Lidar signal de-noising based on wavelet trimmed thresholding technique

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Lidar is an efficient tool for remote monitoring, but the effective range is often limited by signal-to-noise ratio (SNR). By the power spectral estimation, we find that digital filters are not fit for processing lidar signals buried in noise. In this paper, we present a new method of the lidar signal acquisition based on the wavelet trimmed thresholding technique to increase the effective range of lidar measurements. The performance of our method is investigated by detecting the real signals in noise. The experiment results show that our approach is superior to the traditional methods such as Butterworth filter.

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Lidar is meant to determine the range of a target by analyzing the echo of light pulses scattered on the investigated objects. Among many applications, lidar is most widely used in atmospheric research in environments. For instance, lidar can be used to study the high cirrus clouds over equatorial regions[1], high-altitude polar stratospheric clouds[2], stratospheric ozone[3] and stratospheric aerosols or stratospheric aerosols[4].

Lidar uses a laser (emitter) to send a light pulse into the atmosphere and a telescope (receiver) to measure the scattered back intensity (signal). By measuring the scattering and attenuation experienced by the incident light pulse, one can investigate the properties of the scatterers located in the atmosphere. The backscattered radiation detected by lidar can be described by the lidar equation. For a simple case, the equation can be written as

$$p_{\mathrm{r}}(\lambda_{\mathrm{L}}) = \frac{h}{2}\frac{C}{R^2}O(R)\frac{\beta(\lambda_{\mathrm{L,R}})}{4\pi} \times \exp\left(-2\int_0^R k_{\mathrm{e}}(\lambda_{\mathrm{L}}, R')\mathrm{d}R'\right), \qquad (1)$$

where $p_{\mathrm{r}}(\lambda_{\mathrm{L}})$ is the power returned to the lidar at the laser wavelength λ_{L}, C is the lidar constant, R is the range, $h = c \cdot t_{\mathrm{p}}$ with t_{p} being the pulse duration and c the speed of light. The term $O(R)$ describes the overlap between the laser beam and the receiver field of view and it is equal to 1 for ranges where there is a complete overlap. In addition, $\beta(\lambda_{\mathrm{L,R}})$ stands for the combined aerosol and molecular backscattering and $k_{\mathrm{e}}(\lambda_{\mathrm{L}}, R)$ denotes extinction coefficients at the laser wavelength λ_{L}. For an elastic backscattering (one wavelength) lidar, the combined backscattering can be obtained by solving the lidar equation following the method suggested by Ref. [4].

Obviously, all solutions of the lidar equation depend on $p_{\mathrm{r}}(\lambda_{\mathrm{L}})$ because it is the unique parameter that one can detect. So the main limitation of the effective range of a lidar system is caused by the fact that the signal-to-noise ratio (SNR) falls rapidly with the increase of the distance R, which involves all types of lidars[5]. Figure 1 shows a simulated lidar signal without noise and illustrates the characteristic of ideal lidar signal. Figure 2 shows a real lidar signal, which was backscattered by molecular and aerosol in atmosphere, recorded by our own lidar system located in Anhui Institute of Optics and Fine Machine (AIOFM), CAS, Hefei, China. Both figures are plotted in the log axes.

The noise components can strongly affect the results,

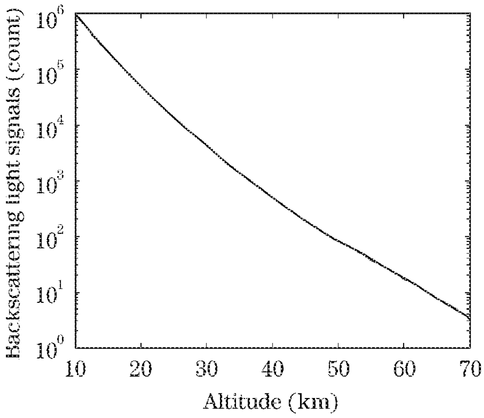

Fig. 1. Simulated lidar signal without noise.

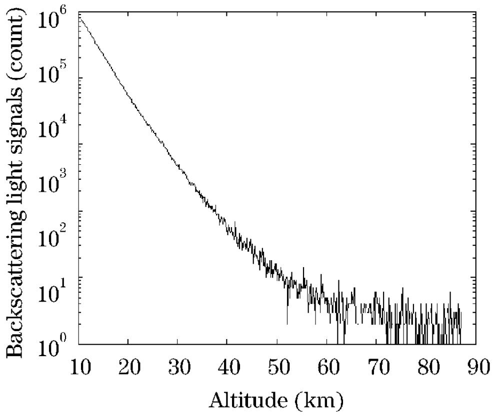

Fig. 2. Real lidar signal with noise.

providing a nonsensical values of concentration, especially at altitude greater than 20 km. By moving average method[5], the signal is only smoothed over distance. So this method cannot eliminate the nonsensical values (especially negative values) produced by noises.

In this paper, we present a new method, which is based on discrete wavelet transform (DWT) with trimmed thresholding technique, to de-noise lidar signal and increase the effective range of lidar measurements. Wavelet de-noising method should not be confused with smoothing technique. Whereas the smoothing technique removes high frequency and retains low ones, the de-noising method attempts to remove noises and retain signals whatever the frequency contents are. Furthermore, wavelet shrinking de-noising technique is considered as a nonparametric method. Thus, it is distinct from parametric method[6] in which we must estimate parameters for a particular model that must be assumed a priori.

In experiment, there are many sources of noises and interferences, which will affect the lidar signal. The distribution of noise is complex, and in fact we have few prior knowledge of noise. Figure 3 shows the power spectral density (PSD) of simulated lidar signal and that of real lidar signal. The aim of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data. Estimation of power spectral is useful in a variety of applications, including the detection of signals buried in wide-band noise. According to spectral estimation, the noise of lidar signal is to distribute in wide band, and the noise and the signal are almost to distribute in the same band interval. So it is impossible to eliminate the noise using digital filters by simply selecting a cut-off frequency.

It is assumed that N samples are given from real lidar signal $f(t)$ observed with noise $y_i = f(t)_i + e_i$, $i = 1, 2, \cdots, N$, where e_i is regarded as Gaussian white noise with zero mean and varied variance.

Wavelet thresholding de-noising technique is designed based on the multiresolution analysis. S. Mallat[7] gave a method of wavelet decomposition according to the principle of multiresolution. In practice, the Gaussian wavelet or Mexican hat wavelet is often used as mother wavelet. The Mexican hat wavelet is compactly supported in the time domain rather than the frequency domain, and is often used in the case where high temporal resolution is required. Because lidar signal is a transient signal in the time domain, Mexican hat wavelet is selected. It is given by

$$w(t) = \frac{2}{\sqrt{3}}\pi^{-1/4}(1-t^2)\mathrm{e}^{-t^2/2}. \quad (2)$$

Let Δt denote the given threshold, the soft threshold can be defined by

$$y = \begin{cases} \mathrm{sign}(x)\cdot(|x|-\Delta t), & |x| \geq \Delta t \\ 0, & |x| < \Delta t \end{cases}. \quad (3)$$

The hard threshold can be defined as

$$y = \begin{cases} x, & |x| \geq \Delta t \\ 0, & |x| < \Delta t \end{cases}. \quad (4)$$

The hard threshold can be described as the usual process of setting, the elements, whose absolute values are lower than the threshold to zero. The soft threshold is an extension of the hard threshold, i.e., first setting the meet elements to zero, and then shrinking the nonzero coefficients towards 0.

Wavelet thresholding de-noising technique provides a new way to reduce noise in signal. However, the soft thresholding method is best in reducing noise but worst in preserving edges, and the hard thresholding technique is best in preserving edges but worst in de-noising. Motivated by finding a more general case that incorporates the soft and hard thresholding schemes, we proposed the following thresholding rule,

$$y = \begin{cases} x * \frac{|x|^\alpha - \Delta t^\alpha}{|x|^\alpha}, & |x| \geq \Delta t \\ 0, & |x| < \Delta t \end{cases}, \quad (5)$$

where Δt is chosen as an estimate of noise level. When $\alpha = 1$, Δt is equivalent to the soft threshold; when $\alpha \to \infty$, it is equivalent to the hard threshold. Figure 4 graphically shows its relationships with soft and hard thresholds. It can be clearly seen that trimmed thresholding technique is something a moderate method between the hard and soft thresholding schemes. With careful tuning of parameter α for a particular signal, one can achieve the best de-noising result within the thresholding framework.

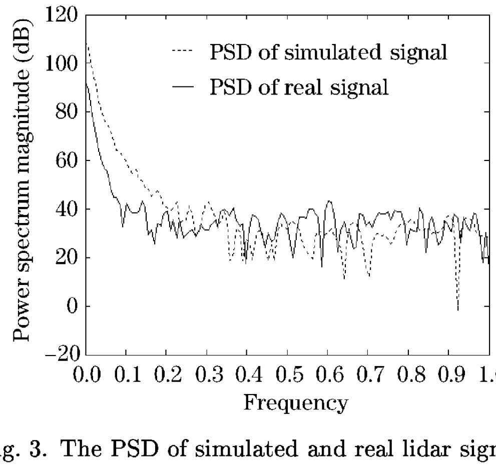

Fig. 3. The PSD of simulated and real lidar signals.

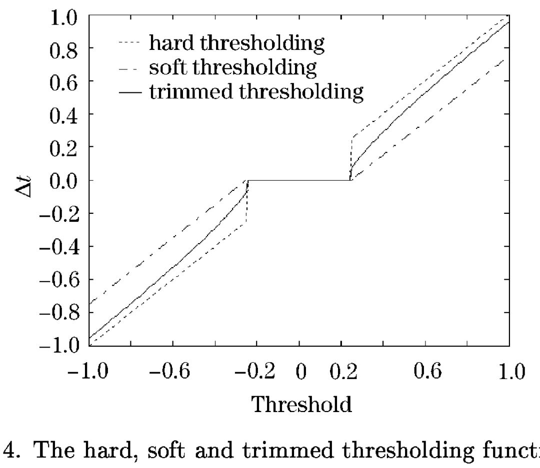

Fig. 4. The hard, soft and trimmed thresholding functions.

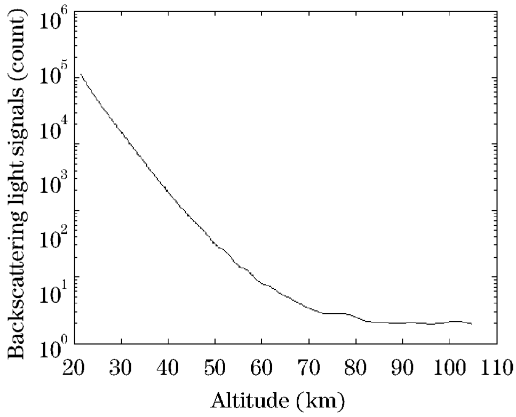

Fig. 5. The de-noised lidar signal by wavelet.

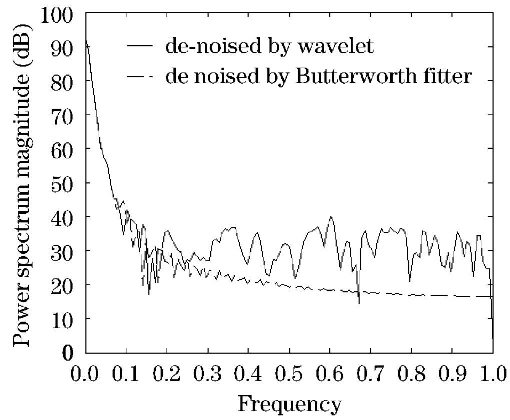

Fig. 6. The PSD of de-noised lidar signals.

According to the noise model, there are four threshold selection rules[8]. The threshold selection rule based on Stein's unbiased risk estimation is used in this paper. Let n be the length of vector x in the algorithm of the following threshold selection rule, we get a risk estimate for a particular threshold value Δt. Minimizing the risks in Δt gives a selection of the threshold value.

Sorting the absolute value of the vector to be estimated from minimum to maximum, and then extracting the root of the sorted vector, a new vector NV is obtained. And the formula of risk at the index k is

$$\text{Risk}(k) = \frac{n - 2k + \sum_{j=1}^{k} NV(j) + (n-k) * NV(n-k)}{n}. \tag{6}$$

The corresponding threshold is $\Delta t = \sqrt{NV(k)}$.

Figure 5 shows the de-noised lidar signal processed by wavelet trimmed thresholding method. The effective range is greater than 80 km. Figure 6 shows the PSD of lidar signal processed by Butterworth filter and that of lidar signal processed by DWT. Obviously, the high-frequency components of lidar signal were lost in filter method and most components of the real lidar signal are retained regardless of the frequency content in wavelet method.

This paper proposed a de-noising method for lidar signal based on the wavelet trimmed thresholding technique. The noisy component in real lidar signal is usually to distribute in wider band while the signal from longer distance (greater than 40 km) with low SNR is almost buried in the noise. So it is impossible to eliminate the noise using conventional digital filter by simply selecting a cut-off frequency. The wavelet coefficient de-noising method using nonlinear trimmed threshold technique can remove the noise and retain the signal components regardless of the signal's frequency content. Consequently, the noise in lidar signal was almost entirely suppressed so that the effective working range for lidar instrument was increased greatly. The experimental results about real data of lidar demonstrate the effectiveness and efficiency of the proposed approach. In particular, the experimental results on real lidar signal show that our approach is still efficient at longer distance (above 80 km) where a poor SNR occurs. Future work is to polish the performance of wavelet trimmed thresholding method more effectively and accurately.

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References

1. A. H. Omar, J. Geophys. Res. **106,** 1227 (2001).
2. V. Santacesaria, A. R. MacKenzie, and L. Stefanutti, Tellus (B) **53,** 306 (2001).
3. L. R. Douglass, M. R. Schoeberl, S. R. Kawa, and E. V. Browell, J. Geophys. Res. **106,** 9879 (2001).
4. F. G. Fernald, Appl. Opt. **23,** 652 (1984).
5. S. Lerkvarnyu, "Moving Average Method for Time Series Lidar Data" http://www.gisdevelopment.net/aars/acrs/1998/ps3/ps3016.shtml.
6. D. S. Huang, *Intelligent Signal Processing Technique for High Resolution Radars* (China Machine Press, Beijing, 2001).
7. S. Mallat, IEEE Trans. Pattern Analysis and Machine Intelligence **11,** 674 (1989).
8. D. L. Donoho, IEEE Trans. Information Theory **41,** 613 (1995).