

# Study on energy up-conversion in Yb, Ho:YAG crystal

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Optic parameters, such as the probabilities of radiative and non-radiative transition and the cross-relaxation probability between Yb<sup>3+</sup> and Ho<sup>3+</sup> ions in Yb,Ho:YAG crystal, are calculated on the basis of Judd-Ofelt and Dexter theories. The energy up-conversion process is analyzed by solving the transition rate-equations. The results show that (1) the intensity of the green fluorescence relates to the square of the concentration of the active ions; (2) the intensity increases with the concentration of sensitive ions as well, but the increasing rate goes rather too slow; (3) the efficiency of the energy up-conversion relates with the speed of the energy up-conversion and the quantum efficiency of the transiting from upper level to lower level.

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The visible laser is widely used in display, optic recording, medical diagnoses and underwater detection etc.<sup>[1]</sup>. Using the energy up-conversion of rare-earth ions is an effective approach to obtain visible laser. Analyses on the efficiency and the rule of energy up-conversion provide an important ground for choosing visible laser crystal.

The energy levels of Yb<sup>3+</sup> and Ho<sup>3+</sup> ions are shown in Fig. 1<sup>[2]</sup>. The energy difference  $\Delta E_1(\text{Ho})$  between the metastable states (<sup>5</sup>I<sub>6</sub>) and the ground state (<sup>5</sup>I<sub>8</sub>) is close to the energy difference  $\Delta E_2(\text{Yb})$  between the ground state (<sup>2</sup>F<sub>7/2</sub>) and the excited state (<sup>2</sup>F<sub>5/2</sub>) of Yb<sup>3+</sup> ions. So Yb<sup>3+</sup> ions accelerate the energy up-conversion of Ho<sup>3+</sup> ions<sup>[3]</sup>.

According to Fig. 1 the transition rate equations are written as follows

$$\begin{aligned} \frac{dN_7}{dt} &= -Q_7N_7 + P_{c2}N_3, \\ \frac{dN_6}{dt} &= Q_{76}N_7 - Q_6N_6, \\ \frac{dN_5}{dt} &= Q_{75}N_7 + Q_{65}N_6 - Q_5N_5 + P_{c0}N_3, \\ \frac{dN_4}{dt} &= Q_{74}N_7 + Q_{64}N_6 + Q_{54}N_5 - Q_4N_4, \\ \frac{dN_3}{dt} &= Q_{73}N_7 + Q_{63}N_6 + Q_{53}N_5 + Q_{43}N_4 \\ &\quad - (Q_3 + P_{c2} + P_{c0})N_3 + (W + P_{c1})N_1, \\ \frac{dN_2}{dt} &= Q_{72}N_7 + Q_{62}N_6 + Q_{52}N_5 + Q_{42}N_4 \\ &\quad + Q_{32}N_3 - (Q_2 + P_{c0})N_2, \\ N &= N_7 + N_6 + N_5 + N_4 + N_3 + N_2 + N_1, \end{aligned} \quad (1)$$

where  $N_i$  is the population of the  $i$ th level of Ho<sup>3+</sup> ion;  $Q_{i,j}$  is the transition probabilities from level  $i$  to  $j$ ,  $Q_{i,j} = A_{i,j} + \omega_{i,i-1}$ , where  $A_{i,j}$  and  $\omega_{i,i-1}$  are the radiant and non-radiant transition probabilities from level

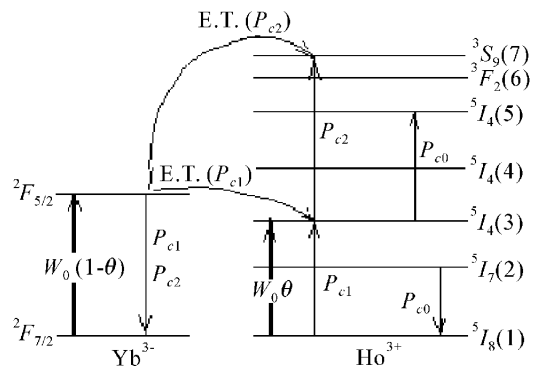


Fig. 1. Scheme of the level of Yb<sup>3+</sup> and Ho<sup>3+</sup> ions.

$i$  to  $j$  respectively and  $Q_i = \sum_j Q_{i,j}$ . Above spectrum parameters have been calculated in Ref. [7].  $\theta W_0$  and  $(1 - \theta)W_0$  represent pump rates for Ho<sup>3+</sup> and Yb<sup>3+</sup>, respectively.  $\theta = \frac{\alpha_H}{\alpha_H + \alpha_Y}$  is the relative absorption cross-section of Ho<sup>3+</sup>,  $\alpha_H$  and  $\alpha_Y$  are the absorption cross-sections of Ho<sup>3+</sup> and Yb<sup>3+</sup> ions, respectively.  $P_{c0}$  represents the probability of the cross-relaxation between Ho<sup>3+</sup> ions,  $P_{ci}$  is the  $i$ th cross-relaxation probability between Ho<sup>3+</sup> and Yb<sup>3+</sup> ions for  $i = 1$  and  $2$ . For YAG crystal  $P_{ci}$  can be calculated on the basis of the Dexter theory with

$$P_{ci} = 1.21 \times 10^{-17} \cdot f_A \cdot S / (R^6 \Delta E_D^4 \tau_D), \quad (2)$$

where  $R^6 = R_0^6 / (xy)$ ,  $x$  and  $y$  are atomic fractions of Ho<sup>3+</sup> and Yb<sup>3+</sup> replaced Y<sup>3+</sup> respectively,  $R_0 = 3.68 \times 10^{-8}$  cm is the smallest distance between Y<sup>3+</sup> ions in YAG crystal. The  $\tau_D$ ,  $f_A$  and  $\Delta E_D$  represent the fluorescence lifetime, the vibrator intensity of the acceptor and the energy difference of donor. If the probability of the cross-relaxation between Ho<sup>3+</sup> self is considered,

then  $R^6 = R_0^6 / x^2$ ,  $S = \int_{-\infty}^{\infty} g_A(\tilde{\nu}) h_s(\tilde{\nu}) d\tilde{\nu}$  is the overlap integral between the normalized linear type functions  $h_s(\tilde{\nu})$  of the emission band of the donor and  $g_A(\tilde{\nu})$  of the absorption band of the acceptor.  $\tilde{\nu}$  is the wave-number. When both the emission and the absorption

band are all the Gaussian distribution and superpose each other, the overlap integral can approximately be determined as  $S \approx \frac{4}{\pi^2 \Delta\nu}$ . When the energy exchange demands phonon to assist, Miyakawa theory<sup>[5]</sup> is required with  $S = S_0 \exp(-\beta\delta E)$  for the modification, where  $\delta E$  is the energy difference between the two transition pair. ( $\delta E = 0, 1.425, 0.3 \times 10^3 \text{ cm}^{-1}$  for three cross-relaxations, respectively).  $\beta = \alpha - \frac{\ln 2}{\hbar\omega'}$  if the multiplicity of the upper level of is equal to that of lower level. The cut-off energy of the phonon is  $\hbar\omega' = 700 \text{ cm}^{-1}$  in YAG crystal; as a token of the no-radiation transition the constant  $\alpha = 3.2 \times 10^{-3} \text{ cm}$  for YAG crystal which relates only to the host. From Refs. [2,6] and author's paper<sup>[7]</sup> the parameters are  $\tau_D = 9.0, 1.1, 1.1 \text{ ms}$ ,  $f_A = 1.03, 1.42, 1.94 \times 10^{-6}$ ,  $\Delta E_D = 0.5, 1.0, 1.0 \times 10^4 \text{ cm}^{-1}$ ,  $S_0 = 2, 5, 5 \times 10^{-4} \text{ cm}$ , respectively. From Eq. (2) the three probabilities of the cross-relaxations are  $P_{c0} = 2.2 \times 10^5 x^2$ ,  $P_{c1} = 1.3 \times 10^4 xy$ ,  $P_{c2} = 2.6 \times 10^5 xy \text{ (s}^{-1}\text{)}$ , respectively.

Inserting the parameters, such as  $Q_i, Q_{ij}, P, P_{ci}$ , to the transition rate-equations (1), and solving rate equations in steady state, we can obtain the population  $N_i$  which versus the concentrations  $x$  and  $y$  of both active and sensitive ions and the pumping rate  $W$ . The intensity of visible fluorescence relates to the interval of the energy levels, the relative population ( $N_{i1} = N_i - N_1$ ) and the transition probability  $Q_{i1}$ . Therefore,  $(N_i - N_1)Q_{i1}/N_0$  can stand for the fluorescence intensity in discussion. The rule of the energy up-conversion is shown as Figs. 2

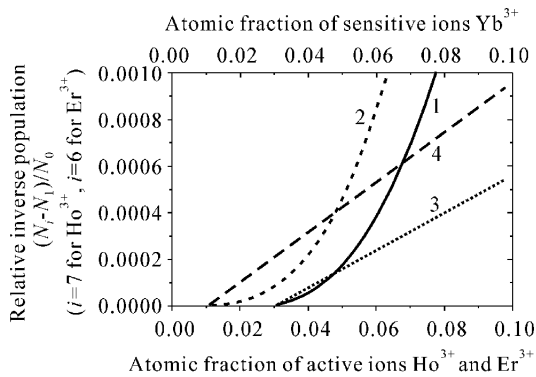


Fig. 2. The relative inverse population versus the concentration of the active and sensitive ions if pump-rate  $W = 10^5 \text{ s}^{-1}$ . Curve 1: 5% Yb, Ho:YAG; 2: 5% Yb, Er:YAG; 3: Yb, 5% Ho:YAG and 4: Yb, 5% Er:YAG.

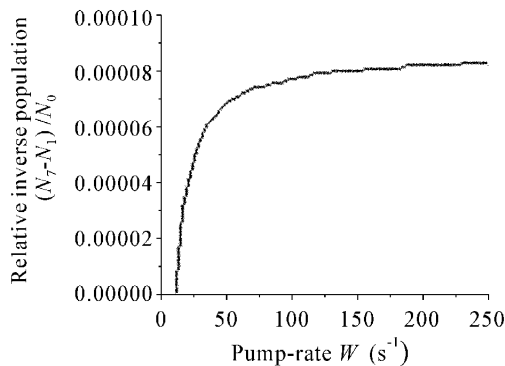


Fig. 3. The relative inverse population versus the pump-rate  $W$  for 3 at.-% Ho, 5 at.-% Yb:YAG.

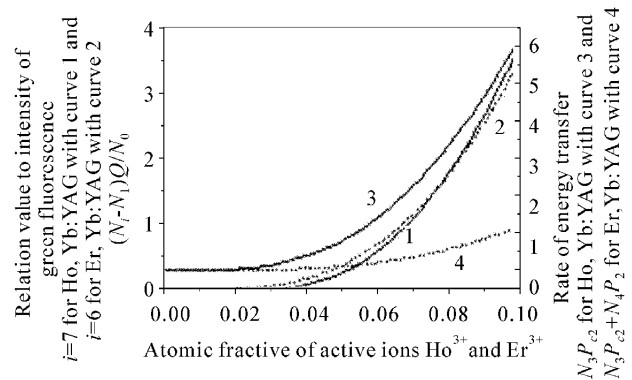


Fig. 4. The intensity of green fluorescence and the rate of energy transfer versus the atomic fraction of active ions for pump-rate  $W = 10^5 \text{ s}^{-1}$  in Ho, 5% Yb:YAG and Er, 5% Yb:YAG.

and 3. The intensity of best strong green fluorescence corresponding to the transition  $^5S_2 \rightarrow ^5I_8$ , as a example, is proportional to the square of the  $\text{Ho}^{3+}$  concentration and increases with the  $\text{Yb}^{3+}$  concentration as well, but the rate of the increasing with the concentration rather too low. The fluorescence intensity strengthens with pumping rate  $W$ , but soon to be saturated. Above results are quite similar to the energy up-conversion in Yb, Er:YAG crystal<sup>[8]</sup> and other solid<sup>[9]</sup>.

The energy up-conversion efficiency  $\eta$  is defined as the ratio of fluorescent energy  $E_s$  and excited energy  $E_{in}$ , namely  $\eta \propto \frac{E_s}{E_{in}}$ ,  $E_s \propto (E_i - E_1)(N_i - N_1)Q_{i1}/N_0$ ,  $E_{in} \propto W$ . It can be also got that the efficiency of energy up-conversion of  $\text{Yb}^{3+} \rightarrow \text{Ho}^{3+}$  is similar to that of  $\text{Yb}^{3+} \rightarrow \text{Er}^{3+}$  in YAG crystal. Because  $(E_i - E_1)$  of the three crystals compared is close,  $(N_i - N_1)Q_{i1}/N_0$  is used to stand for the fluorescence intensity to discuss the relation between the fluorescence intensity and  $x$  (or  $y$ ) in Fig. 4.

The energy up-conversion rate,  $\xi$ , denotes the population  $P_{c2}N_3$  of up-conversion in unit time. The rate of energy up-conversion of  $\text{Yb}^{3+} \rightarrow \text{Ho}^{3+}$  is bigger than that of  $\text{Yb}^{3+} \rightarrow \text{Er}^{3+}$  as Fig. 4 if the concentration of active ions  $\text{Ho}^{3+}$  is the same as that of  $\text{Er}^{3+}$ . It is noticed that most ions that up-converted to the high energy level will transit to metastable level via the radiation and non-radiation process, and only a small part will transit to the ground state and radiate green fluorescence. For this reason the fluorescence intensity of the up-conversion relates not only to energy up-conversion rate  $\xi$ , but also to the quantum efficiency of the radiate transition (namely ratio between the radiative and total transition probability). From the  $Q_j$  and  $Q_{ij}$  data of both  $\text{Ho}^{3+}$  and  $\text{Er}^{3+}$ , we can also estimate that the quantum efficiencies ( $Q_{i1}/Q_i$ ) of the green fluorescence for  $\text{Ho}^{3+}$  and  $\text{Er}^{3+}$  are about 9.2% and 11.8%, respectively. Therefore, the intensity of the green fluorescence in Yb, Ho:YAG crystal is approximately the same as that in Yb, Er:YAG.

The efficiency  $\eta$  and rate  $\xi$  of the energy up-conversion of  $\text{Ho}^{3+}$  are obtained by calculating the optic parameters, such as the probabilities of radiative transition and the cross-relaxation, and by solving the transition rate-equations. These are important foundations for choosing visible laser crystal and determining the ion concentration. It has been demonstrated that Yb, Ho:YAG

is as good laser crystal in energy up-conversion as Yb, Er:YAG.

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