Spatial soliton by cascading $\chi^{(2)}$ effect and its self-induced wave-guide in quasi-phase-matched media

Kezhu Hong (洪克柱), Xianfeng Chen (陈险峰), Guangqie Gao (高光且), and Yingli Chen (陈英礼)

Institute of Optics and Photonics, Dept. of Physics, Shanghai Jiaotong University, Shanghai 200347

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The formation of the spatial solitons in the quadratic nonlinearity $\chi^{(2)}$ media by cascading second harmonic generation (SHG) in quasi-phase-matched (QPM) sample is studied on the basis of nonlinear Schrödinger equation (NLSE). When the solitary wave propagates in the QPM media, it formed optical wave-guides through cascading $\chi^{(2)}$ effect called self-induced soliton wave-guide. Transverse refractive index distribution of the self-induced soliton wave-guide of fundamental and SHG wave is obtained by cascading process. Analysis of guided-mode of such self-induced soliton wave-guide is first proposed to our knowledge. Because the power needed for forming the spatial solitons in cascading process is much lower than that in Kerr media, this kind of self-induced soliton wave-guide shows potential applications in all-optical signal process.

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Wave propagation in materials with substantial dispersion or diffraction and significant $\chi^{(2)}$ nonlinearity can be described by the nonlinear Schrödinger equation (NLSE) and its variants[1]. The NLSE has exact soliton solutions that correspond to a balance between nonlinearity and dispersion in the case of temporal solitons or between nonlinearity and diffraction in the case of spatial solitons. The equations that describe the $\chi^{(2)}$: $\chi^{(2)}$ cascaded nonlinearity can be reduced in the NLSE equation. Solitary waves in quadratic materials have attracted growing attention, because of the possibility to employ large second-order nonlinearities for the needs of all-optical switching. Such spatial solitary waves have been recently observed experimentally in a $LiNbO_3$ slab waveguide[2].

The quasi-phase-matched (QPM) technique is known as an attractive way to obtain good phase matching, and has been studied intensively[3]. The QPM technique relies on the periodic modulation of the nonlinear susceptibility or refractive index, by which an additional wave vector is introduced, which can compensate for the mismatch between the wave vectors of the fundamental and second-harmonic waves. With the QPM technique, phase matching becomes possible at ambient temperatures, and does not introduce spatial walk off. Self-induced trapping of light and formation of spatial solitions in bulk media and in planar wave-guides can utilize such QPM techniques[4]. Formation of solitons in QPM samples could open the possibility of important applications of the solitons in quadratic media, mainly by significantly reducing the power requirements.

In this paper we present the formation of spatial solitons with light beams propagating in QPM media by cascading process. We get numerical solutions for two-wave solitons. And we firstly investigate the soliton-induced wave-guide in a $LiNbO_3$ slab wave-guide of a QPM structure. We consider light beams travelling in a medium with a large quadratic nonlinearity under conditions type I QPM second-harmonic generation (SHG). In the slowly varying envelope approximation, the beam evolution can be described by the reduced normalized equations

$$i\frac{\partial \alpha_1}{\partial \xi} - \frac{r}{2}\frac{\partial^2 \alpha_1}{\partial s^2} + n(\xi)\alpha_1^* \alpha_2 \exp(-i\beta\xi) = 0,$$

$$i\frac{\partial \alpha_2}{\partial \xi} - \frac{\alpha}{2}\frac{\partial^2 \alpha_2}{\partial s^2} - i\delta\frac{\partial \alpha_2}{\partial s} + n(\xi)\alpha_1^2 \exp(i\beta\xi) = 0, \quad (1)$$

where $r = -1$, $\alpha = -k_1/k_2 \approx -0.5$, $\beta = k_1\eta^2\Delta k$ ($\Delta k = 2k_1 - k_2$), $\xi = z/k_1\eta^2$, and η is a characteristic beam width. The parameter δ accounts for the Poynting vector walk-off that occurs in birefringent media when propagation is not along the crystal optical axes and in typical QPM geometries, we set $\delta = 0$. The function $n(\xi)$ stands for the periodic sign reversal of the nonlinear coefficients involved in QPM.

QPM relies on the periodic inversion of the sign of the nonlinear $\chi^{(2)}$ coefficient at given multiples of the coherence length $l_c = \pi/|\Delta k|$, and the so-called mth-order QPM corresponds to a periodic domain inversion with period $2ml_c$. Ideally, the resulting nonlinear coefficient of the material is a steplike function along the longitudinal coordinate, as shown in Fig. 1.

The periodic normalized nonlinear coefficient $n(\xi)$ is expressed by its Fourier series expansion

$$n_m(\xi) = \sum_{l=-\infty}^{\infty} C_l \exp(ilq_m\xi), \quad (2)$$

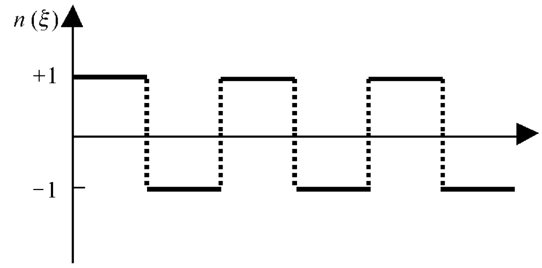

Fig. 1. The fun $n(\xi)$ ideally reverse sign from +1 to −1 periodically.

in terms of the spatial frequency $q_m = 2\pi/\Lambda_m(\Lambda_m = m2\pi/|\beta|)$. In Fig. 1 case, one has

$$C_{2l} = 0, C_{\pm(2l+1)} = \pm\frac{2}{i\pi(2l+1)}. \tag{3}$$

In fact, we should consider QPM domain lengths a bit longer(or shorter)than the nominal QPM length at a given operating frequency, so $\Lambda = \Lambda_m + \Delta\Lambda$. As a consequence, one has $q = q_m - \varepsilon$, with

$$\varepsilon = \frac{2\pi\Delta\Lambda}{\Lambda_m(\Lambda_m + \Delta\Lambda)}. \tag{4}$$

In general the biggest nonlinearity is obtained for 1st-order QPM samples, so we only consider $m = 1$ and we state $\beta \sim 10$[2,3]. Thus we can obtain

$$i\frac{\partial\alpha_1}{\partial\xi} - \frac{r}{2}\frac{\partial^2\alpha_1}{\partial s^2} - i\frac{2}{\pi}\alpha_1^*\alpha_2\exp(-i\varepsilon\xi) \approx 0,$$

$$i\frac{\partial\alpha_2}{\partial\xi} - \frac{\alpha}{2}\frac{\partial^2\alpha_2}{\partial s^2} + i\frac{2}{\pi}\alpha_1^2\exp(i\varepsilon\xi) \approx 0. \tag{5}$$

The global phases of the fundamental and the second-harmonic waves that form the families of stationary solitons in QPM samples satisfy the relation $\theta_2 \approx 2\theta_1 + \varepsilon + \pi/2$.

To reduce Eq. (5), we generally consider

$$\alpha_1 = \alpha_1(s)\exp(i\mu\xi),$$

$$\alpha_2 = i\alpha_2(s)\exp(i\mu\xi). \tag{6}$$

So we get

$$-\alpha_1 + \frac{1}{2}\alpha_1^* + \frac{2}{\pi}\alpha_1\alpha_2 = 0,$$

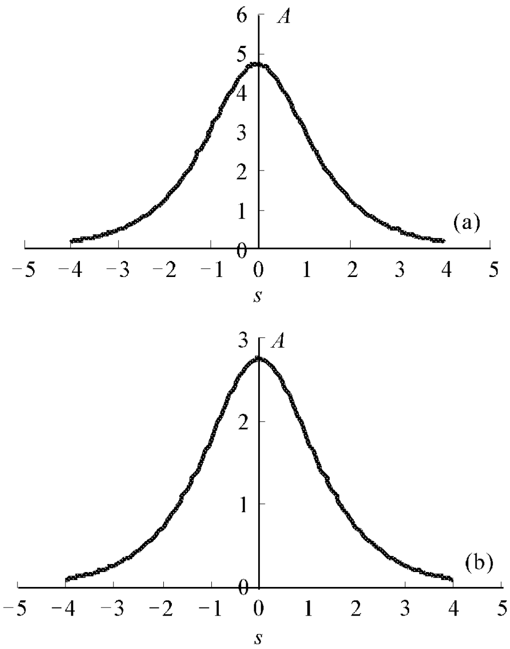

Fig. 2. Stationary soliton solutions of the fundamental and the second-harmonic waves in QMP.

$$-2\alpha_2 + \frac{1}{4}\alpha_2^* + \frac{2}{\pi}\alpha_1^2 = 0. \tag{7}$$

Figure 2 shows the shape of the α_1, α_2 ($\alpha_1(0) = 4.8$, $\alpha_2(0) = 2.8$).

As consider solitons in a concrete slab $LiNbO_3$ WG, we get estimated value of $n_2^{cascad} \approx 2.05 \times 10^{-13}$ cm^2/W. And from the lowest approximation for the value $|n_2^{cascad}| = |[2\chi^2/\pi]^2(4\pi/\lambda c\varepsilon_0)(1/n_{2\omega}n_\omega^2\Delta k)|$, we get the $\chi^{(2)} \approx 17$ pm/V.

We can easily obtain the transverse refractive index distribution of soliton wave-guide of fundamental waves from Fig. 2.

We know the Helmholtz equation on TE modes

$$\frac{d^2\Psi}{dx^2} + [n^2(x)k_0^2 - \beta^2]\Psi(x) = 0, \tag{8}$$

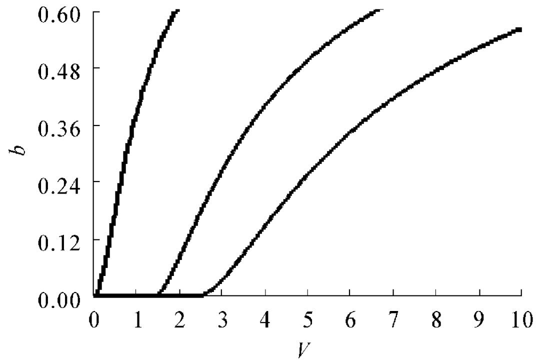

Fig. 3. Dispersion curve of spatial soliton induced wave-guide.

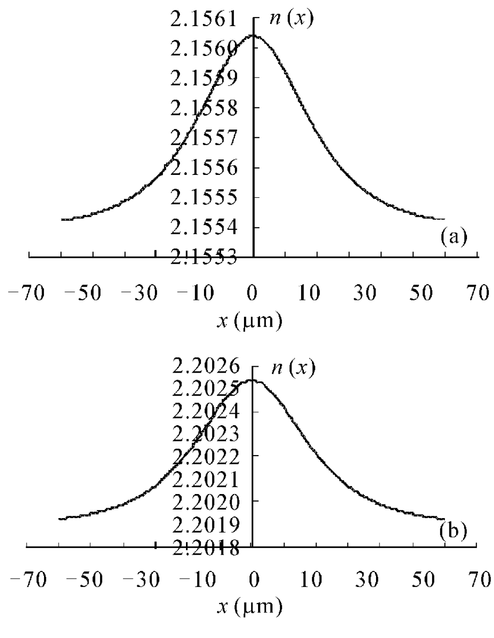

Fig. 4. Transverse refractive index distribution of fundamental soliton wave-guide. (a) $\lambda_0 = 1.064$ μm, $n_0 = 2.1554$; (b) $\lambda_0 = 0.633$ μm, $n_0 = 2.2019$. Internal intensity (after subtraction of reflection losses) is 1.65 GW/cm^2 (about ten times less than that in the KTP).

where x is the propagation direction, β is the propagation constant, and Ψ is the guide-mode E_y. From the above, we can get the relation between the normalized frequency (V) and the normalized refractive index (b) of solitary waves, as shown in Fig. 3.

For example, in Fig. 4(a), we can see the distribution of transverse refractive index , and there are two turning points. We can get the following modes equation

$$\int_{x_2}^{x_1} \kappa(x)\mathrm{d}x = (m + \frac{1}{2})\pi \quad (m = 0, 1, 2, \cdots), \tag{9}$$

where $\kappa(x) = [k_0^2 n^2(x) - \beta^2]^{1/2}$; x_1, x_2 are two turning points, and m is mode number. We get $\beta = 2.025934208$, when $m = 0$ ($\lambda = 1.064$ μm).

While $m = 1, 2, 3, \cdots$, we cannot find the appropriate value of β, so we just get only one self-induced mode in the slab $LiNbO_3$ wave-guide.

We have got the spatial solitons in the SHG by cascading process in QPM. And we also analyzed self-induced guide modes of the fundamental spatial solitary wave in concrete slab $LiNbO_3$ WG of a QPM structure. Because the power needed for the formation of the spatial soliton is greatly reduced, which is about ten times less than that in KTP media, we are easier to obtain the solitary wave-guide mode in PPLN. Thus it is helpful to apply these self-induced modes in the all optics signal process such as all optical switches.

K. Hong's e-mail address is hongkehong@mail.china.com.

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