# Teleportation of an unknown two-atom state using simultaneous interaction of two two-level atoms with cavity field

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Proposal for the teleportation of two-atom state is presented. It is based on the simultaneous interaction of two two-level atoms with a single-mode cavity with a filed of $n$ photons. In the proposed scheme, two pairs of EPR state are used as quantum channel to teleport an unknown two-atom state. The completed time is greatly reduced and cavity field is not required to be detected are shown to be the distinct features of the presented scheme.

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Quantum teleportation[1], a fundamental phenomenon of quantum world, is very representative in quantum communication and information processing. The so-called quantum teleportation is the process that transmits an unknown qubit from a sender (usually called Alice) to a remote receiver (usually called Bob) through quantum channel with the help of classical channel. Recently, teleportations of the polarization state of a single photon[2], the state of single-mode optical field[3], and nuclear spin state[4] have been experimentally realized. Atomic teleportation has attracted many researchers' attention although it was not demonstrated in experiment. Over the past few years, Davidovich *et al.*[5] and Cirac *et al.*[6] have respectively proposed protocols for teleporting an unknown atomic state based on resonant interaction Jaynes-Cummings model. Since then, Zheng *et al.* also presented schemes of atomic state teleportation through the Raman atom-cavity-field interaction[7] and resonant atom-field interaction[8]. In these schemes, atomic teleportation is based on the fact that atoms are sequentially injected into cavity field and the cavity field is required to be detected. These are main obstacles for the realization of atomic teleportation. Motivated by Zheng[9], we present a novel scheme for teleporting an unknown two-atom state to overcome the difficulty. In the proposed scheme, two pairs of EPR state are used as quantum channel linking two distant parties. Alternatively, we divide two teleported atoms and two atoms of EPR states at Alice's site into two pairs and sent them sequentially through cavity field with $n$ photons. After the interaction, we only perform a joint measurement on these four atoms and do not detect cavity field, and we can successfully realize atomic teleportation with certain probability.

Let us review an interesting quantum system composed of two two-level atoms and a single-model cavity with a field of $n$ photons. The effective Hamiltonian of the system in the interaction picture and dipole-dipole interaction may be described by[9]

$$H_{\text{eff}} = \lambda\Big[\sum_{j=1,2}(|e\rangle_{jj}\langle e|aa^{+} - |g\rangle_{jj}\langle g|a^{+}a) + (S_1^{+}S_2^{-} + S_1^{-}S_2^{+})\Big], \tag{1}$$

where $|e\rangle_j$ and $|g\rangle_j$ stand for the excited and ground state of $j$th atom, $S_j^{+}$ and $S_j^{-}$ are the raising and lowering operators for $j$th atom. $a^{+}$ and $a$ are respectively the creation and annihilation operators for the cavity mode. The Rabi frequency is $\lambda = g^2/\delta$, where $g$ is the atom-cavity coupling strength, $\delta$ is the detuning between the atomic transition frequency $\omega_0$ and cavity frequency $\omega$. We assume $\delta \gg g$ and $\hbar = 1$.

For convenience, we define the basis of two-atom and cavity field as

$$|\phi\rangle_1 = |gg\rangle_{12}|n+1\rangle; \quad |\phi\rangle_2 = |ge\rangle_{12}|n\rangle;$$
$$|\phi\rangle_4 = |ee\rangle_{12}|n-1\rangle; \quad |\phi\rangle_3 = |eg\rangle_{12}|n\rangle. \tag{2}$$

Here $|n\rangle$ stands for $n$ photons in cavity field. The wave function of the system may be thus expressed by

$$|\phi(t)\rangle = C_1|\phi\rangle_1 + C_2|\phi\rangle_2 + C_3|\phi\rangle_3 + C_4|\phi\rangle_4. \tag{3}$$

The time evolution of the vector $\vec{C} = (C_1, C_2, C_3, C_4)$ is given by

$$i\frac{\mathrm{d}}{\mathrm{d}t}\vec{C} = \lambda V^{I}\vec{C} \tag{4}$$

with

$$V^{I} = \begin{pmatrix} -2(n+1) & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2n \end{pmatrix}.$$

Assume that two atoms are initially in the different states, we simultaneously inject these atoms into cavity field, after an interaction time $t$, two atoms are in the following states:

$$|gg\rangle_{12} \to |gg\rangle_{12}, \tag{5}$$
$$|ee\rangle_{12} \to |ee\rangle_{12}, \tag{6}$$
$$|ge\rangle_{12} \to -i\sin(\lambda t)|ge\rangle_{12} + \cos(\lambda t)|eg\rangle_{12}, \tag{7}$$
$$|eg\rangle_{12} \to \cos(\lambda t)|ge\rangle_{12} - i\sin(\lambda t)|eg\rangle_{12}. \tag{8}$$

Here and throughout, we have discarded the factors $\exp(2ni\lambda t)$, $\exp(-2(n+1)i\lambda t)$ and $\exp(-i\lambda t)$, respectively. The time evolution of two atoms implies that two atoms and cavity field are in the disentangled state. Namely, the time evolution of the system is independent of the photon number of the cavity field. Hence, our scheme allows the cavity field to be in arbitrary state with a few photons such as a thermal state.

Now we return to teleport an unknown two-atom state using the simultaneous interaction Jaynes-Cummings model. Suppose Alice has an unknown atomic state of the form

$$|\psi\rangle_{12} = \alpha|eg\rangle_{12} + \beta|ge\rangle_{12}, \qquad (9)$$

where $\alpha$, $\beta$ are unknown to Alice except that $|\alpha|^2+|\beta|^2 = 1$. She wants to teleport the state $|\psi\rangle_{12}$ to a remote receiver Bob. To do so, she firstly sets up a quantum channel between herself and Bob. Here we use two distant EPR pairs as such a quantum channel, which may be expressed as

$$|\psi\rangle_{34} = \frac{1}{\sqrt{2}}(|eg\rangle_{34} - i|ge\rangle_{34}), \qquad (10)$$

$$|\psi\rangle_{56} = \frac{1}{\sqrt{2}}(|eg\rangle_{56} - i|ge\rangle_{56}). \qquad (11)$$

These states can be produced by the above method. For example, two atoms with atomic Rydberg state $|g\rangle_3$ and $|e\rangle_4$ are simultaneously sent into the cavity, we choose interaction time by controlling two-atom velocity such that $\lambda t = \pi/4$, and we thus obtain the state $|\Psi\rangle_{34}$. Suppose atoms 3 and 5 belong to Alice, atoms 4 and 6 to Bob. Thus the initial state of the system is

$$|\phi\rangle = |\psi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{56}. \qquad (12)$$

Before quantum teleportation, Alice divides her four atoms into two pairs and makes atom 1 and 3 one pair while atoms 2 and 5 constitute the other pair. After that, she injects two pairs sequentially into the cavity. While four atoms exit the cavity, according to Eqs. (5) – (8), the evolution of the whole system is the following state:

$$\begin{aligned}|\Psi\rangle = {} & \frac{1}{4}[\alpha|eg\rangle_{46} + \beta|ge\rangle_{46}] \otimes |eegg\rangle_{1235} \\ & + \frac{i}{4}[-\alpha|eg\rangle_{46} + \beta|ge\rangle_{46}] \otimes |geeg\rangle_{1235} \\ & + \frac{i}{4}[\alpha|eg\rangle_{46} - \beta|ge\rangle_{46}] \otimes |egge\rangle_{1235} \\ & + \frac{1}{4}[\alpha|eg\rangle_{46} + \beta|ge\rangle_{46}] \otimes |ggee\rangle_{1235} \\ & + \frac{\alpha}{2\sqrt{2}}|gg\rangle_{46}|eeeg\rangle_{1235} - \frac{\beta}{2\sqrt{2}}|ee\rangle_{46}|ggge\rangle_{1235} \\ & - \frac{i\alpha}{2\sqrt{2}}|gg\rangle_{46}|egee\rangle_{1235} + \frac{i\beta}{2\sqrt{2}}|ee\rangle_{46}|gegg\rangle_{1235} \\ & - \frac{\alpha}{2\sqrt{2}}|ee\rangle_{46}|ggeg\rangle_{1235} + \frac{\beta}{2\sqrt{2}}|gg\rangle_{46}|eege\rangle_{1235} \\ & + \frac{i\alpha}{2\sqrt{2}}|ee\rangle_{46}|eggg\rangle_{1235} - \frac{i\beta}{2\sqrt{2}}|gg\rangle_{46}|geee\rangle_{1235} \\ & - \frac{i\alpha}{2}|ge\rangle_{46}|egeg\rangle_{1235} - \frac{i\beta}{2}|eg\rangle_{46}|gege\rangle_{1235}. \end{aligned} \qquad (13)$$

Subsequently, a joint measurement on Alice's four atoms is performed by four state-selective field ionization detectors $D$. If Alice's four atoms are detected and found to be in the states $|e\rangle_1$, $|e\rangle_2$, $|g\rangle_3$, $|g\rangle_5$ or $|g\rangle_1$, $|e\rangle_2$, $|e\rangle_3$, $|g\rangle_5$ or $|e\rangle_1$, $|g\rangle_2$, $|g\rangle_3$, $|e\rangle_5$ or $|g\rangle_1$, $|g\rangle_2$, $|e\rangle_3$, $|e\rangle_5$, respectively, the state of atoms 4 and 6 at Bob's hand collapses into

$$|\Psi\rangle_{46}^{I} = \alpha|eg\rangle_{46} + \beta|ge\rangle_{46}, \qquad (14)$$

$$|\Psi\rangle_{46}^{II} = -\alpha|eg\rangle_{46} + \beta|ge\rangle_{46}, \qquad (15)$$

$$|\Psi\rangle_{46}^{III} = \alpha|eg\rangle_{46} - \beta|ge\rangle_{46}. \qquad (16)$$

Finally, Alice sends the outcome of her measurement to Bob via classical channel, Bob performs a unitary transformation on atoms 4 and 6 according to Alice's measurement outcomes and can then reconstruct the teleported state. For example, Alice's measurement outcome on atoms 1, 2, 3, and 5 is the states $|e\rangle_1$, $|e\rangle_2$, $|g\rangle_3$ and $|g\rangle_5$. Bob does nothing and reconstructs the unknown two-atom state of Eq. (9) at his site without necessitating the movement of an information carrier. The two-atom teleportation is thus successfully realized. However, if Alice measures the remaining states, the teleportation fails to success.

We should take into account the feasibility of atomic state teleportation in experiment with the existing techniques. Obviously, if the time required to complete atomic state teleportation is much shorter than that of atomic excited lifetimes and cavity dissipation, the proposed scheme may be realizable. We can choose four Rydberg atoms with principal quantum number 50 and 51, whose radioactive time is $T_r = 3 \times 10^{-2}$ s. The coupling constant of atom-cavity-field is set as $g = 2\pi \times 24$ kHz, when we choose $\delta = 10g$[10], the completed teleportation time is thus about $2\times\left(\frac{\pi\delta}{4g^2}\right) = 1.0\times10^{-4}$ s, much shorter than $T_r$, and also much shorter than the completed teleportation via interaction of four atoms, one-by-one, with a single-mode cavity. On one hand, high-quality cavity is considered to reduce cavity dissipation like superconducting microwave with a quality factor $Q > 10^{11}$ reported in Ref. [11]. For the cavity field frequency $V_c = 50$ GHz, the efficient decay time of cavity is on the order of $T_{cav} = \frac{Q}{n\omega_c} > 0.1$ s, much longer than the required time. More recently, Osnagli *et al.*[12] have experimentally realized two-atom entangled maximally state by using the technique proposed by Zheng[9]. Therefore, we think the present scheme can be realized in experiment with the existing techniques.

In conclusion, we have employed the method provided by Zheng to present a scheme for teleporting an unknown two-atom state from Alice to Bob with the help of classical information. Unlike previous schemes, the realization of atomic teleportation in our scheme is based on simultaneous interaction of two two-level atoms with a single-mode cavity with a field of $n$ photons and two pairs of EPR state function as quantum channel. Before the teleportation, we divide the teleported atoms and two atoms of EPR state at Alice's location into two pairs and inject these atoms sequentially into cavity, teleportation of unknown two-atom state with certain probability has been realized successfully while four atoms is required to be detected.

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