A new method for angular displacement measurement

Caini Zhang (张彩妮) and Xiangzhao Wang (王向朝)

Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800

Received April 18, 2003

We describe a new method for angular displacement measurements that is based on a Fabry-Perot interferometer. A measurement accuracy of 10^{-8} rad is obtained by use of the sinusoidal phase modulating interferometry. Another Fabry-Perot interferometer is used to obtain the key initial angle of incidence.

OCIS codes: 120.3180, 070.6020.

A Fabry-Perot (F-P) interferometer has a simple structure. The optical path difference (OPD) between two reflections from the two faces of a F-P interferometer is a function of the incident angle on the F-P interferometer. A F-P interferometer is difficult to be used in angular displacement measurements because the initial angle of incidence must be obtained before the measurements[1]. We propose a new method for the measurement of angular displacements based on double F-P interferometers to solve this problem in this paper.

In recent years, high-precision optical interferometry has played an important role in the fields of information storage systems, robotics, micromechanics, and so on[2–4]. Among the interferometry, the advantage of the sinusoidal phase-modulating (SPM) interferometry is obvious. The SPM can be realized simply by modulating the injection current of a laser diode (LD).

In this paper, the LD-SPM interferometry is used in the double F-P interferometers to improve the measurement accuracy of angular displacements.

Figure 1 shows a F-P interferometer made of two parallel plates of glass. An object reflects a beam to it. When the object is rotated, the OPD between two transmitted beams 1 and 2 varies. The variation of the OPD is different with the different initial angle of incidence on the F-P interferometer. Figure 2 shows the setup we present for measuring the angular displacement. The light source is a LD whose wavelength is modulated sinusoidally through varying the injection current. The laser beam reflected by the object is divided into two parts by the beam splitter. The transmitted and reflected beams are incident on the F-P interferometers 1 and 2, respectively. The difference between the angles of incidence on the F-P interferometers 1 and 2 is γ. Two beams transmitted through the F-P interferometer 1 interfere with each other. When the object is rotated a angle of $\delta(t)$, the interference signal is given by

$$s_1(t) = \cos[z_1 \cos(\omega t + \theta) + \alpha_1], \quad (1)$$

where z_1 is the phase modulation depth of the interference signal, ω and θ are the frequency and the initial phase of injection current of the LD, respectively. The phase variation α_1 of the interference signal is given by

$$\alpha_1 = g\{\cos i - \cos[i + 2\delta(t)]\}, \quad (2)$$

where $g = 4\pi h/\lambda$, h is the distance between two flat plates of the F-P interferometers, λ is the central wavelength of the laser beam, and i is the initial angle of incidence on the F-P interferometer 1. The interference signal $s_2(t)$ detected by PD2 can be written as

$$s_2(t) = \cos[z_2 \cos(\omega t + \theta) + \alpha_2], \quad (3)$$

where z_2 is the phase modulation depth of the interference signal. The phase variation α_2 can be written as

$$\alpha_2 = g\{\cos(i + \gamma) - \cos[i + \gamma + 2\delta(t)]\}. \quad (4)$$

When the angular displacement $\delta(t)$ is smaller than 5°, the approximations for trigonometric functions, such as $\sin\delta(t) = \delta(t)$, $\cos\delta(t) = 1$, can be used. Equation (2) is reduced to

$$\delta(t) = \frac{\alpha_1}{2g\sin i}. \quad (5)$$

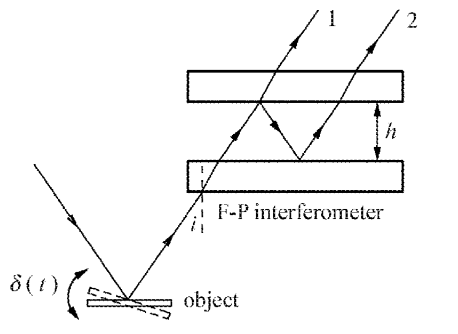

Fig. 1. A F-P interferometer for angular displacement measurement.

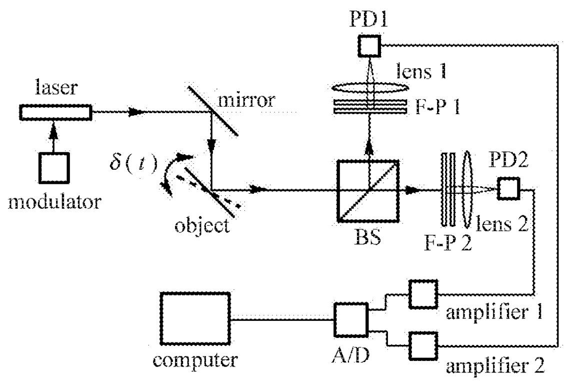

Fig. 2. Double F-P interferometers for angular displacement measurement.

$\delta(t)$ does not only depend on the phase variation α_1 but also on the initial angle i. From Eqs. (2) and (4), the initial angle of incidence i is obtained as

$$\sin i = \left(-b + \sqrt{b^2 - 4c}\right)/2, \tag{6}$$

where

$$b = \frac{\alpha_2}{g\sin\gamma} - \frac{\alpha_1}{g\tan\gamma}, \tag{7}$$

$$c = \frac{b^2 + (\alpha_1/g)^2}{4} - \frac{(\alpha_1/g)^2}{b^2 + (\alpha_1/g)^2}, \tag{8}$$

where g and γ are constants. The angular displacement $\delta(t)$ depends on the phase variations α_1 and α_2. α_1 and α_2 can be obtained through Fourier transform of the interference signals $s_1(t)$ and $s_2(t)$, respectively[5,6].

From α_1 and α_2, $\delta(t)$ can be calculated according to Eqs. (5)−(8). Therefore, the accuracy of $\delta(t)$ depends on the accuracy with which the phase variations α_1 and α_2 are measured.

Suppose that the central wavelength and the modulation frequency of the light source in Fig. 2 are 660 nm and 1000 Hz, respectively. The angular displacement of the object is

$$\delta(t) = 10^{-4}\sin(6\pi t/0.512). \tag{9}$$

Choosing $z_1 = 2.23$, $z_2 = 2.33$, $\theta = 0$, $\gamma = \pi/60$ rad, $h = 10$ mm, and $i = \pi/60$ rad, we got the interference signals $s_1(t)$ and $s_2(t)$ from Eqs. (1) and (2), respectively. Adding noises to the interference signals, we obtained the interference signals $s'_1(t)$ and $s'_2(t)$, which are shown in Figs. 3(a) and (d), respectively. The noises were the random numbers with a normal distribution. Their rms values were 3% of those of the interference signals. The amplitude distribution of a discrete Fourier transform of the noisy interference signals are shown in Figs. 3(b) and (e), respectively. The phase variations α_1 and α_2 were obtained through an inverse Fourier transform of the amplitude distributions and shown in Figs. 3(c) and (f), respectively. The initial values of α_1 and α_2 were set to zero. The amplitudes of α_1 and α_2 were 1.01192 and 2.02001 rad, respectively. The initial angle of incidence i was calculated from Eq. (6) to be 0.05233 rad. From Eq. (5) the angular displacement was obtained and shown in Fig. 4(a). Figure 4(b) is a plot of Eq. (9). The difference between rms values of the angular displacements shown in Figs. 4(a) and (b) was 3.33×10^{-8} rad.

A possible error source of this method is the measurement error of i. The differentiation of Eq. (5) gives the error of angular displacement $\mathrm{d}\delta(t)$. By using the approximations for trigonometric function, we got

$$\mathrm{d}\delta(t) = [(\alpha_1/2g)/\tan i]\mathrm{d}i, \tag{10}$$

$\mathrm{d}i = \pi/60 - 0.05236 = 3 \times 10^{-5}$ rad is the measurement error of i. The error $\mathrm{d}\delta(t)$ for $\mathrm{d}i$ was $0.52‰\delta(t)$, which was obtained from Eqs. (5) and (10).

The measurement of angular velocities of a rotating object is a possible application of this method. The angular

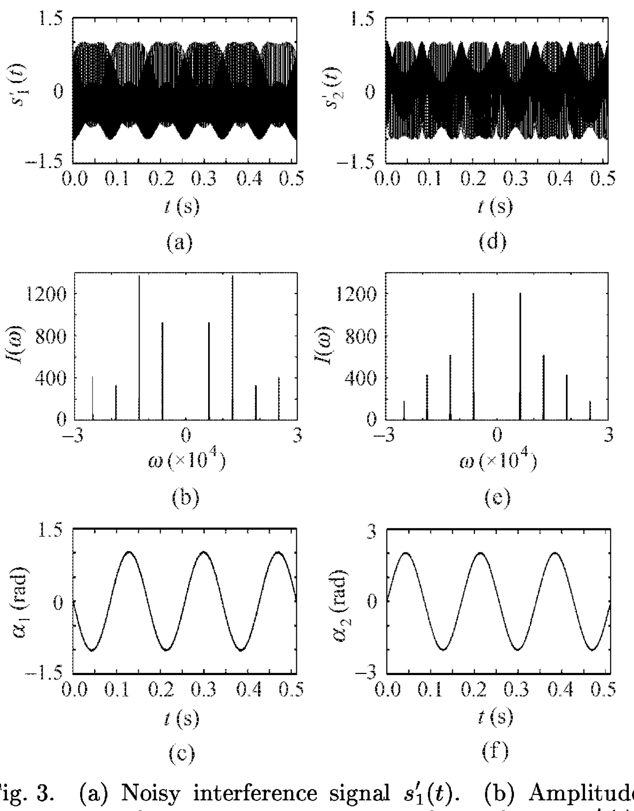

Fig. 3. (a) Noisy interference signal $s'_1(t)$. (b) Amplitude distribution of the discrete Fourier transform of signal $s'_1(t)$. (c) Phase variation α_1 obtained by computer simulations. (d) Noisy interference signal $s'_2(t)$. (e) Amplitude distribution of the discrete Fourier transform of signal $s'_2(t)$. (f) Phase variation α_2 obtained by computer simulations.

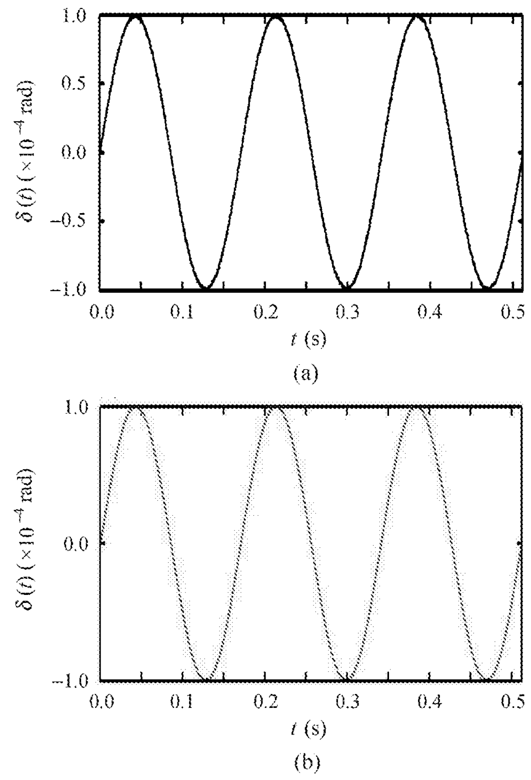

Fig. 4. Angular displacement changes with time obtained by computer simulations (a) and described by Eq. (9) (b).

velocity Ω can be determined by measuring the angular displacements of the object, using the formula

$$\Omega = \delta(t)/t. \quad (11)$$

A new measurement method for angular displacement has been proposed. The initial angle of incidence was obtained by using double F-P interferometers. The linear relationship between the phase variation of the interference signals and the angular displacement of the object has been given. The computer simulations make it clear that with this method the angular displacement can be measured with an accuracy of 10^{-8} rad.

This work was supported by the National High Technology Development 863 Program (2002AA404050) of China and the Foundation for Development of Science of Shanghai (0114nm013). C. Zhang's e-mail address is zhangcn@siom.ac.cn.

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