Influence of cladding layer field of slab waveguide on M^2 factor

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Based on the theory of semiconductor laser pattern and the non-paraxial vectorial moment theory of light beam propagation, the beam quality factor M^2 of TE_0 propagating mode is analyzed and calculated. The result shows that when both core layer and cladding layer are considered, $M^2 > 1$ is always obtained. Moreover, by analyzing the characteristic of real beams, this result is generalized to the multilayer isotropic linear slab waveguides.

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In recent years, the propagation of laser and the beam propagation factor M^2 are one of the focuses of laser technology[1]. The concept of second intensity moment and optical beam propagation factor, which Prof. A. E. Siegman introduced, advances the research in this area[2]. The second intensity moment theory expands the classical ABCD theory to the non-paraxial beam, while the beam propagation factor provides a standard to evaluate the beam quality. The combination of the two produces a general method to evaluate practical non-paraxial beams[3−5]. According to the above theory, the fundamental Gaussian mode is the limitation of diffraction, which deserves the best beam propagation factor. The M^2 factor is defined as the ratio of beam quality of practical beam to fundamental Gaussian beam. From this point of view, the M^2 factor should be greater than 1, while Ref. [6] presents a semiconductor whose M^2 of TE_0 mode is less than 1 when the core layer thickness of waveguide cavity is tiny and the amplitude in cladding layer of cavity is neglected. In fact, when the core layer thickness of waveguide cavity is tiny, the power of the optical field is mainly concentrated in the cladding layers. That is to say, the optical field in the cladding layers has determinant effect on M^2 factor and can not be neglected under this condition. Adopting the same data in Ref. [6], the results, taking cladding layers into account, are different with those in Ref. [6] when the core layer is thin. In the last part of this paper, by analyzing the contribution of optical field, the result is expanded to the beam produced by the multilayer waveguides cavity.

The cavity of semiconductor is a three-layer planar waveguide, with the thickness of core layer $2d$, refractive index n_1 and the refractive index of cladding layer n_2. When working, the electromagnetic waves are propagating and resonating along z axis in form of the eigen-mode of the waveguides cavity, getting gain in the active layer and outgoing from one end. For the semiconductor lasers, the TE_0 mode is the most important mode and when the thickness of core layer is tiny, other modes are cut off. So the following analysis and calculation only takes TE_0 mode into account. According to the Fourier theory, the far field distribution of the beam is totally decided by the outgoing wave. According to the waveguide theory, the distribution of TE_0 mode can be written as

$$E_y(x) = \begin{cases} \cos(Ux/d)/\cos(U) & |x| \le d \\ \exp(-W|x|)/\exp(-W) & |x| > d \end{cases}, \tag{1}$$

where $U^2 = (k^2n_1^2 - \beta^2)d^2$, $W^2 = (\beta^2 - k^2n_2^2)d^2$ are respectively core parameter and cladding parameter of the waveguide structure. $k = 2\pi/\lambda$ represents the wave number in vacuum and β is the propagation constant of TE_0 mode. For the convenience of analysis and without discount of validity, the $\exp(-i\omega t)$ and $\exp(i\beta z)$ items in the above formula are neglected. The angular spectrum of the outgoing wave is

$$\begin{aligned} \tilde{E}_y(\alpha) &= \tilde{E}_1(\alpha) + \tilde{E}_2(\alpha) \\ &= 2A[U\sin(U)\cos(k\alpha d)/d - \alpha k\cos(U)\sin(k\alpha d)] \\ &\quad \times \left[\frac{1}{(U/d)^2 - (k\alpha)^2} + \frac{1}{(W/d)^2 + (k\alpha)^2}\right], \end{aligned} \tag{2}$$

where A is constant and α is the spatial frequency. $|\alpha| \le 1$ represents those plane waves that can propagate along z axis with an angle of $\arcsin\alpha$, and $|\alpha|$ represents evanescent waves whose effect area is only near the outgoing end and can be neglected when analyzing the far field distribution. From the above formula, it is obvious that the angular spectrum of the outgoing wave is a combination of the angular spectrum of core layer with subscript 1 and cladding layers with subscript 2.

According to the definition, M^2 factor is decided by the multiple of the waist radius of beam W and the far-field divergence angle θ. For the TE_0 mode of the waveguide,

$$M^2 = \pi W_x(0)\tan\theta_x/\lambda_0, \tag{3}$$

where

$$W_x^2(0) = -\sqrt{\frac{\varepsilon}{\mu}}\frac{\lambda_0}{\pi^2 P}\int_{-1}^{1}\tilde{E}_y(\alpha)\frac{\partial^2\tilde{E}_y(\alpha)}{\partial\alpha^2}\mathrm{d}\alpha, \tag{4}$$

$$\tan^2\theta_x = \sqrt{\frac{\varepsilon}{\mu}}\frac{4}{\lambda_0 P}\int_{-1}^{1}\tilde{E}_y^2(\alpha)\frac{\alpha^2}{\sqrt{1-\alpha^2}}\mathrm{d}\alpha, \tag{5}$$

$$P = \sqrt{\frac{\varepsilon}{\mu}}\frac{1}{\lambda_0}\int_{-1}^{1}\tilde{E}_y^2(\alpha)\sqrt{1-\alpha^2}\mathrm{d}\alpha. \tag{6}$$

For the convenience of comparison, the same parameters in Ref. [6] are adopted. The wavelength $\lambda_0 \approx 0.9$ μm, the refractive index of core layer $n_1 = 3.950$, and the refractive index of cladding layer $n_2 = 3.385$. For symmetrical planar waveguide, there is no cutoff wavelength for TE_0 mode. For random core layer thickness $2d$ and wavelength λ, there exists TE_0 mode. Under the condition of single mode, other modes must be cut off. So the following condition must be satisfied:

$$2d < \frac{\lambda_0}{2\sqrt{n_1^2 - n_2^2}}. \quad (7)$$

The calculation shows that under this circumstance, the core layer thickness of semiconductor laser waveguides must satisfied $2d < 0.367$ μm to achieve single mode.

Figure 1 displays the result of M^2 factor. The dashed line is the result only concerns the effect of core layer, and the solid line represents the resulting concerning cladding layers. The dashed really reaches 1 when $d \approx 0.144$ μm, that is coincided with the result of Ref. [6]. While the effect of cladding layers is concerned, the result is different. M^2 factor reaches a peak at $d \approx 0.02$ μm, then decreases with the increase of d, and reaches its nadir at $d \approx 0.1$ μm. On the whole, M^2 is always greater than 1. The two curves are totally different when d is tiny. That is because when d is tiny, W/d is also tiny, and according to Eq. (2), the angular frequency of cladding layers is dominant and cannot be neglected. When d is larger, W/d increases, which causes the effect of cladding layers to weaken and the two curves incline to coincide.

Figure 2 compares the divergence angle under the two conditions. When d trends to zero, W/d follows it. According to Eq. (1), the waist of the beam maximizes. And according to Fourier optics, the divergence angle reaches its peak. As the increase of d, the divergence angle is at its minimum. The increase of d will cause the optical field in the core layer contributing and the divergence angle decrease. As d increases farther, the core layer field dominates, the divergence angle decreases. The result in this paper coincides well with FAHP divergence angle in Ref. [9]. The result shows that maximum divergence angle is a little smaller than 63.435° which is the result in Ref. [5]. It is about 60° when d is approximately 0.11 μm.

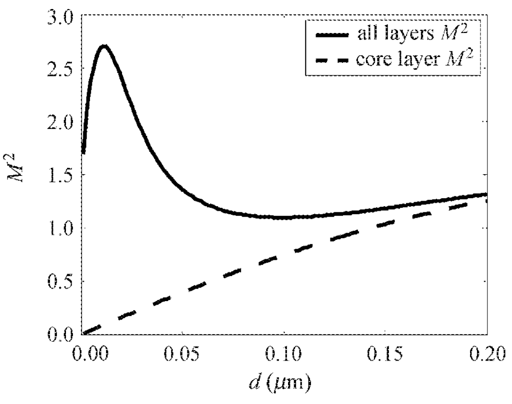

Fig. 1. Relationship between M^2 factor and the core layer thickness of cavity.

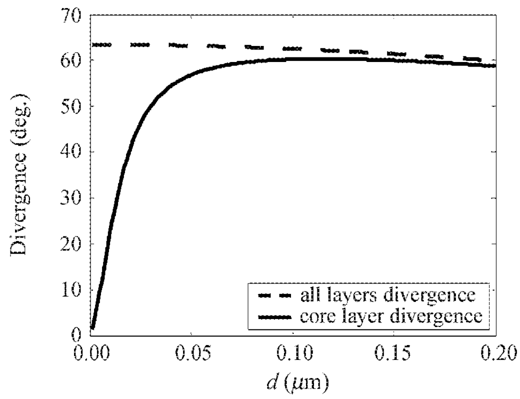

Fig. 2. The comparison of divergence angle.

Figures 3(a) and (b) respectively display the beam waist of core layer field and all layers. Neglecting the cladding layer, the waist is proportional to the thickness of the core layer. When d goes to zero, the beam waist also fades away. While the cladding layers are concerned, the beam waist never reaches zero, because of the descending field in the cladding layer. The result shows that when d goes to zero, the beam waist is remarkable. When d is close to zero, the acute variety of the beam waist and the divergence angle cause M^2 factor vibrating in this range. Overall M^2 is always larger than 1.

Under practical conditions, the step refractive index is unachievable. And four or five layers of heterostructure waveguides are used as resonance cavity. Suppose $f(x)$

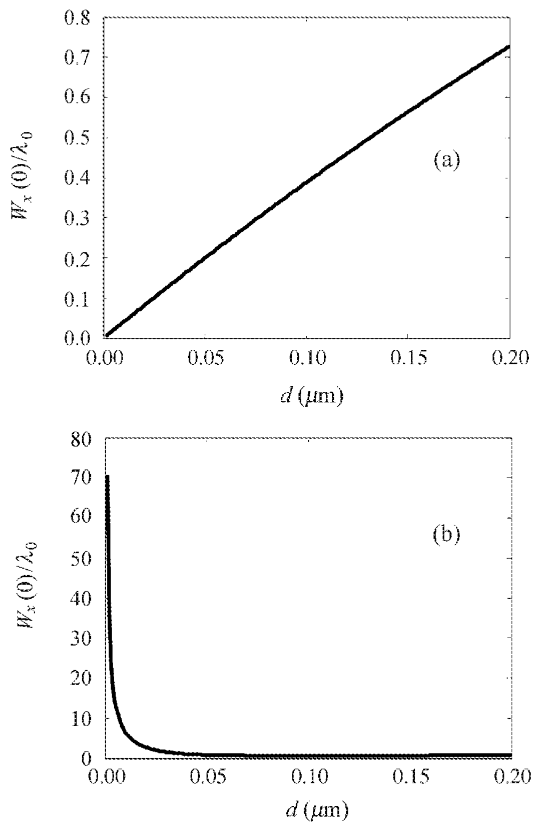

Fig. 3. (a) The beam waist of core layer field; (b) the beam waist of all layers.

is the optical field distribution in the core layers and the distribution in the cladding layers is still in the form of Eq. (2). Then the distribution and angular frequency of optical field are

$$\begin{aligned} E(x) &= f(x)\mathrm{rect}(\frac{x}{2d} + A\exp(-\frac{W_1|x|}{d})H(x-d) \\ &\quad + B\exp(-\frac{W_2|x|}{d})H(-x-d), \end{aligned} \tag{8}$$

$$\begin{aligned} \tilde{E}(\alpha) &= \tilde{E}_1(\alpha) + \tilde{E}_2(\alpha) \\ &= 2d\sin c(2d\alpha) \times F(\alpha) + \frac{2AW_1}{W_1^2+\alpha^2} \\ &\quad \times[\pi\exp(jd\alpha)\times\delta(\alpha) - j\frac{\cos d\alpha}{\alpha} + d\sin c(\frac{d\alpha}{\pi})] \\ &\quad + \frac{2BW_2}{W_2^2+\alpha^2}\times[\pi\exp(-jd\alpha)\delta(\alpha) \\ &\quad + j\frac{\cos d\alpha}{\alpha} + d\sin c(\frac{d\alpha}{\pi})], \end{aligned} \tag{9}$$

where A and B are constants, W_1 and W_2 are respectively the upper and lower cladding parameters, $H(x)$ is the Heaviside function, and $F(\alpha)$ is the Fourier transform of $f(x)$. Because the imaginary part of spatial frequency represents the change of phase, only the real part is concerned here. The first couple of $\sin c$ function appears when $\alpha = \pm\pi/2d$, so when d is tiny, this function becomes flat, and the amplitude is very small with the maximum $2d$. Under this condition, whatever $F(\alpha)$ be, the result of convolution integral has the characteristics of flatness and tiny amplitude. And when d is tiny, W_1 and W_2 both trend to zero. From Eq. (9), the amplitude of spatial frequency in cladding layers is distinctly larger than that in core layers. According to the Parseval's theorem, the energy of the optical field is mainly concentrated in the cladding layers. Thus the diffraction field is mainly decided by the cladding layers field. And the M^2 of the descending cladding field is always greater than 1. As d increases, the maximum of $\sin c$ function increases, and becomes sharper. The energy in core layers increases, the effect of cladding layers weakens and finally the result is coincided with the ones which neglect the cladding layers.

The above analysis and numerical calculation prove that when the thickness of core layer is tiny, the field in cladding layers dominates, and when d is great, the field in core layer dominates. In the process of the change of role, M^2 is always greater than 1. And through the analytical consequence, this result is generalized to multilayer semiconductor lasers.

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