

Self-deflection of bright soliton in a separate bright-dark screening soliton pair based on higher-order space charge field

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Based on the interaction of the separate soliton pair, the self-deflection of the bright screening soliton in a bright-dark pair is studied by taking the higher order space charge field into account. Both numerical and analytical methods are adopted to obtain the result that the higher order of space charge field can enhance the deflection process of the bright soliton and varying the peak intensity of the dark soliton can influence the self-deflection strongly. The expression of the deflection distance with the dark soliton's peak intensity is derived, and some corresponding properties of the self-deflection process are figured out.

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Photorefractive (PR) spatial solitons have been extensively studied during the last three decades^[1]. Many interesting topics on the PR solitons, soliton pair and soliton interaction were investigated thoroughly in a single PR crystal^[2-13]. Very recently, we suggested a crystal circuit in which two PR crystals are connected in a chain by electrode leads with or without a voltage source and predicted theoretically that each crystal could support a steady-state one-dimensional bright or dark spatial soliton which we call separate spatial soliton pair^[14-16]. It is worthy to be noted that the two solitons in such a soliton pair can affect each other when the initial conditions changed. The coupling effects on the intensity profiles, dynamical evolutions, stabilities and self-deflection of the two solitons were investigated in detail on such a biased^[14,15] and unbiased^[16] PR circuit and many new phenomena were revealed, in which the most interesting one was that in a bright-dark (B-D) soliton pair, the dark soliton could affect the bright one by light-induced current whereas the bright soliton could not affect the dark one in the limit that the spatial extent of the optical wave is much less than the width of the crystal. We know that the bright soliton can deflect from its original trajectory in a single biased PR crystal considering the diffusion effect^[17-22]. The stronger the biased field is, the larger the deflection range of a PR bright soliton is because the higher-order space charge fields enhance the process of the self-deflection^[20,21]. In this paper, we investigate how to use the dark soliton to control the self-deflection of the bright soliton in a separate B-D soliton pair supported by a biased serial crystal circuit in which the crystals are non-photovoltaic PR crystals. Emphasis is put on the effects of higher-order space charge field on the self-deflection of a bright soliton.

As shown in Fig. 1, two non-photovoltaic PR crystals denoted by P and \hat{P} , are connected electrically in a chain with an external voltage source V_A . For each crystal, two electrodes are made on the surfaces with their normal parallel to the crystals' c -axis. Two optical beams I and \hat{I} are input vertically into the two crystals. In the limit

in which the optical wave has a spatial extent Δx much less than the x width of the crystal, we predicted that each crystal could support a spatial bright (dark) screening soliton^[14], which we call a separate spatial screening soliton pair.

We have known that there are three types of separate spatial soliton pairs: bright-bright (B-B), B-D and dark-dark (D-D). In this paper, we only consider the B-D soliton pairs. We assume that the bright soliton forms in the crystal P and the dark one forms in the crystal \hat{P} . Each crystal is oriented with its c -axis perpendicular to the direction of the laser beam, and the beams are polarized parallel to the c -axis. Each beam propagates in the crystal along the z -direction and is permitted to diffract only along the x -direction. The theoretical model about the separate spatial soliton pairs formed in such a configuration was built in Ref. [14], in which the diffusion process and the higher-order space charge fields were not taken into account. The results obtained in Refs. [14] and [15] for a B-D soliton pair are as follows. The induced space charge field E_{sc} in crystal P and \hat{E}_{sc} in crystal \hat{P} are: $E_{sc} = gE_A \frac{I_d}{I+I_d}$, $\hat{E}_{sc} = \hat{g}\hat{E}_A \frac{\hat{I}_\infty + \hat{I}_d}{\hat{I} + \hat{I}_d}$, where $g = \hat{\delta}(\hat{I}_\infty + \hat{I}_d) / [\delta I_d + \hat{\delta}(\hat{I}_\infty + \hat{I}_d)]$, $\hat{g} = \delta I_d / [\delta I_d + \hat{\delta}(\hat{I}_\infty + \hat{I}_d)]$, $\delta = S\mu s_i(N_D - N_A) / (\gamma R N_A W)$,

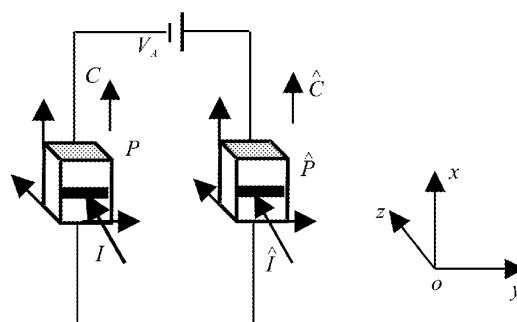


Fig. 1. Illustration of the biased series PR crystal circuit consisting of two PR crystals and a voltage source. C and \hat{C} denote c -axis.

$\hat{\delta} = \hat{S}\hat{\mu}\hat{s}_i(\hat{N}_D - \hat{N}_A)/(\hat{\gamma}_R\hat{N}_A\hat{W})$, $E_A = V_A/W$ and $\hat{E}_A = V_A/\hat{W}$. Here N_D is the donor density, N_A is the acceptor density, s_i is the photoexcitation cross section, γ_R is the carrier recombination rate, μ and e are, respectively, the electron mobility and the charge, S is the surface area of the electrodes of the crystals, W is the distance between the two electrodes, I_d is the so-called dark irradiance, $I = I(x, z)$ is the power density profile of the beam, and it attains asymptotically a constant value at $x \rightarrow \pm\infty$, that is, $I(x \rightarrow \pm\infty, z) = I_\infty$. The parameters with a symbol $\hat{\cdot}$ for the crystal \hat{P} have the same physical meaning as those without the symbol for the crystal P . For simplification, we take $\delta = \hat{\delta}$ and $I_d = \hat{I}_d$, which result in the simplified expressions $g = \frac{\hat{\rho}+1}{2+\hat{\rho}}$, $\hat{g} = \frac{\hat{\rho}}{2+\hat{\rho}}$, where $\hat{\rho} = \hat{I}_\infty/I_d$, denoting the peak input intensity of the dark soliton normalized to the dark irradiance.

Taking diffusion term and higher-order space charge field into account, expressing the bright soliton beam $I = I(x, z)$ in terms of the normalized envelope $I(x, z) = I_d|U(x, z)|^2$, U obeys the following dynamical evolution equation^[20,21],

$$\begin{aligned}
 iU_\xi + \frac{1}{2}U_{ss} - \beta\frac{U}{1+|U|^2} + \gamma\frac{(|U|^2)_s U}{1+|U|^2} \\
 + \gamma_1\frac{(|U|^2)_s U}{(1+|U|^2)^3} = 0, \quad (1)
 \end{aligned}$$

where $s = x/x_0$, $\xi = z/(kx_0^2)$, $\beta = \frac{\hat{\rho}+1}{\hat{\rho}+2}\sigma EA$, $\gamma = \sigma\tau$, $\gamma_1 = \zeta\sigma\left(\frac{\hat{\rho}+1}{\hat{\rho}+2}EA\right)^2$, $\sigma = (k_0x_0)^2(n_e^4r_{33}/2)$, $\tau = K_B T/(ex_0)$, $\zeta = \varepsilon_0\varepsilon_r/(eN_Ax_0)$, $U_\xi = \partial U/\partial\xi$, $U_{ss} = \partial^2 U/\partial s^2$. Here x_0 is an arbitrary spatial width, $k = k_0n_e$, $k_0 = 2\pi/\lambda_0$ and λ_0 is the free-space wavelength of the lightwave employed, n_e is the unperturbed extraordinary index of refraction, γ is the first-order diffusion term, and γ_1 is the higher-order space charge field.

Taking the same method as in Ref. [14], the two normalized profiles of the B-D soliton pair can be solved from the following integral equations,

$$(2\beta)^{1/2}s = \pm \int_{y(s)}^1 \frac{r^{1/2}d\tilde{y}}{[\ln(1+r\tilde{y}^2) - \tilde{y}^2 \ln(1+r)]^{1/2}}, \quad (2)$$

$$(-2\hat{\beta})^{1/2}\hat{s} = \pm \int_{\hat{y}(\hat{s})}^0 \frac{d\tilde{y}}{\left[(\tilde{y}^2 - 1) - \frac{1+\hat{\rho}}{\hat{\rho}} \ln\left(\frac{1+\hat{\rho}\tilde{y}^2}{1+\hat{\rho}}\right)\right]^{1/2}}, \quad (3)$$

where $y(s)$ and $\hat{y}(\hat{s})$ are normalized real function bounded between 0 and 1. $U = r^{1/2}y(s)\exp(i\nu\xi)$ for the bright soliton, where $r = I_0/I_d = I(0,0)/I_d$, and $\hat{U} = \hat{\rho}^{1/2}\hat{y}(\hat{s})\exp(i\hat{\nu}\hat{\xi})$ for the dark one. ν and $\hat{\nu}$ represent nonlinear shift of the propagation constant.

We can see that only under the condition that $E_A > 0$ and $\hat{E}_A < 0$, the B-D soliton pair can be formed in the circuit. The bright soliton is related to the peak intensity of the dark one through the parameter $\beta = \sigma g EA$, where $g = \frac{\hat{\rho}+1}{2+\hat{\rho}}$. So varying the input intensity of the dark soliton $\hat{\rho}$ can change the properties of the bright one.

Here we take two SBN (strontium barium niobate)

crystals as P and \hat{P} with the following parameters: $\gamma_{33} = 237 \times 10^{-12}$ m/V, $n_e = 2.33$, $N_A = 4 \times 10^{16}$ cm⁻³, and $\varepsilon_r=880$. The x -width of the two crystals $W = \hat{W}=1$ cm. Moreover, we have $\lambda_0 = 0.5$ μ m, $x_0 = 40$ μ m, and $T = 300$ K. We then get the first-order diffusion term $\gamma = 0.56$ and $\sigma = 8.82 \times 10^{-4}$. We take the applied voltage V_A to be 1000 V, which equals to $E_A = 1 \times 10^5$ V/m and $\hat{E}_A = -1 \times 10^5$ V/m. In this paper, we only investigate the dynamical self-deflection evolution of the bright screening soliton when the diffusion term and the higher-order space charge fields are taken into account. If we take a bright soliton incident upon the crystal P , its dynamical evolution can be obtained by numerically solving Eq. (1) when the values of $r, \hat{\rho}, \beta, \gamma$ and γ_1 are given. First, we take the input peak intensity of bright soliton $r = 10$ and $\hat{\rho} = 10$ for the dark, and we get $g = 11/12, \beta = 80.9$, and $\gamma_1 = 0.225$. Using the above parameters, we determine a bright soliton $U_0 = r^{1/2}y_0(s)$ from Eq. (2) as the incident beam, and solve Eq. (1) numerically to obtain the dynamical evolutions of the bright soliton as shown in Fig. 2. From the figure we can see that the bright soliton bends opposite to the direction of the c -axis of the crystal. The spatial shift of the beam center varying with the propagation distance is illustrated in Fig. 2(b) (solid line). We can see that when considering the higher order space-charge field, the spatial shift becomes larger than only γ is concerned, and we say that the higher-order term γ_1 enhances the self-deflection process of the bright screening soliton.

Since the main purpose of this paper is to investigate the effect of the input peak intensity of the dark soliton on the bright one's self-deflection, now we change the input intensity of the dark soliton to some other values and get the corresponding self-deflection processes of the bright one. Taking $\hat{\rho}$ to be 1, 10, 100, and 1000, we get four B-D soliton pairs in which the four bright screening solitons have the same value of the normalized peak input intensity, i.e., $r = 10$. Figure 3 gives the ξ

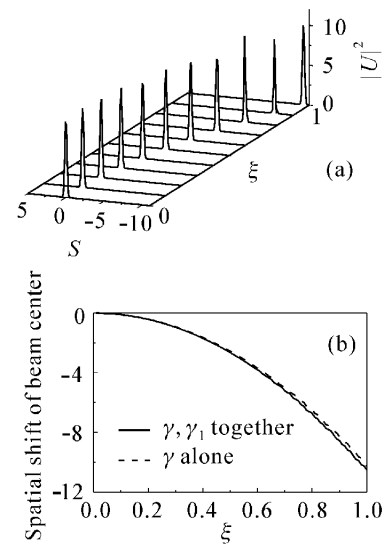


Fig. 2. (a) Dynamical evolution of screening bright soliton under the case of $r = 10, \hat{\rho} = 10, \gamma = 0.56, \gamma_1 = 0.225$ and $E_A = -\hat{E}_A = 1 \times 10^5$ V/m; (b) Trajectories of the beam center considering the higher-order term (solid) and not (dashed).

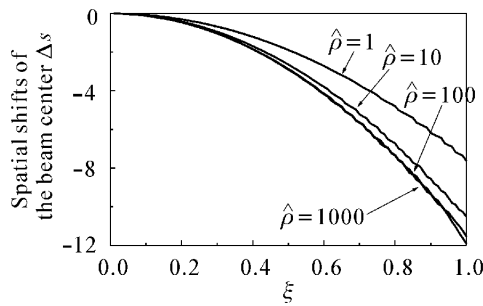


Fig. 3. Spatial shifts of the bright beam center with $r = 10$ and $E_A = -\hat{E}_A = 1 \times 10^5$ V/m under different values of $\hat{\rho}$.

dependence of Δs under different values of $\hat{\rho}$. Δs is the spatial shift of the beam center of the bright soliton from its origin. We can see that in this case, the spatial shift of the beam center increases as $\hat{\rho}$ increases. The larger $\hat{\rho}$ is, the larger the spatial shift is.

We now use the perturbative procedure to derive an approximate analytical expression for Δs varying with $\hat{\rho}$, from which we can obtain more comprehensive information than that from the numerical results. The solution of Eq. (1) is made as^[4,21]

$$U = r^{1/2}y[s + u(\xi)] \times \exp\{i[\nu\xi + \omega(\xi)(s + u(\xi)) + \sigma(\xi)]\}, \quad (4)$$

while $U = r^{1/2}y(s)\exp(i\nu\xi)$ is the steady-state bright soliton of Eq. (1) with $\gamma, \gamma_1 = 0$. Here $u(\xi)$ represents a shift in the position of the beam center, $\omega(\xi)$ is associated with the angle between the central wave vector of this beam and the propagation axis ξ , and $\sigma(\xi)$ is a phase factor that is allowed to vary during propagation. As in Ref. [22], we get $du(\xi)/d\xi = -\omega(\xi)$ and $d\omega(\xi)/d\xi = 4\beta[\gamma K(r) + \gamma_1 K_1(r)]$, where the dimensionless functions $K(r)$ and $K_1(r)$ are given by

$$K(r) = \int_{-\infty}^{+\infty} ds \frac{2y^2(s)}{1 + ry^2(s)} \times \{y^2(s) \ln(1 + r) - \ln[1 + ry^2(s)]\} \left(\int_{-\infty}^{+\infty} ds y^2(s) \right)^{-1}, \quad (5)$$

$$K_1(r) = \int_{-\infty}^{+\infty} ds \frac{2y^2(s)}{[1 + ry^2(s)]^3} \times \{y^2(s) \ln(1 + r) - \ln[1 + ry^2(s)]\} \left(\int_{-\infty}^{+\infty} ds y^2(s) \right)^{-1}. \quad (6)$$

The spatial shifts $\Delta s(\xi) = -u(\xi)$ and $u(\xi)$ can be directly integrated, we can obtain

$$\begin{aligned} \Delta s(\xi) &= 2\beta[\gamma K(r) + \gamma_1 K_1(r)]\xi^2 \\ &= 2\sigma \frac{r\hat{h}_0 + 1}{r\hat{h}_0 + 2} E_A [\gamma K(r) + \gamma_1 K_1(r)]\xi^2. \end{aligned} \quad (7)$$

We can see from the above expressions directly that $K(r)$ and $K_1(r)$ depend on the peak intensity of the

bright soliton r . Moreover, the normalized envelop $y(s)$ is related to the parameter β which depends on $\hat{\rho}$ and E_A . So $K(r)$ and $K_1(r)$ are also functions of $\hat{\rho}$. Here we discuss the dependences of the $K(r), K_1(r)$ and Δs on $\hat{\rho}$. We define $\Delta s_0 = |\Delta s(\xi = 1)|$, denoting the deflection distance at $\xi = 1$. Figure 4 shows the curves of $K(r), K_1(r)$ and Δs_0 varying with $\hat{\rho}$. The results show that the three parameters are all nonlinear functions of $\hat{\rho}$. $K(r)$ and $K_1(r)$ have the same changing rule with $\hat{\rho}$, that is, increasing monotonously with $\hat{\rho}$ change from 0.001 to 1000 which results in a monotonous increase of the deflection distance Δs_0 with $\hat{\rho}$. When $\hat{\rho}$ is very small or big enough, $K(r)$ and $K_1(r)$ almost keep unchanged and therefore Δs_0 approaches constant values at its two edges.

Figure 5 gives the dependence of Δs_0 on $\hat{\rho}$ under different values of r . We can see from it that the potential deflection ranges of the bright soliton caused by $\hat{\rho}$, i.e., $\Lambda_s = |\Delta s_0(\hat{\rho} \rightarrow 0) - \Delta s_0(\hat{\rho} \rightarrow \infty)|$ are different dramatically, for example, $\Lambda_s(r = 10) \gg \Lambda_s(r = 1000)$. The result means that under some conditions, varying $\hat{\rho}$ can change the value of Δs_0 broadly, but under some other conditions narrowly. It is a nonlinear function of r . If we draw out the curves of Λ_s with r as shown in Fig. 6, we can see that there is a peak value of $\Lambda_s = 5.43$ at $r = 7.58$. This means that if we want to modulate the deflection distance of the bright soliton largely, we should select the value of r near 7.58 and otherwise we can let r be very small or very large where the deflection range almost approaches zero, and where varying $\hat{\rho}$ can hardly change the original deflection of the bright soliton. Based on this result, we can select appropriate values of r

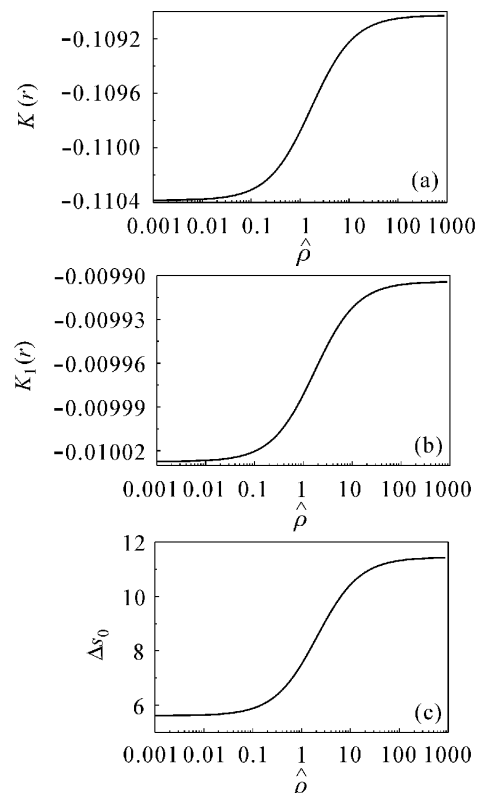


Fig. 4. Dependences of $K(r)$ (a), $K_1(r)$ (b) and Δs_0 (c) on $\hat{\rho}$ with $r = 10, E_A = -\hat{E}_A = 1 \times 10^5$ V/m.

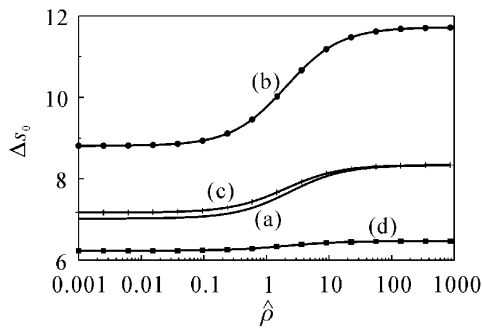


Fig. 5. Dependence of Δs_0 on $\hat{\beta}$ under different values of r . (a) $r=1$; (b) $r=10$; (c) $r=100$; (d) $r=1000$.

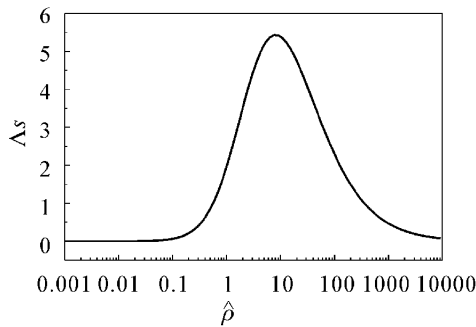


Fig. 6. Dependence of Δs on r .

according to this curve to satisfy our demand of a broad, a narrow or a mediate range of the deflection.

In conclusion, we have investigated the self-deflection of a bright soliton in a B-D soliton pair formed in a biased serial non-photovoltaic PR crystal circuit based on higher order space charge field with emphasis on the effect of the peak intensity of dark soliton on it. Both numerical and analytical methods are adopted to obtain the result that the higher order of space charge field can enhance the deflection process of the bright soliton and varying the peak intensity of the dark soliton can influence the self-deflection strongly. Based on this, a novel optically controlled beam deflector or scanner can be possibly made up.

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