Optimization of holey fiber for dispersion compensation

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Hermite-Gaussian functions were used to calculate the dispersion of holey fibers, when pitch was more than 0.6 μm, there was dispersion inflexion at 1.55 μm (wavelength), and the relation of the dispersion inflexion versus fiber structure was given. Calculations show that the dispersion of holey fiber could reach -2300 ps/(nm·km), or they could compensate (to within $\pm 4\%$) the dispersion of 120 times of their length of standard single mode fiber over a 100-nm range.

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Group velocity dispersion (GVD) in single mode fibers limits the data transmission rate in broadband WDM system. To compensate the dispersion in transmission fibers, dispersion compensation fiber (DCF) is widely used. The minus GVD of DCF is about 10 times of that of transmission fiber, therefore, several km DCF is needed to compensate 80-km transmission fiber, and cost of system with DCF is increased observably because of the high price of DCF. In recent years, holey fiber (HF) provides some new approaches to dispersion compensation for its unique characters.

The HF is made from a single material (for example, pure silica) with an array of air holes running along their entire length. A region where one hole is missing acts as the core along which light can be guided. Since the same material is used throughout the fiber, the effective index difference between core and cladding is not limited by material incompatibilities, it is determined by the size and structure of holes in cladding. Since a large range of index (air and silica) is available, the dispersion can be flexible in a wide range.

The potential of HF used as a dispersion-compensating fiber was demonstrated in Ref. [1]. Their calculation showed that the dispersion of HF could exceed -2000 ps/(nm·km), or they could compensate (to within $\pm 0.2\%$) the dispersion of 35 times of their length of standard fiber over a 100-nm range. But their mode structure was quite simple, they used a silica core in air to resemble a HF with large air holes, which is clearly different from real HF.

Hermite-Gaussian functions were used to model the light propagation in HF[2,3], following this method, we developed our software and calculated the dispersion of HF with hexagonal lattice. Calculations show that if pitch is more than 0.6 μm, there is dispersion inflexion around 1.55 μm, where the dispersion slope changes from positive to negative value, and the dispersion can reach -2300 ps/(nm·km), or compensate the dispersion of 120 times of their length of standard single mode fiber over a 100-nm range around 1.55 μm.

The cross section of the fiber is shown in Fig. 1, d is the diameter of the air hole and Λ is the pitch of the lattice.

We assume that the HF is uniform in propagation (z) direction, therefore, the electric field can be written as

$$E_j(x,y,z) = (e_j^t(x,y) + e_j^z(x,y)\hat{z})\exp(i\beta_j z), \quad (1)$$

and assume the transverse electric field as

$$e_j^t = e_x\hat{x} + e_y\hat{y}. \quad (2)$$

Then we can obtain the following equations

$$\begin{aligned} &\left[\frac{\nabla^2}{k^2} - \frac{\beta^2}{k^2} + n^2\right] e_x \\ &= \frac{-1}{k^2}\frac{\partial}{\partial x}\left(e_x\frac{\partial \ln n^2}{\partial x} + e_y\frac{\partial \ln n^2}{\partial y}\right), \\ &\left[\frac{\nabla^2}{k^2} - \frac{\beta^2}{k^2} + n^2\right] e_y \\ &= \frac{-1}{k^2}\frac{\partial}{\partial y}\left(e_x\frac{\partial \ln n^2}{\partial x} + e_y\frac{\partial \ln n^2}{\partial y}\right), \end{aligned} \quad (3)$$

where $k = 2\pi/\lambda$ is the wave number and $n = n(x,y)$ is the transverse refractive index profile.

The transverse electric field can be expanded as

$$e^t(x,y) = \sum_{a,b=0}^{F-1}\left(\varepsilon_{ab}^x \varphi_a^m(x)\varphi_b^m(y)\hat{x} + \varepsilon_{ab}^y \varphi_a^m(x)\varphi_b^m(y)\hat{y}\right), \quad (4)$$

where F is the number of terms retained in this expansion, and φ_a^m is the elements of the orthonormal set of Hermite-Gaussian basis functions as

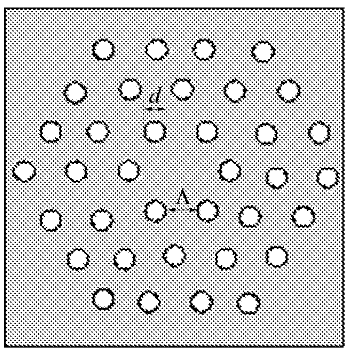

Fig. 1. Cross section of HF.

$$\varphi_a^m(x) = \frac{2^{-a}\pi^{-1/4}}{\sqrt{(2a)!w_m}} \exp\left(\frac{-x^2}{2w_m^2}\right) H_{2a}\left(\frac{x}{w_m}\right), \quad (5)$$

where H_i is the ith-order Hermite polynomial, and w_m is the characteristic width, which is taken as $\Lambda/2$.

We decompose the squared index distribution n^2 in the following way

$$n^2(x,y) = \sum_{a,b=0}^{P-1} p_{ab} \cos\left(\frac{2a\pi x}{l}\right) \cos\left(\frac{2b\pi y}{l}\right), \quad (6)$$

where P is the expanding terms, and l is the transverse extent of the structure. Substituting Eqs. (4)−(6) into (3), we obtain the following eigenvalue equation

$$M\hat{v} = \frac{\beta^2}{k^2}\hat{v}, \quad (7)$$

where $\hat{v}$ is the eigenvector, and its components is the coefficient ε_{ab} in Eq. (4), then

$$\hat{v} = (\varepsilon_{00} \cdots \varepsilon_{0F} \varepsilon_{10} \cdots \varepsilon_{FF})'. \quad (8)$$

The matrix M in Eq. (7) takes the form

$$\begin{bmatrix} M_{0000} & \cdots & M_{000F} & M_{0010} & \cdots & M_{00FF} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{0F00} & \cdots & M_{0F0F} & M_{0F10} & \cdots & M_{0FFF} \\ M_{1000} & \cdots & M_{100F} & M_{1010} & \cdots & M_{10FF} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{FF00} & \cdots & M_{FF0F} & M_{FF10} & \cdots & M_{FFFF} \end{bmatrix}. \quad (9)$$

When the decomposition in Eqs. (4) and (6) is used, the element of M becomes

$$M_{abcd} = \frac{1}{k^2} I_{abcd}^{(1)} + I_{abcd}^{(2)}, \quad (10)$$

and $I^{(1)}$ and $I^{(2)}$ are overlap integrals as

$$I_{abcd}^{(1)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi_a^m(x)\varphi_b^m(y)\nabla^2[\varphi_c^m(x)\varphi_d^m(y)]\mathrm{d}x\mathrm{d}y, \quad (11)$$

$$I_{abcd}^{(2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n^2(x,y)\varphi_a^m(x)\varphi_b^m(y)\varphi_c^m(x)\varphi_d^m(y)\mathrm{d}x\mathrm{d}y$$

$$= \sum_{f,g=0}^{P-1} p_{fg} I_{fac}^{(21)} I_{gbd}^{(21)}, \quad (12)$$

$$I_{i_1 i_2 i_3}^{(21)} = \int_{-\infty}^{\infty} \cos\left(\frac{2i_1\pi x}{l}\right) \varphi_{i_2}^m(x)\varphi_{i_3}^m(x)\mathrm{d}x. \quad (13)$$

Then the electric field distribution $E(x,y)$ can be obtained by solving the eigenvalue equation (7), and the dispersion can be calculated using the following prescription

$$\mathrm{GVD} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{1}{v_g}\right), \quad (14)$$

$$v_g = \frac{c\beta}{k} \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E^2 \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n^2 E^2 \mathrm{d}x\mathrm{d}y}. \quad (15)$$

Calculations show that the dispersion of HFs can be less than zero if air holes are large and core is small enough, whereas the dispersion slope can be negative or positive value. Figure 2 shows two examples, both Λ in the figure are 0.8 μm. In Figs. 2 − 4, the x-axis is wavelength (in μm), and the y-axis is dispersion (in ps/(nm·km)).

Figure 2 shows that, for the same pitch Λ, the sign of dispersion slope can be changed if the air hole diameter d changed. Calculations show that there is an inflexion of dispersion, where the slope (changing from negative to positive) is zero (shown in Fig. 3). If d/Λ is less than that of inflexion, the slope is positive; and if d/Λ is larger than that of inflexion, the slope is negative.

We calculated the dispersion of HF with different Λ (from 0.1 to 10 μm). It is shown that if $\Lambda < 0.6$ μm, the dispersion slope is always positive (Fig. 4(a)); and if $\Lambda > 2.5$ μm, the dispersion is always positive (Fig. 4(b)), which is useless for dispersion compensation.

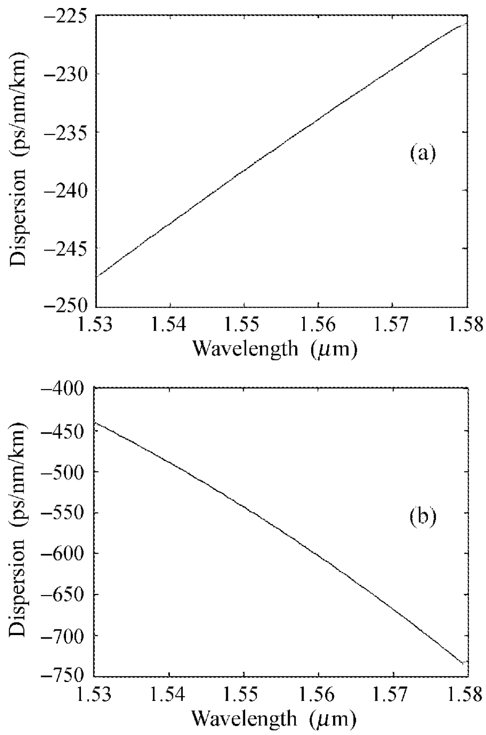

Fig. 2. Dispersion of hexagonal HF. (a) $d/\Lambda = 0.6$; (b) $d/\Lambda = 0.9$.

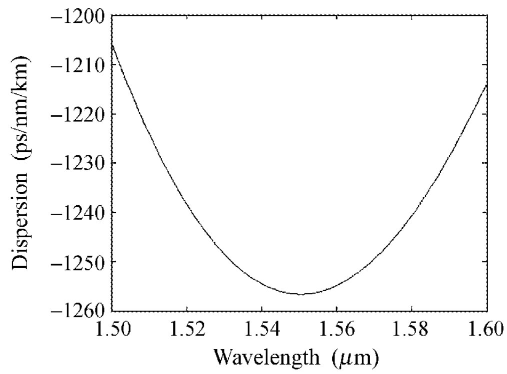

Fig. 3. Inflexion of dispersion slope.

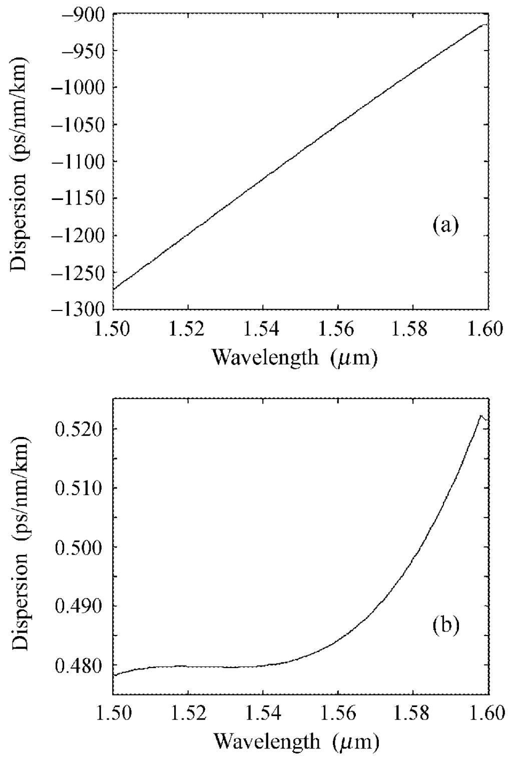

Fig. 4. (a) $\Lambda = 0.5$, $d/\Lambda = 0.999$ positive dispersion slope; (b) $\Lambda = 2.6$, $d/\Lambda = 0.32$ positive dispersion.

Therefore, the available range of Λ is $0.6 \leq \Lambda \leq 2.5$ μm. The critical d/Λ at dispersion inflexion and the dispersion value versus Λ are shown in Fig. 5. We can see that the dispersion reaches -2300 ps/(nm·km), exceeding that of Ref. [1] (-2000 ps/(nm·km)).

Finding the dispersion inflexion is significant to dispersion compensation using HF. For standard single mode fiber, the dispersion and dispersion slope are both positive at 1.55 μm, since the dispersion and dispersion slope of HF are both negative if d/Λ is larger than the critical value. If we choose d/Λ to be more than the critical value, the dispersion and dispersion slope can both be compensated. For example, if we choose $\Lambda = 0.8$ μm, from Fig. 5, the corresponding d/Λ is 0.82, here we choose d/Λ=0.835. Calculations show that $D = -2050$ ps/(nm·km) at 1.55 μm, which means that it can compensate the dispersion of 120 times of its length of standard single mode fiber ($D = 17$ ps/(nm·km)). Furthermore, it can do this to within $\pm 4\%$ over the entire 100-nm range centered at 1.55 μm (dispersion slope is about -6 ps/(nm^2·km)), its compensated

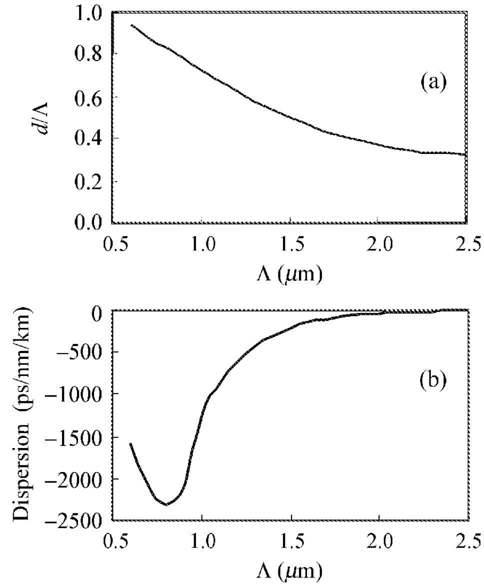

Fig. 5. (a) The critical d/Λ at dispersion inflexion versus Λ; (b) the dispersion D at dispersion inflexion versus Λ.

length is about 3 times of that of Ref. [1]. In Ref. [1], only the core diameter can be adjusted. However, in this paper, both the pitch Λ and the hole diameter d/Λ can be adjusted, the freedom of design is increased.

We calculated the dispersion of HF, and found that for hexagonal HF, there were dispersion inflexions when $\Lambda \geq 0.6$ μm, the relation of the dispersion inflexion versus fiber structure was also given. For certain Λ, when d/Λ is less than critical value, the dispersion slope is positive; when d/Λ is larger than critical value, the dispersion slope is negative. Calculations show that the dispersion of HF can exceed -2300 ps/(nm·km), or they can compensate (to within $\pm 4\%$) the dispersion of 120 times of their length of standard single mode fiber over a 100-nm range. We can see that HF has considerable potential in dispersion compensation.

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