Time evolution of atomic inversion in a standing wave light field

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The interaction between an atomic beam of two-level atoms and a standing wave light field has been studied by the exact solution of a time-dependent quantum system developed recently. When the initial atomic state is choosen to be ground, we find that with the limit of zero detuning the atoms will oscillate between the upper and the lower levels with a decaying amplitude. The most interesting result obtained in this paper is when the initial atomic state is a particular superposition of the two levels, now the system does not oscillate at any time.

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The experimental realization of Bose-Einstein condensates [1−3] has sparked a considerable amount of theoretical and experimental efforts[4,5] in creating atom laser and exploring its optic properties. In this paper, we consider the interaction between an atomic beam of two-level atoms and a classical standing wave light field. The incident atomic beam is perpendicular to the standing wave and experiences an exchange of momentum with the photons in the light wave. We first derive the quantum state of a two-level atom in the standing wave field in terms of exact solution of a time-dependent Schrödinger equation based on the invariant Hermitian operator [6,7] and then we study the time evolution of atomic inversion and the population occupation in the upper state.

The Hamiltonian describing the interaction is[8]

$$\widehat{H} = \hbar\omega_0\sigma_z + \frac{P^2}{2m} + \hbar\Omega(\sigma_+ \mathrm{e}^{-i\omega t} + \sigma_- \mathrm{e}^{i\omega t}) \cos kx, \quad (1)$$

where P is the momentum of the centre mass of the atom along the transverse direction (x-direction), m is the mass of atom, σ_z and $\sigma_\pm$ are the pseudo spin operators, ω_0 and ω are respectively the atomic and field frequencies, $k = \omega/c$ is the wave number of the standing wave, and $\Omega = \mu\varepsilon_0/\hbar$ is the Rabi frequency with μ being the dipole moment and ε_0 the field amplitude. We shall assume that the interaction time is sufficiently small such that the transverse kinetic energy absorbed by the atom during the interaction can be neglected.

The Hamilton which we consider is

$$\widehat{H} = \omega_0\sigma_z + \Omega(\sigma_+ \mathrm{e}^{-i\omega t} + \sigma_- \mathrm{e}^{i\omega t}) \cos kx, \quad (2)$$

where natural unit $\hbar = 1$ is used throughout.

The time-evolution of quantum states is governed by Schrödinger equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left|\Psi(t)\right\rangle = \widehat{H}(t)\left|\Psi(t)\right\rangle. \quad (3)$$

Applying the following unitary transformation

$$\left|\Psi(t)\right\rangle = \widehat{R}(t)\left|\Psi'(t)\right\rangle, \quad (4)$$

$$\widehat{R}(t) = \exp\left[\frac{\gamma(t)}{2}(\sigma_+ \mathrm{e}^{-i\beta(t)} - \sigma_- \mathrm{e}^{i\beta(t)})\right], \quad (5)$$

where $\gamma(t)$, $\beta(t)$ are the time-dependent real parameters. The Schrödinger Eq. (3) becomes

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left|\Psi'\right\rangle = (\widehat{R}^+\widehat{H}\widehat{R} - \widehat{R}^+ i\frac{\mathrm{d}}{\mathrm{d}t}\widehat{R})\left|\Psi'\right\rangle. \quad (6)$$

If the parameters $\gamma(t)$ and $\beta(t)$ are related to $\Omega\cos kx, \omega_0$ and ω in the Hamiltonian by the following auxiliary equations

$$\dot{\gamma} = 2\Omega\cos kx \sin(\beta - \omega t), \quad (7)$$

$$\frac{1}{2}(\dot{\beta} - \omega_0)\sin\gamma = \Omega\cos kx\cos\gamma\cos(\beta - \omega t). \quad (8)$$

Equation (6) can be substantially simplified

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left|\Psi'\right\rangle = [\omega_0\cos\gamma - 2\Omega\cos kx\sin\gamma\cos(\beta - \omega t) + 2\dot{\beta}\sin^2\frac{\gamma}{2}]\sigma_z\left|\Psi'\right\rangle. \quad (9)$$

We now consider one of solutions of Eqs. (7) and (8)

$$\beta = 2n\pi + \omega t,$$
$$\gamma = \tan^{-1}\left(\frac{2\Omega\cos kx}{\omega - \omega_0}\right). \quad (10)$$

Substituting β and γ into Eq. (9), we obtain

$$i\frac{\mathrm{d}}{\mathrm{d}t}\left|\Psi'\right\rangle = (\omega_0\cos\gamma - 2\Omega\cos kx\sin\gamma + 2\omega\sin^2\frac{\gamma}{2})\sigma_z\left|\Psi'\right\rangle. \quad (11)$$

For two-level atoms, let $|\pm\rangle$ be the eigenstates of σ_z,

$$\sigma_\pm\left|\pm\right\rangle = \pm\frac{1}{2}\left|\pm\right\rangle. \quad (12)$$

According to Refs. [9] and [10], the general solution of Eq. (3) is written as

$$|\Psi\rangle = C_+ \mathrm{e}^{i\alpha_+(t)} \widehat{R}(t)\,|+\rangle + C_- \mathrm{e}^{i\alpha_-(t)} \widehat{R}(t)\,|-\rangle\,, \quad (13)$$

with the phase $\alpha_\pm$ defined by

$$\alpha_\pm = \int_0^t dt'\,\langle \pm|\,[\widehat{R}^+ i\frac{d}{dt}\widehat{R} - \widehat{R}^+ H \widehat{R}]\,|\pm\rangle\,, \quad (14)$$

the first part of which is known as Berry phase.

With the help of Eqs. (13) and (14), the general solution of Eq. (3) can be rewritten as

$$|\Psi(t)\rangle = \widehat{U}(t)\,|\Psi(0)\rangle\,. \quad (15)$$

The time-evolution operator is obviously

$$\widehat{U}(t) = \widehat{R}(t)\mathrm{e}^{-i\chi(t)\sigma_z}\widehat{R}^+(0)\,|\Psi(0)\rangle\,, \quad (16)$$

where

$$\chi(t) = \int_0^t \mathrm{d}t'\,(\omega_0\cos\gamma - 2\Omega\cos kx\sin\gamma + 2\omega\sin^2\frac{\gamma}{2})$$
$$= (\omega_0\cos\gamma - 2\Omega\cos kx\sin\gamma + 2\omega\sin^2\frac{\gamma}{2})t. \quad (17)$$

We first assume that the atoms are initially in the ground state with a Gaussian wavefunction

$$|\Psi(\xi,0)\rangle = (\pi\sigma^2)^{-\frac{1}{4}}\exp(-\frac{\xi^2}{2\sigma^2})\,|-\rangle\,, \quad (18)$$

where $\xi = kx$, and σ is proportional to rms transverse position spread of the input beam.

The atomic inversion is given by

$$\rho_{22} - \rho_{11}$$
$$= 2\langle\sigma_z\rangle$$
$$= 2(\pi\sigma^2)^{-\frac{1}{2}}\exp\left(-\frac{\xi^2}{\sigma^2}\right)$$
$$\times\{\langle -|\,\widehat{R}(0)\mathrm{e}^{i\chi(t)\sigma_z}\widehat{R}^+(t)\sigma_z\widehat{R}(t)\mathrm{e}^{-i\chi(t)\sigma_z}\widehat{R}^+(0)\,|-\rangle\}$$
$$= -(\pi\sigma^2)^{-\frac{1}{2}}\exp\left(-\frac{\xi^2}{\sigma^2}\right)$$
$$\times\{\cos^2\gamma + \sin^2\gamma\cos[(\omega-\omega_0)\cos\gamma + 2\Omega\sin\gamma\cos\xi]t\}.$$
$$(19)$$

We assume that the frequencies of the light and atom are the same i.e. resonance condition

$$\Delta = \omega - \omega_0 = 0. \quad (20)$$

Substituting the above equations into Eq. (19) we obtain

$$\langle 2\sigma_z\rangle = -(\pi\sigma^2)^{-\frac{1}{2}}\exp\left(-\frac{\xi^2}{\sigma^2}\right)\cos[2\Omega(\cos\xi)t]. \quad (21)$$

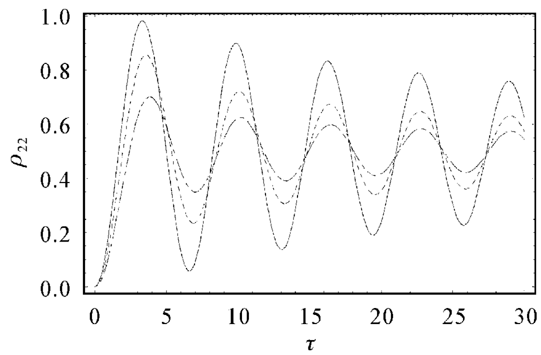

Fig. 1. Time evolution of $\langle\rho_{22}\rangle$ for the different value of the initial transverse position spread of the input atomic beam. $\sigma = 0.1$ (solid curve); $\sigma = 1$ (dashed curve); $\sigma = 10$ (dashed dotted curve).

By integrating ξ from -∞ to ∞, we get the following equation which is the exact time evolution of the atomic inversion

$$\langle 2\sigma_z\rangle = -J_0(\tau) - 2\sum_{n=1}^{\infty}(-1)^n J_{2n}(\tau)\mathrm{e}^{-n^2\sigma^2}\,, \quad (22)$$

where $\tau = 2\Omega t$ and J_m denotes the Bessel functions.

With the help of the commutation relation of the pseudospin operators, the probability of atom in the upper state can be obtained easily

$$\langle\sigma_+\sigma_-\rangle = \frac{1}{2} - \frac{1}{2}J_0(\tau) - \sum_{n=1}^{\infty}(-1)^n J_{2n}(\tau)\mathrm{e}^{-n^2\sigma^2}. \quad (23)$$

As expected, atoms coupled with light field will oscillate between the upper and the lower states. However, this oscillation can not maintain if the light field is a classical standing wave. The atomic probability in the upper or lower states will turn to be a constant for the attenuation of the amplitude of the oscillation, as shown in Fig. 1. It is also shown that the degree of this attenuation is determined by the initial transverse position spread of the input beam σ.

We then assume that the atoms are initially in the superposition state of ground and the excited states with the same probability

$$|\Psi(\xi,0)\rangle = \frac{1}{\sqrt{2}}(\pi\sigma^2)^{-\frac{1}{4}}\exp(-\frac{\xi^2}{2\sigma^2})(|+\rangle + \mathrm{e}^{i\theta}\,|-\rangle). \quad (24)$$

In the resonance excitation that

$$\Delta = \omega - \omega_0 = 0, \quad (25)$$

the atomic inversion is given by

$$\rho_{22} - \rho_{11} = 2\langle\sigma_z\rangle$$
$$= (\pi\sigma^2)^{-\frac{1}{2}}\exp\left(-\frac{\xi^2}{\sigma^2}\right)\sin\theta\sin[2\Omega t\cos\xi]. \quad (26)$$

Integrating ξ from $-\infty$ to ∞, we obtain

$$2\langle\sigma_z\rangle = 2\sin\theta\sum_{n=0}^{\infty}(-1)^n J_{2n+1}(\tau)\mathrm{e}^{-\frac{1}{4}(2n+1)^2\sigma^2}. \quad (27)$$

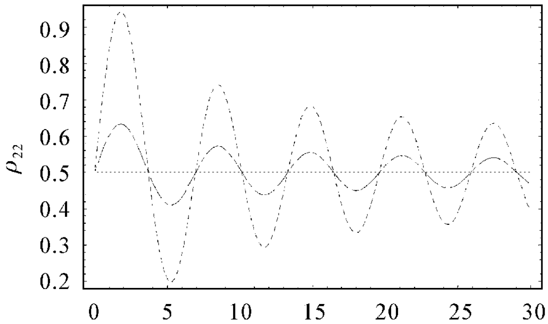

Fig. 2. Time evolution of $\langle\rho_{22}\rangle$ with the initial transverse position spread of the input atomic beam $\sigma = 1$ for the different value of the initial phase. $\theta = 0$ (solid curve); $\theta = \frac{\pi}{2}$ (dashed curve); $\theta = \arcsin(0.3)$ (dashed dotted curve).

The probability of being in the upper state is following

$$\langle\sigma_+\sigma_-\rangle = \frac{1}{2} + \sin\theta\sum_{n=0}^{\infty}(-1)^n J_{2n+1}(\tau)\mathrm{e}^{-\frac{1}{4}(2n+1)^2\sigma^2}. \tag{28}$$

It is shown that if the atoms are initially in a maximum coupled state, the atoms also will oscillate between the upper and the lower states with a decaying amplitude as they are initially in ground state. However, the degree of this attenuation is determined by initial phase θ besides the initial transverse position spread of the input beam σ, as shown in Fig. 2. If the initial phase $\theta = n\pi$, n is an integer, the atom can be localized by a classical standing wave light field.

In this paper, we have studied the interaction between an atomic beam of two-level atoms and a classical standing wave light field. Starting with the exact solution of a time-dependent Schrödinger equation, the atomic inversion is evaluated and we have found that (under the resonance condition $\Delta = 0$) the atoms beginning with various initial states will oscillate between the upper and the lower states with a decaying amplitude because of the transverse position spread of the beam; In particular, if the atoms are initially in the maximum coupled state with a relative phase $\theta = n\pi$, they will not oscillate between the upper and the lower levels at any time. This effect that atomic population occupation in the upper or lower states is trapped is called "atomic localization". Thus we find the condition which permits the atomic localization.

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