Particle digital in-line holography with spherical wave recording

Baozhen Ge (葛宝臻), Qieni Lü (吕且妮), and Yimo Zhang (张以谟)

College of Opto-Electronics & Precision Instrument Engineering, Tianjin University; Key Laboratory of Opto-Electronics Information and Technical Science, Ministry of Education, Tianjin University, Tianjin 300072

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In this paper, we propose a method of digital in-line holography of particle. A diverging spherical beam is used for illumination in recording hologram, the complex amplitude distribution generated by particle field at a single plane located in the Fresnel diffraction region is recorded by CCD, and a plane beam for reconstructing hologram, then, the magnified image can be obtained by numerical reconstruction in computer. This procedure can be interpreted by Fourier optical theory and the theoretical analysis have been done in detail, the experimental results, the air freshener being subject, are also given.

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Holography is ideally suitable for the study of dynamic three-dimensional particle field, and its applications for this purpose are almost limitless[1]. With the development of high-resolution CCD and fast computer, digital holography, digital recording of hologram and numerical reconstruction of the wavefront, have become possible, and have been widely applied in various fields of science and engineering. One of the potential applications of the digital holography is particle analysis, such as the measurement of size, position, velocity, and distribution of particle[2−12]. In-line holography is the simplest type of holography and should always be firstly considered in particle field studies. Digital in-line holography now has been developed into a new tool and applied widely in various fields such as droplet, cell biology, micro-particle imaging and tracking, polymer crystallization, etc. In digital in-line holography, the reference wave can be collimated as well as divergent wave. The most common approach is to pass a collimated laser beam through to study volume of particle. The hologram of particle is recoded by a CCD sensor and then numerically reconstructed in computer. As plane wave is used for recording and reconstruction, the magnification of reconstructed image is unity. When a diverging beam is used for illumination in recording or reconstructing hologram, the magnified image can be available. This technology has been applied to holographic microscopy[11,12]. Xu *et al.*[12] reported the application in micrometer-sized spherical latex as well as ferromagnetic beads. In this paper, we propose a method that a diverging spherical beam is used to illuminate particle field, the hologram of particle is recorded by CCD sensor, and the original field is reconstructed numerically. The mathematical description of recording hologram with spherical incoming wave is given. And the magnified image can be obtained.

The diagram of the used in-line hologram recording optical system is shown in Fig. 1. A collimated laser beam is focused with a convergent lens. The focus acts as the point light source which emanates a diverging spherical wave. The particle fields are illuminated with this spherical wave, a part of the beam is diffracted by the particles and forms the a geometrically magnified diffraction pattern on recording plane and the other part passes the particle zone without being diffracted serving as a reference wave. The waves interfere and produce in-line holography on CCD and are recorded by CCD, this procedure can be interpreted by Fourier optical theory.

Supposing that the amplitude of laser radiation is unit amplitude, the amplitude transmittance of particle field is expressed as $[1 - o(x, y)]$. The spherical wave illuminates particle field and generates a magnified diffraction pattern on CCD. According to the Huygens-Fresnel approximation and the formula of lens phase, the spherical wave illuminating the particle field can be described as

$$u_1(x_1, y_1) = \frac{f}{f - z_1} \exp(jkz_1) \times \exp\left[j\frac{k}{2(z_1 - f)}(x_1^2 + y_1^2)\right], \tag{1}$$

where $k = 2\pi/\lambda$, f is the focus length of the lens, λ is the wavelength of the illumination light. Through particle field, the scalar component of the field in the recording plane with a distance z_2 from the particle plane is given by

$$u(x, y) = \frac{\exp(jkz_2)}{j\lambda z_2} \int\int [1 - o(x_1, y_1)] u_1(x_1, y_1) \exp\{j\frac{k}{2z_2}[(x - x_1)^2 + (y - y_1)^2]\} \mathrm{d}x_1 \mathrm{d}y_1. \tag{2}$$

Substituting Eq. (1) into Eq. (2), yielding

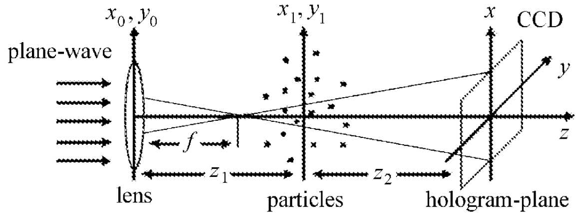

Fig. 1. The diagram of digital holographic recording system.

$$u(x,y) = -\frac{f}{z_2+z_1-f}\exp[jk(z_2+z_1)] \times \exp[j\frac{k}{2(z_2+z_1-f)}(x^2+y^2)] \times\{1-\frac{m_0}{j\lambda z_2}\int\int o(x_1,y_1) \times \exp\{j\frac{k}{2m_0 z_2}[(x-m_0x_1)^2+(y-m_0y_1)^2]\} \mathrm{d}x_1\mathrm{d}y_1\}. \quad (3)$$

The resultant intensity distribution in the hologram record $I(x,y)$ is given by

$$I(x,y) = I_0\left\{1-\frac{m_0}{j\lambda z_2}\int\int o(x_1,y_1) \times \exp\{j\frac{k}{2m_0z_2}[(x-m_0x_1)^2+(y-m_0y_1)^2]\}\mathrm{d}x_1\mathrm{d}y_1 + \frac{m_0}{j\lambda z_2}\int\int o^*(x_1,y_1) \times \exp\{-j\frac{k}{2m_0z_2}[(x-m_0x_1)^2+(y-m_0y_1)^2]\}\mathrm{d}x_1\mathrm{d}y_1 + \frac{m_0^2}{\lambda^2 z_2^2}|\int\int o(x_1,y_1) \times \exp\{j\frac{k}{2m_0z_2}[(x-m_0x_1)^2+(y-m_0y_1)^2]\} \mathrm{d}x_1\mathrm{d}y_1|^2\right\}, \quad (4)$$

where superscript $*$ denotes complex conjugation, $m_0 = \frac{z_2+z_1-f}{z_1-f}$ is geometric magnification of the recording system, and $I_0 = (\frac{f}{z_2+z_1-f})^2$ is the average intensity on holographic plane. For $||o|| \ll 1$, the last term in Eq. (4) can be neglected. The image complex-amplitude distribution reconstructed with a plane wave, located at distance z' from the holographic plane, is given by

$$u(x',y') = \frac{1}{j\lambda z'}\exp(j\frac{2\pi}{\lambda}z') \times\int\int t(x,y)\exp\{\frac{j\pi}{\lambda z'}[(x'-x)^2+(y'-y)^2]\}\mathrm{d}x\mathrm{d}y, \quad (5)$$

where $t(x,y) = 1 - I(x,y)$ is the amplitude transmittance of hologram. After performing some mathematical operations, the complex-amplitude distribution in the reconstruction plane of the real image, located at distance $z' = m_0 z_2$, is described as

$$u(x',y') = I_0\exp(jkz')\left\{\frac{1-I_0}{I_0}+o^*(\frac{x'}{m_0},\frac{y'}{m_0}) + \frac{m_0^2}{2j\lambda z'}\times\int\int o(x_1,y_1)\exp\{j\frac{k}{2}\frac{1}{2m_0z_2} \times[(x'-m_0x_1)^2+(y'-m_0y_1)^2]\}\mathrm{d}x_1\mathrm{d}y_1\right\}. \quad (6)$$

Equation (6) represents the well-known complex-amplitude distribution of a reconstructed real image plane in the case of conventional in-line holography. The second term corresponds to the real image of the particle, $o^*(\frac{x'}{m_0},\frac{y'}{m_0})$ means that the reconstructed image is magnified by the factor m_0.

The particle diffraction pattern of geometric magnification is obtained and recorded on CCD, and transmitted to a computer by frame grabber card, and then, numerically reconstructed. Let the hologram amplitude transmittance $t(x,y)$ be sampled on a rectangular raster of $M\times N$ matrix points, with steps Δx and Δy along the coordinates. Here Δx and Δy are the distances between neighboring pixels on the recording medium, x' and y' are replaced by $m'\Delta x'$ and $n'\Delta y'$, $m' = 0, 1, \cdots, M-1$, $n' = 0, 1, \cdots, N-1$, $\Delta x'$ and $\Delta y'$ are the pixel size in the reconstructed image. The discrete complex amplitude distribution $u(m',n')$ is given, aside from constant factors, by the following discrete Fresnel transformation

$$u(m',n') = \exp[j\frac{\pi}{\lambda z}(m'^2\Delta x'^2+n'^2\Delta y'^2)] \times\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} t(m,n)\exp[j\frac{\pi}{\lambda z}(m^2\Delta x^2+n^2\Delta y^2)] \times\exp[-j2\pi(\frac{m'm}{M}+\frac{n'n}{N})]. \quad (7)$$

Equation (7) can be solved by the convolution approach, and rewritten as

$$u(m',n') = \mathrm{F}^{-1}\{\mathrm{F}[t(m,n)] \times\exp[-j\pi\lambda z(\frac{\alpha^2}{(\Delta xM)^2}+\frac{\beta^2}{(\Delta yN)^2})]\}, \quad (8)$$

where F denotes the 2D discrete Fourier transformation and α and β are transverse discrete spatial frequencies. The magnified real image can be obtained by numerical reconstruction using Eq. (8).

The set-up described above is used to analyze the particles of the air freshener sprayed manually. The power of He-Ne laser is 10 mW, λ = 632.8 nm, f = 350 mm, z_1 = 550 mm, z_2 = 50 mm, the magnified factor $m_0 = 1.25$. The magnified diffraction pattern is recorded with a CCD sensor (DALSA 1M30 camera, 1024 × 1024 pixels; pixel size 12 × 12 μm^2), the obtained hologram is shown in Fig. 2(a). For protection from damage, the thin glass plate is placed on the front of the CCD. If illuminated with coherent light, the plate produces an interference pattern. This interference fringes are even visible. Figure 2(b) shows the reconstructed real image obtained by numerical reconstruction of hologram shown in Fig. 2(a). Although the particles are visible, a great deal of background noise obscures the image, the background needs to be removed. The two methods have been presented to remove the unwanted background in Ref. [12]. One is numerical background subtraction and the other is double exposure. The method of double exposure is used in experiments. Figure 3 shows the background intensity before particle is sprayed. Figure 4 shows the contrast hologram obtained by subtracting Fig. 3 from Fig. 2(a). The reconstructed image is shown in Fig. 5. As the background information has been subtracted, the resultant reconstruction is of better quality. Spheres lying out of the focal plane in Fig. 5 are visible as gray

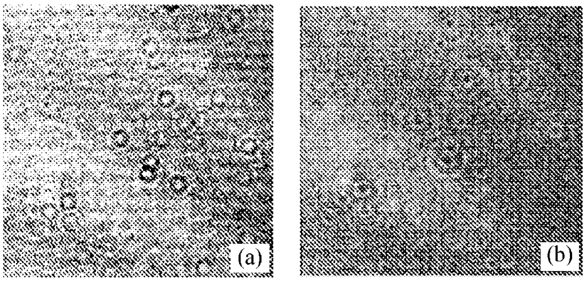

Fig. 2. Holograms and reconstruction of the air freshener. (a) The hologram; (b) the reconstructed image from (a).

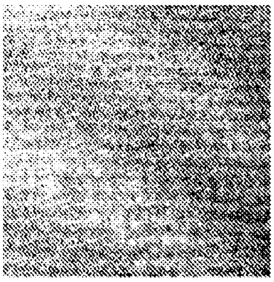

Fig. 3. The background intensity without particles sprayed.

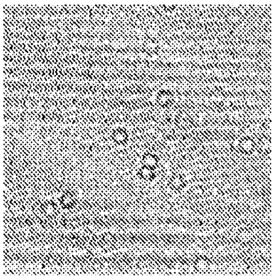

Fig. 4. The contract hologram obtained by subtracting Fig. 3 from Fig. 2(a).

patches without sharp features.

In summary, we have been investigated image formation in digital in-line holography with spherical wave recording and plane wave reconstructing, and experiment results are given. The theory analysis has been done in detail based on Fourier optical theory. Substantially, the magnified images can also be reconstructed by use of a divergent wave with different wavelength between recording and reconstructing processes. Although this property is well known in conventional holography, it can be used more flexibly in digital holography. This result indicates promising capability of microscopy and can be applied to various fields involving in cell biology, micro-particle imaging and tracking, and so on.

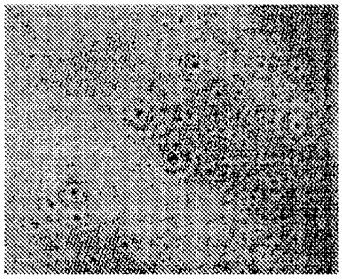

Fig. 5. The reconstructed image from Fig. 4.

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References

1. J. D. Trolinger, Opt. Eng. **14,** 383 (1975).
2. M. Adams, T. Kreis, and W. Jüptner, Proc. SPIE **3823,** 38 (1999).
3. V. Kebbel, M. Adams, H. J. Hartmann, and W. Jüptner, Meas. Sci. Technol. **10,** 893 (1999).
4. S. Murata and N. Yasuda, Opt. Laser Technol. **32,** 567 (2000).
5. R. B. Owen and A. A. Zozulya, Opt. Eng. **39,** 2187 (2000).
6. C. B. Lefebvre, S. Coëtmellec, D. Lebrun, and C. Özkul, Opt. Laser Eng. **33,** 409 (2000).
7. S. Coëtmellec, C. B. Lefebvre, D. Lebrun, and C. Özkul, Meas. Sci. Technol. **12,** 1392 (2001).
8. H. J. Kreuzer, M. J. Jericho, I. A. Meinertzhagen, and W. Xu, J. Phy.: Condens. Matter **13,** 10729 (2001).
9. Q. N. Lü, Y. M. Zhang, B. Z. Ge, H. Y. Zhao, W. G. Luo, and P. Qiao, Proc. SPIE **4929,** 481 (2002).
10. B. Z. Ge, Q. N. Lü, Y. M. Zhang, H. Y. Zhao, W. G. Luo, and P. Qiao, Proc. ISTP **3,** 451 (2002).
11. I. Yamaguchi, J. Kato, S. Ohta, and J. Mizuno, Appl. Opt. **40,** 6177 (2001).
12. W. Xu, M. H. Jericho, A. Meinertzhagen, and H. J. Kreuzer, Appl. Opt. **41,** 5367 (2002).