The optimum scheme of a static Fourier-transform spectrometer based on birefringent crystal

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An optimum design of static Fourier-transform spectrometer based on Savert prisms is presented in this paper. A new method of increasing path difference and resolution of spectrometer is given. When the angle between the crystal optical axis of the first Savert prism and the incident interface is 58° and the angle between the crystal optical axis of the second Savert prism and the incident interface is 28°, the maximum path difference will be 0.63 mm, the maximum resolution will be 15.8 cm^{-1}, and the whole field-of-view will reach 6°.

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Fourier-transform spectrometers based on Michelson interferometers are frequently used in both industrial and scientific applications[1]. But the spectrometer has two drawbacks. One is that the interferometer requires a complicated scanning mechanism with exceptionally high quality and well isolated from external perturbation. The other is that the measurements of transient phenomena, such as short laser pulses or explosion spectra, are not easy to be realized. In order to overcome the difficulty, SMII (Spatially Modulated Imaging Interferometer) has been invented in 1980's[2]. In the paper, a kind of SMII, static Fourier-transform spectrometer based on birefringent crystal, has been discussed. Its operating principle is using some polarizer devices to split the object into two virtual sources. By Fourier transform, spectrum can be obtained from the interferogram. The instrument is small, light, stable and integrated. In this paper, We describe a new optimum design of the spectrometer based on Savert prism[3].

Resolution and field-of-view are two important parameters of a spectrometer. Resolution can be indicated by wave number[4]

$$R = \frac{\nu}{\Delta\nu}, \tag{1}$$

where ν is average wave number of two adjacent spectral lines which can just be separated; $\Delta\nu$ is wave number difference between the two adjacent spectral lines, it can be described by

$$\Delta\nu = \frac{1}{\Delta_{\max}}, \tag{2}$$

where $\Delta_{\max}$ is the largest path difference of the spectrometer. We can see from Eqs. (1) and (2) that the larger the path difference is, the higher the resolution will be.

For the goal of seeking a method to enlarge optical path difference, we analyze the propagation direction of o-ray and e-ray in crystals by ray-tracing method[5-7] and work out the path difference of two coherent beams.

Figure 1 indicates the propagation path of e-ray and o-ray in Savert prism. OA and CE are the paths of o-ray, OB and AF are the directions of e-wave normal, OC and BD are the paths of e-ray. The unit direction vector of incident light is $S_0(\cos\alpha_0, \cos\beta_0, \cos\gamma_0)$, where α_0, β_0, and γ_0 are the angles between incident light and X, Y, Z axis. $\cos\alpha_0$, $\cos\beta_0$, and $\cos\gamma_0$ are direction numbers. YOZ is the incidence plane, and we can get $\gamma_0 = 90°$, so the direction vector of the incident light is $(\cos\alpha_0, \cos\beta_0, 0)$, $\alpha_0 + \beta_0 = 90°$. In general, light beams are not symmetrical about system light axis, so in order to discuss the light beams propagation on both sides of the axis, we agree to the following rules: from the light beam to the normal line, if the rotation direction is clockwise, the angle between the two vectors is positive; if it is counter clockwise, the angle is negative.

In the first Savert prism, the crystal optical axis is in XOY plane and the angle between the optical axis and OY is 45°. In order to have a general discussion and produce an optimum design, we suppose the variable angle denoted with δ_1, and call the device as a parallel polarization splitter. Its crystal optical axis is $\mathbf{C_1} = (\sin\delta_1, \cos\delta_1, 0)$, $\delta_1 \subset [0, 90°]$. In anisotropic crystal, a ray will be split into o-ray and e-ray.

O-ray: In the first prism, the direction vector $\mathbf{K_{1o}}$ of o-ray is $(\cos\alpha_{1o}, \cos\beta_{1o}, \cos\gamma_{1o})$. According to Snell's law, we get

$$\begin{aligned} n_i \sin\alpha_0 &= n_o \sin\alpha_{1o}, \\ \beta_{1o} &= 90° - \alpha_{1o}, \\ \gamma_{1o} &= 0. \end{aligned} \tag{3}$$

E-ray: We suppose the e-wave normal vector is $\mathbf{K_{1e}}$ $(\cos\alpha_{1e}, \cos\beta_{1e}, \cos\gamma_{1e})$. Refractive index of e-wave is[8]

$$n_{1e} = \frac{n_o n_e}{\sqrt{n_o^2 \sin^2\theta_1 + n_e^2 \cos^2\theta_1}}, \tag{4}$$

where θ_1 is the angle between e-wave normal $\mathbf{K_{1e}}$ and crystal optical axis $\mathbf{C_1}$, given by

$$\begin{aligned} \cos\theta_1 &= \cos(\mathbf{K_{1e}}, \mathbf{C_1}) \\ &= \cos\alpha_{1e}\sin\delta_1 + \sin\alpha_{1e}\cos\delta_1. \end{aligned} \tag{5}$$

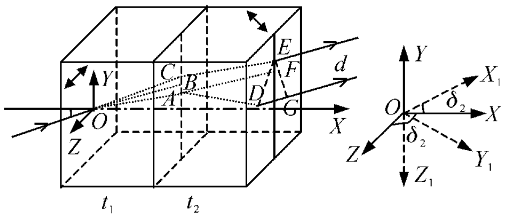

Fig. 1. Trace followed by ordinary and extraordinary rays in the Savert splitter.

From Eqs. (4), (5) and Snell's law, we can get the refraction angle of e-wave

$$\begin{aligned}\alpha_{1e} = a\cot\{&(n_o^2 - n_e^2)\sin 2\delta_1 \\ &+2n_o n_e[(n_e^2 \sin^2\delta_1 \\ &+n_o^2\cos^2\delta_1)/(n_i^2\sin^2\alpha_0) - 1]^{1/2}\} \\ &/2(n_e^2\sin^2\delta_1 + n_o^2\cos^2\delta_1),\end{aligned} \quad (6)$$

where $n_i = 1$ is refractive index in the air. Then we can get the direction vector of e-ray $\mathbf{S_{1e}}$ $(\cos\alpha_{1et}, \cos\beta_{1et}, \cos\gamma_{1et})$, where $\alpha_{1et} = \alpha_{1e} + \eta_1$, η_1 is e-ray's wall-off angle in the first prism[8].

The crystal optical axis of the second prism is in XOZ plane and the angle between optical axis and OZ is denoted by δ_2. The direction vector $\mathbf{C_2}$ of the crystal optical axis is $(\sin\delta_2, 0, \cos\delta_2)$. When o-ray and e-ray transmit into the second prism, the o-ray turn into e-ray and the e-ray turn into o-ray.

O-ray: Its direction vector is $(\cos\alpha_{2o}, \cos\beta_{2o}, \cos\gamma_{2o})$, from Snell's law, we know its propagation direction is identical with the o-ray's in the first prism. That is to say OA is parallel to CE.

E-ray: The e-wave is in incident plane XOY, and its direction vector is $\mathbf{K_{2e}}$ $(\cos\alpha_{2e}, \cos\beta_{2e}, \cos\gamma_{2e})$. In the same way, we can get the refraction angle of e-wave in the second prism by

$$\alpha_{2e} = \arccos\left[\left(\frac{n_o^2(n_e^2 - \sin^2\alpha_0)}{(n_e^2 - n_o^2)\sin^2\delta_2\sin^2\alpha_0 + n_o^2 n_e^2}\right)^{1/2}\right]. \quad (7)$$

The e-ray direction vector in the second prism is $\mathbf{S_{2e}}$ (S_X, S_Y, S_Z). In order to get S_X, S_Y and S_Z, we need to establish a new coordinate system $X_1Y_1Z_1$ (see Fig. 1). In the new coordinate, the direction vector of $\mathbf{S_{2e}}$ is (S_{X1}, S_{Y1}, S_{Z1}); the direction vector of $\mathbf{K_{2e}}$ is $(k_{eX1}, k_{eY1}, k_{eZ1})$; the direction vector of crystal optical axis is $(0, 1, 0)$. By coordinate conversion[9], we get

$$\begin{bmatrix} k_{eX1} \\ k_{eY1} \\ k_{eZ1} \end{bmatrix} = \begin{bmatrix} \cos\delta_2 & 0 & -\sin\delta_2 \\ \sin\delta_2 & 0 & \cos\delta_2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} k_{eX} \\ k_{eY} \\ k_{eZ} \end{bmatrix}. \quad (8)$$

The e-ray's wall-off angle in the second prism is

$$\begin{aligned}\cos\eta_2 &= \cos(\mathbf{K_{2e}}, \mathbf{S_{2e}}) \\ &= k_{eX1}S_{X1} + k_{eY1}S_{Y1} + k_{eZ1}S_{Z1}.\end{aligned} \quad (9)$$

From Eq. (9) and considering $\mathbf{K_{2e}}$, $\mathbf{S_{2e}}$, $\mathbf{C_2}$ in one plane, we get

$$\begin{cases} S_{X1} = k_{eX1}\cos\eta_2 - \frac{k_{eX1}k_{eY1}\sin\eta_2}{\sqrt{k_{eX1}^2 + k_{eZ1}^2}} \\ S_{Y1} = (\sin^2\eta_2 + k_{eY1}^2\cos 2\eta_2 \\ \qquad + \sqrt{k_{eX1}^2 + k_{eZ1}^2}\,k_{eY1}\sin 2\eta_2)^{1/2} \\ S_{Z1} = k_{eZ1}\cos\eta_2 - \frac{k_{eY1}k_{eZ1}\sin\eta_2}{\sqrt{k_{eX1}^2 + k_{eZ1}^2}} \end{cases}. \quad (10)$$

So in the original coordinate, the direction vector of e-ray is given by

$$\begin{bmatrix} S_X \\ S_Y \\ S_Z \end{bmatrix} = \begin{bmatrix} \cos\delta_2 & \sin\delta_2 & 0 \\ 0 & 0 & -1 \\ -\sin\delta_2 & \cos\delta_2 & 0 \end{bmatrix} \begin{bmatrix} S_{X1} \\ S_{Y1} \\ S_{Z1} \end{bmatrix}. \quad (11)$$

By Snell's law, we know the two emergent rays are parallel to incident light whatever the incident angle is. In Fig. 1, two eye points are D and E, respectively, and DE is the shear difference of two parallel emergent beams. Now we can get path difference

$$\begin{aligned}\Delta = L_1 - L_2 = &(n_o \cdot OA + n_{2e} \cdot AF + DG) \\ &-(n_{1e} \cdot OB + n_o \cdot CE).\end{aligned} \quad (12)$$

By using the direction vectors obtained above, we can calculate the path difference with Matlab.

When designing an optimum polarization splitter of polarization interferometer, we have three purposes. 1) For higher resolution, path difference should be larger. 2) According to the definition of field-of-view of polarization interferometer: supposing the path difference is M when the incident angle is α, and the path difference is N when the incident angle is 0, if the difference between M and N is half of the wavelength of the incident light, then α is half field-of-view. So, for large field-of-view, the change of path difference should be small when the incident angle alters. 3) For symmetrical sampling and symmetrical field-of-view, the path difference should be symmetrical when the incident angle alters in two sides of the normal line.

According to o-ray's and e-ray's trace analyzed above, we make a program with Matlab to get a series of Δ-α_0 curves about incident angles and path differences when the direction of crystal optical axis altered. Parts of

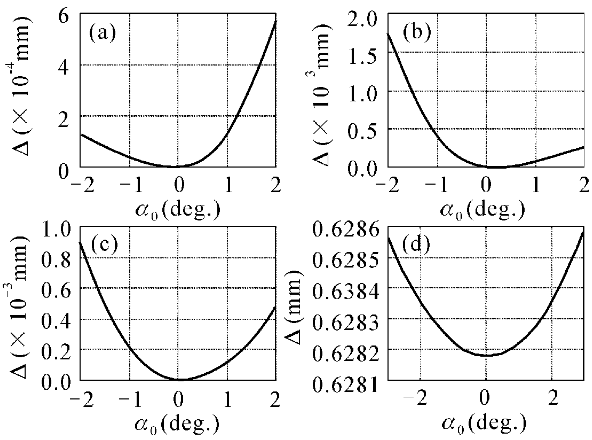

Fig. 2. The Δ-α_0 curves on the different direction of crystal optical axis. Abscissa axis denotes incident angles (in degree), and vertical axis denotes path difference (in mm). (a) $\delta_1 = 30°$, $\delta_2 = 30°$; (b) $\delta_1 = 60°$, $\delta_2 = 60°$; (c) $\delta_1 = 45°$, $\delta_2 = 45°$; (d) $\delta_1 = 58°$, $\delta_2 = 28°$.

curves are presented in Fig. 2. According to the three purposes listed above, we hope the curves should be flat and symmetrical, so a satisfying design scheme of polarization splitter is picked out from a lot of curves calculated. That is, if the angle between crystal optical axis of the first Savert prism and the incident interface is 58°, and the angle between optical axis of the second Savert prism and the incident interface is 28°, the instrument would be much better. The curve of this design can be seen in Fig. 2(d).

From Fig. 2(d), some advantages will be seen. Firstly, when lights transmit into the splitter from both sides of the normal line in the incident plane, the curve is fairly symmetrical. So we can get symmetrical sampling and symmetrical field-of-view.

Secondly, the maximum path difference will reach 0.63 mm and the maximum resolution will be 15.8 cm^{-1}. It is ten times of the resolution (230 cm^{-1}) mentioned in Ref. [10], and nearly two times of the resolution (30 cm^{-1}) mentioned in Ref. [3].

Thirdly, according to the definition of field-of-view of spectrometer, for the wavelength of 0.6 μm, we can see from Fig. 2(d) that when the incident angles are within $\pm 3°$, the path difference alters only 0.3 μm. It means the whole field-of-view could reach 6°. For infrared and far-infrared radiation the field-of-view will be wilder. The experimental work to test the method will be done in future.

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References

1. R. R. Korb, P. Dybwad, W. Wadsworth, and J. W. Salisbury, Appl. Opt. **35,** 1679 (1996).
2. C. M. Zhang, L. B. Xiang, B. C. Zhao, and L. Y. Liu, Optical Technique (in Chinese) **26,** 232 (2000).
3. M. Hashimoto and S. Kawata, Appl. Opt. **31,** 6096 (1992).
4. C. Y. Yin, *Modern Interferometric Measuring Technique,* (in Chinese) (Publishing house of Tianjin University, Tianjin, 1999) pp. 409 – 476.
5. M. C. Simon, Appl. Opt. **22,** 354 (1983).
6. W. Q. Zhang, Appl. Opt. **31,** 7328 (1992).
7. M. C. Simon and R. M. Echarri, Appl. Opt. **25,** 1935 (1986).
8. M. H. Jiang, *Crystal Optics,* (in Chinese) (Publishing House of ShangDong Science Technology, Jinan, 1980) pp. 261 – 271.
9. Q. T. Liang, Appl. Opt. **29,** 1008 (1990).
10. S. Prunet, B. Journet, and G. Fortunato, Opt. Eng. **38,** 983 (1999).