delta_p - \Delta_p)^2/2D^2], \tag{5}$$

where D represents Doppler width. The gain averaged over the Doppler distribution can be represented as

$$G = -\mathrm{Im} \int_{-\infty}^{\infty} \rho_{13}(\delta_p)\rho(\delta_p)\mathrm{d}\delta_p. \tag{6}$$

So, if $G > 0$ and $\rho_{11} - \rho_{33} < 0$, the probing field gets gain without inversion. In the following we will discuss the effect of Doppler broadening on the inversionless gain by the numerical results. The numerical algorithm consists of calculating $\mathrm{Im}\rho_{13}$ and $\rho_{11} - \rho_{33}$ in the presence of Doppler broadening, i.e., we replace Δ_p and Δ_d in Eqs. (2) with $\delta_p(v)$ and $\delta_d(v)$. For a given D and Δ_p, we generate a probability distribution of $\delta_p(v)$ by using Eq. (5). The resulting values of δ_p are centered at the selected value of Δ_p. By choosing optimal intervals in Doppler width D to assure that there are sufficient atoms to participate in producing consistent and accurate results. For each value of δ_p, we determine δ_d from Eq. (4b). Then, for each δ_p and δ_d, Eqs. (2) are used to calculate $\mathrm{Im}\rho_{13}$ and $\rho_{11} - \rho_{33}$. To determine the Doppler averaged signal as given by Eq. (6), we simply sum the signal from the contributions due to the various values of δ_p, distributed in accordance with Eq. (5).

In Fig. 2, we give the numerical results of the inversionless gain G (i.e. Eq. (6)) as a function of Δ_p for copropagating probing and driving fields when the driving field is on-resonance (i.e. $\omega_d = \omega_{13}$), the values of Doppler width and other parameters are given in MHz. Here we use $D = 0.0001$ as the approximations of the zero Doppler width.

Figure 2 shows that the gain maximum value $G_{\max}$ does not monotonously vary with Doppler width D. When $D < 0.1$ and $D > 5$, $G_{\max}$ decrease with D increasing; when $0.1 < D < 5$, $G_{\max}$ increases with D increasing; when $D = 5$, we can get the largest $G_{\max}$ which is much larger than that obtained when Doppler broadening is absent. Figure 3 plots the inversionless gain G as a function of Doppler width D, where Δ_p takes the value corresponding to the largest $G_{\max}$ in Fig. 2, and the values of other parameters are same as those in Fig. 2.

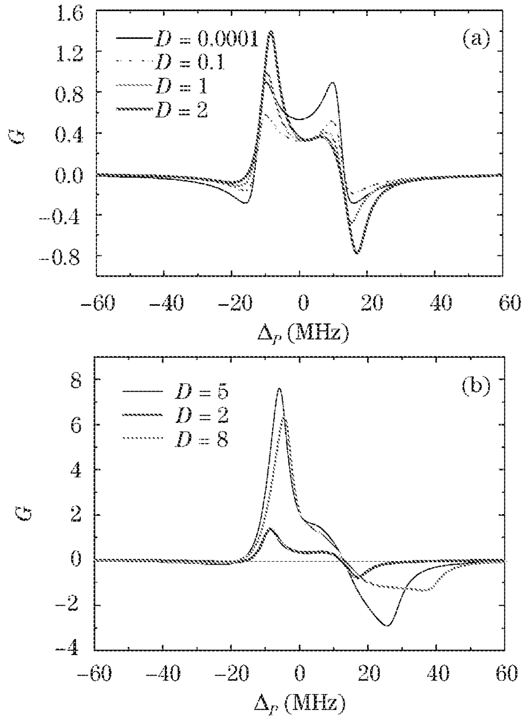

Fig. 2. Gain versus the probe detuning for different Doppler width $\gamma_{13} = 0.02$, $\gamma_{23} = 2$, $\Lambda = 2$, $\alpha = 0.2$, $\Omega = 4$, $r_0 = 0.4$, $J_3 = 0.1$, $J_2 = 0.3$, $\Delta_d = 0$.

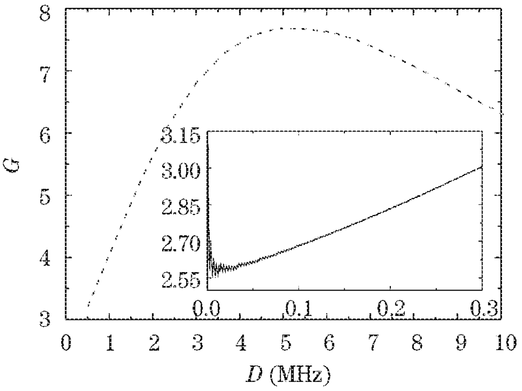

Fig. 3. Gain versus Doppler width. Δ_p takes the value corresponding to the largest $G_{\max}$ in Fig. 2, and the values of the other parameters are same as those in Fig. 2.

Our numerical calculation results show that when the probing and driving fields are copropagating and $\Delta_d \neq 0$, and when the probing and driving fields are counterpropagating and $\Delta_d = 0$, the gain maximum value $G_{\max}$ does not monotonously decrease with D increasing, in some region of Doppler width, $G_{\max}$ increasing with D, but the largest $G_{\max}$ being still smaller than that obtained when Doppler broadening is absent; when the probing and driving fields are counterpropagating and $\Delta_d \neq 0$, $G_{\max}$ does monotonously decrease with D increasing.

Why the Doppler broadening may lead to an obvious enhancement of gain for some case in an open V-type LWI system? We will give a physical explanation as follows.

Quantum coherence (QC) and interference in atomic system lead to LWI. The present of Doppler broadening will produce a destructive or constructive contribution for QC and this is mainly determined by the Doppler width, propagating direction and detunings of the driving and probing fields. In some case, the constructive QC appears, leading to gain increasing; and in another case, the destructive QC arises, leading to gain decreasing; and in some special case (e.g., for copropagating the probe and driving fields, $\Delta_d = 0$ and $D = 5$), the constructive QC is very strong and will leads to get a gain much larger than that without Doppler broadening.

In conclusion, we have studied the effect of the Doppler broadening on the gain in an open V-type LWI system by using numerical results from the stationary analytical solution. We found that in most cases the gain without inversion does not monotonously decrease or increase with the Doppler width; when the probing and driving fields are copropagating, the driving field is resonant, at a suitable Doppler width it is possible to get a gain without inversion much larger than that obtained when Doppler broadening is absent.

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