Effect of phase fluctuation on Hopf bifurcation in a ladder type atomic system

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Hopf bifurcation inducing lasing without inversion has been analyzed by taking into account the effect of phase fluctuation in the driving field based on a closed three level ladder-type atomic model. It is shown that due to the phase fluctuation of the driving field, the necessary threshold increases significantly. Furthermore the area domain to get lasing without inversion decreases as the driving field's linewidth increases.

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Since the seminal works of Kocharovskaya, Harris and Scully *et al.*[1−3], lasing without inversion (LWI) has attracted tremendous attention, beacause it can provide a new tool in the pursuit of tabletop UV or γ -ray laser and has other interesting statistical properties[4−13]. Recently Mompart *et al.* studied instability in the closed resonant lower-ladder system and found that the destabilization of the nonlasing solution can occur through a Hopf bifurcation giving rise to self-pulsing emission, but in the study the phase of the driving field is fixed[14]. However, in fact the phase of the driving field is generally variable. It is well known that phase fluctuation of the driving field can significantly influence the optical properties of driven atomic systems[15,16]. Fleischhauer *et al.* studied the influence of pump-field phase diffusion on laser gain in a closed V three level system, and found that the phase diffusion leads to a decay of the coherent trapping state and the gain is reduced[17]. Gong *et al.* found that in a simple three-level atomic system, a change from an inversion laser to a noninversion laser action can occur as the driving-field linewidth increases[18] and the linewidth of the driving field precludes the medium from becoming transparent[19,20]. In this paper, we investigate the nonlinear dynamic behavior of LWI by taking into account phase fluctuation of the driving field based on a closed three level atomic model. We find that due to the phase fluctuation of the driving field, the necessary threshold is increased significantly. Furthermore the area domain to get LWI is decreased.

We consider the closed lower-ladder system. The medium levels are named 1, 2, 3. The driving field with Rabi frequency Ω is resonant with transition 1-2, and the medium placed inside a unidirectional ring laser cavity is resonant with the lasing transition 1-3. An incoherent pump field with a pumping rate Λ and a weak coherent laser field with Rabi frequency α are coupled between levels 1 and 2. The transition between levels 2 and 3 is forbidden. We seek solutions of the problem so that the slowly varying density matrix element amplitudes of the medium in the interacting picture have the forms

$$\rho_{21} = -ix_{21}, \rho_{31} = -ix_{31}, \rho_{32} = x_{32}, \qquad (1)$$

where x_{21}, x_{31} and x_{32} are real time-dependent variables. In our notation, if $x_{31} > 0$, the system exhibits gain for the probe field; If $x_{31} < 0$, the probe field is attenuated. In the rotating-wave slowly varying envelope and mean field approximations, this scheme is governed by the set of Maxwell-Bloch equations

$$\begin{aligned}
\mathrm{d}\rho_{11}/\mathrm{d}t &= R_1 + 2\Omega x_{21} + 2\alpha x_{31}, \\
\mathrm{d}\rho_{22}/\mathrm{d}t &= R_2 - 2\Omega x_{21}, \\
\mathrm{d}x_{21}/\mathrm{d}t &= -\gamma_d x_{21} - \Omega(\rho_{11} - \rho_{22}) + \alpha x_{32}, \\
\mathrm{d}x_{31}/\mathrm{d}t &= -\gamma_a x_{31} + \alpha(1 - 2\rho_{11} - \rho_{22}) + \Omega x_{31}, \\
\mathrm{d}x_{32}/\mathrm{d}t &= -\gamma_b x_{32} - \Omega x_{31} - \alpha x_{21}, \\
\mathrm{d}\alpha/\mathrm{d}t &= -k\alpha + g x_{31}. \qquad (2)
\end{aligned}$$

In above equations, the closure relation $\rho_{11}+\rho_{22}+\rho_{33} = 1$ has been used. k and g signify the damping rate of the lasing field due to cavity loss and the unsaturated gain of the lasing transition, respectively. In the radiative limit, the atomic polarization damping rates γ_a, γ_b, and γ_d can be expressed as

$$\begin{aligned}
\gamma_a &= (w_{31} + w_{21} + w_{13})/2, \\
\gamma_b &= (w_{12} + w_{13})/2, \\
\gamma_d &= (w_{31} + w_{21} + w_{12})/2. \qquad (3)
\end{aligned}$$

The expressions of R_1 and R_2 are

$$\begin{aligned}
R_1 &= P\rho_{11} + Q\rho_{22} + w_{13}, \\
R_2 &= w_{21}\rho_{11} - w_{12}\rho_{22}, \qquad (4)
\end{aligned}$$

where

$$P = -(w_{21} + w_{31} + w_{13}), Q = w_{12} - w_{13}. \qquad (5)$$

w_{12}, w_{21}, w_{13} and w_{31} in expressions (3), (4) and (5) depend upon the particular choice of the schemes. For the lower-ladder scheme (Fig. 1),

$$w_{12} = 0, w_{21} = \gamma_{12}, w_{13} = \gamma_{31} + \Lambda, w_{31} = \Lambda. \qquad (6)$$

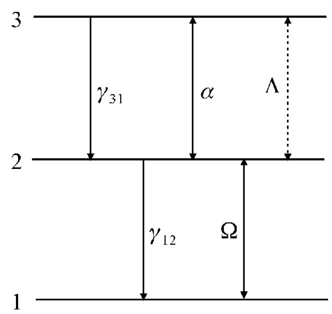

Fig. 1. The closed three level ladder-type atomic model. γ_{31} and γ_{12} are the spontaneous decay rates, Λ is the incoherent pump rate and $\alpha(\Omega)$ is Rabi frequency of the probe (driving) field.

Here γ_{ij} denotes the decay rate from level i to j, and Λ is the incoherent pump rate.

The above analysis assumed that the phase of the driving field is fixed. However this is impractical. Let $\phi(t)$ represent the phase fluctuation of the driving field, i.e.

$$\Omega' = \Omega \exp[i\phi(t)], \quad (7)$$

here Ω is assumed to be real. The phase is characterized by the following random equation of motion[21],

$$\dot{\phi}(t) = \mu(t) \quad (8)$$

with zero average, i.e., $\langle \mu(t) \rangle = 0$. Here $\mu(t)$ is a δ-correlated Langevin-noise term, whose diffusion coefficient gives the linewidth $2R_L$ of the driving field, i.e.

$$\langle \mu(t)\mu(t') \rangle = 2R_L \delta(t - t'). \quad (9)$$

In this case, the Maxwell-Bloch equations should be averaged over the random fluctuation phase. That is, the density-matrix elements ρ_{ii}, ρ_{ij} must be replaced by their stochastic averaged values $\langle \rho_{ii} \rangle, \langle \rho_{ij} \rangle$ respectively. By using the method of Refs. [19, 20, 22 − 24], it can be known that the phase fluctuation in the driving field modifies the off-diagonal rates γ_d and γ_b to $\gamma_d' = \gamma_d + R_L$ and $\gamma_b' = \gamma_b + R_L$ respectively. In other words, because of the phase fluctuation of the driving field, the off-diagonal decay rates now have additional diffusion terms along with the usual decay rates. In the subsequent section, we will discuss the effect of phase fluctuation on the driving field.

Setting α and all time-derivatives in Eq. (2) be zero, the nonlasing solution is

$$x_{31} = x_{32} = 0, x_{21} = x_{21}^0 = \Omega(\rho_{22}^0 - \rho_{11}^0)/\gamma_d',$$

$$\rho_{11}^0 = [2\Omega^2 w_{13} + \gamma_d/w_{12}w_{13}]/D,$$

$$\rho_{22}^0 = [2\Omega^2 w_{13} + \gamma_d' w_{21} w_{13}]/D,$$

$$\rho_{33}^0 = 1 - \rho_{22}^0 - \rho_{11}^0, \quad (10)$$

where

$$D = -2\Omega^2(P + Q + w_{21} - w_{12})$$
$$-\gamma_d'(w_{12}P + w_{21}Q). \quad (11)$$

In the following, the population difference defined by

$$n_{ij} = \rho_{ii}^0 - \rho_{jj}^0, \quad (12)$$

is used. The noninversion condition implies $n_{13} > 0$.

From the nonlinear dynamics viewpoint, a lasing solution always corresponds to a loss of stability of the nonlasing stationary solution. Equation (2) is linearized about the solution (10). The resulting Jacobian matrix can be split into two independent submatrices. One of them governs the stability of the variables α, x_{31} and x_{32} and therefore the generation of the lasing field. The characteristic polynomial of this sub-matrix is

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0, \quad (13)$$

with the coefficients

$$A_2 = k + \gamma_a + \gamma_b',$$

$$A_1 = k(\gamma_a + \gamma_b') + \gamma_a\gamma_b' + \Omega^2 + gn_{13},$$

$$A_0 = K(\Omega^2 + \gamma_a\gamma_b') + g(\gamma_b' n_{13} + n_{21}\Omega^2/\gamma_a). \quad (14)$$

We apply the Hurwitz criteria for determining the instabilities associated with the above polynomial: $A_2, A_1, A_0 > 0$ and $H_2 = A_2A_1 - A_0 > 0$ signify negative real parts of all eigen- values which means the stability of the nonlasing solution. From Eq. (14), we know that A_2 and A_1 are always positive (for noninversion $n_{13} > 0$). The destabilization of the trivial solution occurs through a pitchfork bifurcation (static instability) if $A_0 < 0$, or, alternatively, through a Hopf bifurcation (self-pulsing instability) if $H_2 < 0$. In this case, $\sqrt{A_1}$ gives the angular pulsation frequency of the lasing field at the destabilization point. Here

$$H_2 = (\gamma_a + \gamma_b')[k(k + \gamma_a + \gamma_b') + \gamma_a\gamma_b' + \Omega^2]$$
$$+g[(k + \gamma_a)n_{13} - \Omega x_{21}]. \quad (15)$$

Mompart *et al.* pointed out that for a closed lower-ladder scheme, the destabilization of the nonlasing solution occurs only via a Hopf bifurcation, giving rise to the self-pulsing emission[14]. From Eq. (15), as $n_{13} > 0$ (noninversion), in order to satisfy the condition of Hopf bifurcation ($H_2 < 0$), $x_{21} > 0$ (noninversion between levels 1 and 2) is necessary. We can suitably select parameters' values of the system to make $H_2 < 0$. For $\Omega = 0$ as well as for very large value of Ω^2, H_2 is positive and the nonlasing solution is stable. Figure 2 illustrates different curves $H_2 = 0$ as a function of the linewidth of the driving field, where the value of g is given in MHz^2 and the other parameters are given in MHz. For a given value of the linewidth, for example $R_L = 0$, LWI is obtained within a closed curve in the $\Omega - \Lambda$ plane ($H_2 < 0$). Outside this curve the nonlasing solution is stable. With the increment of the linewidth, the threshold of the pump field or the driving field clearly increases. Furthermore, the area domain to get LWI is significantly decreased. For given parameters in this paper, LWI will be impossible when linewidth excesses 5.0, no matter how strong the driving field or the pump field is. So the effect of phase fluctuation in driving field is not neglectable. In practical experiment for LWI, it is necessary to reduce the phase diffusion of the driving field.

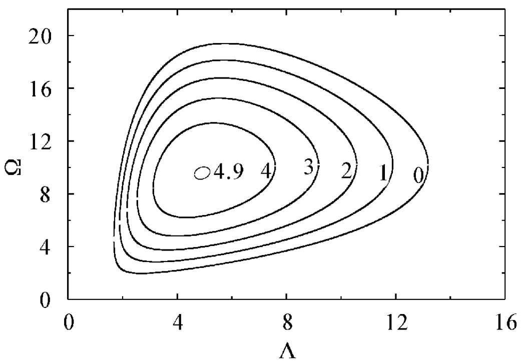

Fig. 2. LWI regions in the $\Omega - \Lambda$ plane for several values of linewidth. Every curve corresponds to R_L=0, 1, 2, 3, 4 and 4.9 from outside to inside, respectively. The other parameters' values are $k = 0.5, g = 10000, \gamma_{31} = 3.5, \gamma_{12}$=19.

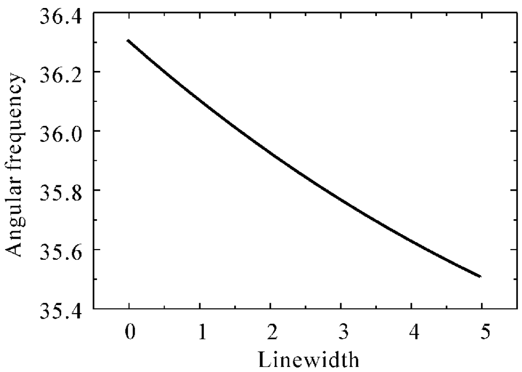

Fig. 3. The angular frequency of the lasing field versus the linewidth of the driving field. $\Omega = 10, \Lambda = 5$. The other parameters' values are the same as those in Fig. 2.

Mompart *et al.* investigated the evolution of the LWI laser field amplitude using a direct numerical integration of Eq. (2) and found that the laser field oscillates around zero with an angular frequency after a transient time. Now we study the effect of the phase fluctuation on the angular frequency and get the angular pulsation frequency as the function of the linewidth of the driving field, as shown in Fig. 3. It shows that the angular frequency is monotonously lowered with the increment of the linewidth. So the phase fluctuation in the driving field will decrease the oscillating angular frequency of the self-pulsing LWI laser output.

In conclusion, we investigate the nonlinear dynamics behavior of LWI by taking into account the effect of phase fluctuation in the driving field based on a closed three level ladder-type atomic model. It is shown that due to the phase fluctuation of the driving field, the necessary threshold is increased significantly. Furthermore the area domain to get LWI is decreased. The fact that the phase fluctuation on the driving field decreases the angular frequency of output laser is also found.

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