Cross-phase modulation-induced penalties in multichannel DWDM optical transport networks

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In dense wavelength division multiplexing (DWDM) optical transmission systems, cross-phase modulation (XPM) due to Kerr effect causes phase shift on each channel, which will ultimately be transformed to amplitude noise that leads to power penalties. In this letter, the XPM-induced penalty in multi-channel DWDM systems is investigated theoretically and an applied algorithm that can be practically used in engineering design is proposed.

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The impact of cross-phase modulation (XPM) on the transmission performance of dense wavelength division multiplexing (DWDM) systems has been studied theoretically and experimentally[1−4] in the angle-modulated and intensity-modulated systems. The theoretical derivations, however, only gave XPM-induced pulse broadening[5] or phase shift[6] that was not explicit enough to be compared with experimental results. In this letter, a simple algorithm to calculate the XPM-induced power penalty in the presence of fiber dispersion, in the intensity-modulated and directly-detected (IM-DD) DWDM transmission systems is derived based on Ref. [6] and phase noise-intensity noise conversion theory in Ref. [7]. Numerical results show that XPM leads to significant penalties especially in systems with high transmission rates and a large number of wavelengths.

According to Ref. [6], the XPM-induced phase shift on probe wavelength j at length L is given by

$$\phi_j(L,t) = \frac{1}{\pi}\sum_{k\neq j}\int_0^\infty |P_k(0,\omega)|H_{jk}(\omega) \times \exp\left[i\omega\left(t-\frac{L}{V_{gj}}\right)\right]\mathrm{d}\omega, \quad (1)$$

where $P_k(0,\omega)$ is the spectral intensity of the pump wavelength k, V_{gj} is the group velocity of wavelength j, $H_{jk}(\omega)$ is the equivalent system function that describes the XPM-induced phase response on channel j by k. It is given by

$$|H_{jk}(\omega)| = 2\gamma_j\sqrt{\eta_{\mathrm{XPM}}(\omega)}L_{\mathrm{eff}}. \quad (2)$$

Nonlinear coupling coefficient γ_j, effective fiber length L_{eff}, XPM efficiency η_{XPM} in Eq. (2) can be expressed by physical parameters of wavelength channels such as attenuation coefficient α, angular frequency of optical signal ω_j, nonlinear refractive index n_2, dispersion coefficient D, effective core area of the fiber A_{eff}, wavelength separation of channels $\Delta\lambda_{jk}$, and fiber length L

$$\gamma_j = n_2\omega_j/cA_{\mathrm{eff}}, \quad L_{\mathrm{eff}} = (1-\mathrm{e}^{-\alpha L})/\alpha,$$

$$\eta_{\mathrm{XPM}}(\omega) = \frac{\alpha^2}{\omega_j^2 d_{jk}^2+\alpha^2}\left[1+\frac{4\sin^2(\omega_j d_{jk}L/2)\mathrm{e}^{-\alpha L}}{(1-\mathrm{e}^{-\alpha L})^2}\right],$$

where $d_{jk} \approx D\Delta\lambda_{jk}$ is the walk-off parameter.

Equation (1) assumes that all the pump wavelengths modulate the probe wavelength j independently, and the interferences between pump wavelengths are ignored. Thus the spectrum for the XPM-induced phase noise of probe signal caused by all pump channels can be represented as

$$S_{\Delta\phi_{jk}}(f) = \sum_{j\neq k}|H_{jk}(f)|^2 S_p(f), \quad (3)$$

where $S_p(f)$ is the power spectrum of probe wave k.

In IM-DD systems, especially in system with CW semiconductor lasers, the phase noise is expected to be converted to the intensity fluctuation through chromatic dispersion[7], inducing phase modulation to amplitude modulation (PM-AM) conversion noise that degrades bit-error-rate (BER) performance and brings power penalties. Applying the PM-AM noise expression given in Ref. [7] to Eq. (3), the XPM-induced amplitude noise of the photocurrent in the optical receiver can be written as

$$N_{\mathrm{PM-AM}}(f) = \frac{1}{2}I_0\left\{\sum_{n=0}^{\infty}4J_n\left(\sqrt{2S_{\Delta\phi_{jk}}(f)}\right) \times J_{n+1}\left(\sqrt{2S_{\Delta\phi_{jk}}(f)}\right) \times \sin\left[\frac{1}{2}(2n+1)(2\pi f)^2\ddot{\beta}L\right]\right\}^2, \quad (4)$$

where J_n is the n order first class Bessel function, I_0 is the direct current, and $\ddot{\beta} = \frac{\mathrm{d}^2\beta}{\mathrm{d}\omega^2} = -\frac{\lambda^2}{2\pi c}D$. Then the variance (power) of PM-AM noise is given by

$$\sigma^2_{\mathrm{PM-AM}} = R_I^2\int_{-\infty}^{\infty}N_{\mathrm{PM-AM}}(f)|H_I(f)|^2\mathrm{d}f, \quad (5)$$

where $H_I(f)$ is the normalized response function of main amplifier of the optical receiver circuit, R_I is its low frequency transimpedance.

Gaussian approximation algorithm is used to calculate the receiver's sensitivity deterioration (i.e. power penalty) and to evaluate the impact of XPM-induced amplitude noise on the transmission performance. Non-return-to-zero (NRZ) pulse is considered as the optimum option for signal modulation in long haul DWDM transmission systems, and has already applied in most of systems throughout the world. In the letter all the wavelength channels are assumed to be NRZ coded, both in probe and pump waves. The power spectrum of an NRZ signal with pulse width T can be expressed as

$$S_p(f) = \frac{A^2 T}{2} \frac{\sin^2(\pi f T)}{(\pi f T)^2} + \frac{A^2}{4}\delta(f). \quad (6)$$

A^2 is twice as large as the average power of the received signal P_0, i.e., $P_0 \equiv A^2/2$.

The first item on the right-hand side of Eq. (6) is the fluctuation element that causes the XPM effect. Substituting Eq. (6) into Eq. (3) yields

$$S_{\Delta\phi_{jk}}(f) = |H(f)|^2 S_p(f)$$
$$= 4\gamma_1^2 \eta_{\mathrm{XPM}}(f) L_{\mathrm{eff}}^2 P_0 T \left(\frac{\sin \pi f T}{\pi f T}\right)^2. \quad (7)$$

The XPM-induced amplitude noise can be obtained through Eqs. (4), (5) and (7).

Assuming Gaussian receiver noise, it can be shown that the BER at a receiver output is

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right), \quad (8)$$

where the Q factor is given by

$$Q = \frac{V(b_{\max}) - V(b_{\min})}{\sigma(b_{\max}) + \sigma(b_{\min})}. \quad (9)$$

$b_{\max}, b_{\min}$ are the optical power for the corresponding logical ONE's and ZERO's. $V(b)$ and $\sigma(b)$ are the mean power and variance, respectively, which are expressed as

$$V(b_{\min}) = GRb_{\min}R_I + GI_d R_I H_i(0),$$
$$V(b_{\max}) = GRb_{\max}R_I + GI_d R_I H_i(0), \quad (10)$$

$$\sigma^2(b_{\min}) = G^{2+x} \mathrm{e} R_I^2 f_b \Sigma_2 + R_I^2 Z + \sigma^2_{\mathrm{PM-AM}},$$

$$\sigma^2(b_{\max}) = G^{2+x} \mathrm{e} R_I^2 f_b I_4 + R_I^2 Z + \sigma^2_{\mathrm{PM-AM}}. \quad (11)$$

Equations (8)−(11) are expressions needed by Gaussian approximation approach to calculate receiver sensitivity. Note that the PM-AM noise is already included in Eq. (11). G is the average multiplicative gain of the avalanche photodiode, and x is the excess noise factor ($G = 1, x = 0$ for PIN). R, I_d, E, Z, and f_b represent responsivity, dark current, extinction ratio, noise factor of the receiver and signal bit rate, in turn. Σ_2 can be expressed by the equalized Personic coefficients.

The dependence of receiver sensitivity under XPM consideration on the fiber length, wavelength separation, and data rate is investigated numerically as follows. Firstly Eqs. (1)−(7) are used to calculate the power of XPM-induced amplitude noise, secondly Gaussian recursive algorithm is applied to get $b_{\max}$, then the receiver sensitivity can be obtained by $P_{\min} = b_{\max}(1 + E)/2$.

A 0.8 nm spaced DWDM point to point transmission system is considered here with $\lambda = 1550$ nm, $\alpha = 0.2$ dB/km, dispersion coefficient $D = 2$ ps/(nm·km) (G.655 fiber), nonlinear refractive index $n_2 = 3.2 \times 10^{-20}$ m^2/W, effective core area $A_{\mathrm{eff}} = 5.5 \times 10^{-11}$ m^2, $P_0 = 0.9$ mW, $I_0 = 0.505$ μA, $R = 0.8$, $E = 1/20$, and $I_d = 0.02$ μA. Suppose the rolloff factor of the output rised cosine wave be 1. Figure 1 shows the XPM-induced penalty versus bit rate of the modulated signal in a two-channel system at $L = 60$ km. When the bit rate is 2.5 Gb/s, penalty caused by XPM is about 0.7 dB and can be neglected in power budget, but for 10-Gb/s systems, penalty increases for almost three times and becomes an important factor in the design of physical aspects. In Fig. 2, receiver sensitivity decreases with the channel separation. Figure 2 is also obtained in a two-channel system with fiber length 60 km.

The receiver sensitivity versus fiber length in multichannel DWDM system is shown in Fig. 3. The wavelength channels are indexed in numerical sequence from 1 to W, W is the number of wavelengths (channel) per fiber. Clearly, the more W is, the larger the penalty will be caused. Meanwhile, the penalties on the middle channel (e.g. channel No. 8 when $W = 16$, as shown in Fig. 3) are the largest. The physical reason is that the spacing with other channels of the middle channel is the smallest.

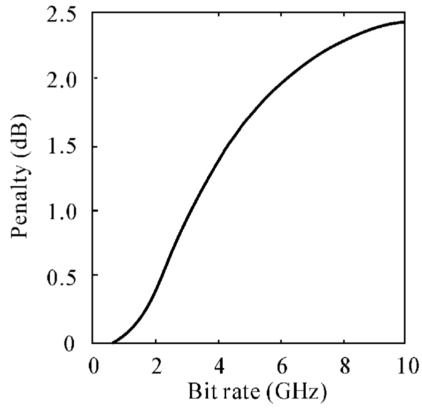

Fig. 1. XPM-induced penalty versus bit rate of the modulated signal in a two-channel system. L=60 km.

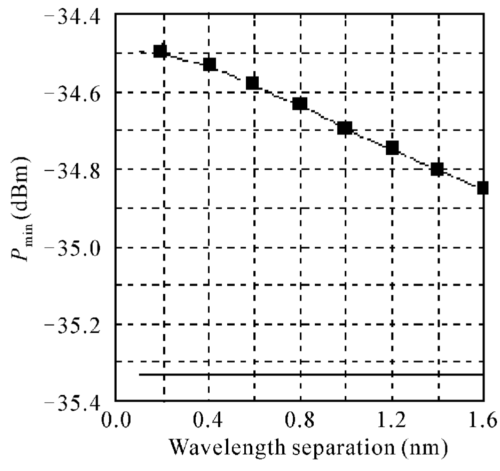

Fig. 2. Receiver sensitivity vs. channel separation. Also in two-channel system, bit rate is 2.5 Gb/s, L=60 km.

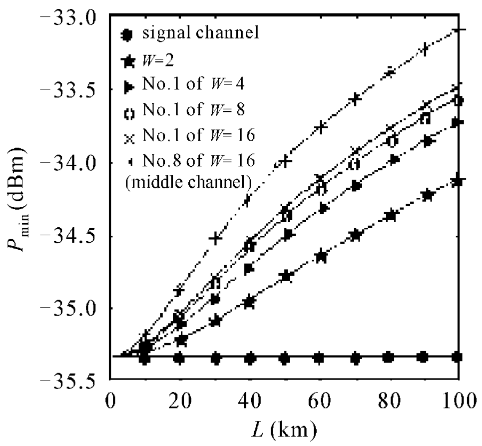

Fig. 3. XPM-induced penalties. Wavelength separation $\Delta\lambda =$ 0.8 nm and signal bit rate is 2.5 Gb/s. The channels are numbered in accordance with the order of wavelengths. Signal's waveform is supposed to be ideal rectangle waves with NRZ coding.

In conclusion, we have applied the PM-AM noise conversion theory to evaluate the XPM-induced power penalties and used Gaussian recursive algorithm to calculate receiver sensitivity as a function of fiber length, wavelength separation and bit rate of signal. Numerical results show that XPM leads to significant penalties especially in systems with high transmission rates and a large number of multiplexed wavelengths.

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