Laser beam characteristic for laser resonators with diffraction optical elements

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Received July 14, 2002

The matrix eigenvalue method is used to analyze a laser resonator composed of diffraction optical elements. The results show that this type of resonator can separate fundamental mode and high order modes effectively. The output beams can be designed for different requests.

OCIS codes: 140.4780, 260.1960, 030.4070.

Laser resonators play an important role in the output characteristics of a laser. A resonator with a low gain lasing medium is required to have a very low loss for the fundamental mode. A stable Fabry-Perot cavity is usually used in a low gain lasing medium cavity. Although this cavity has a very low loss for the fundamental mode, it has many inherent disadvantages[1].

Recently, the resonators with diffraction element DMSM (diffraction mode-selecting mirror) become a new technique for changing output beam distributions[2]. We can design desired output beams, such as plane top beam and doughnut laser beam, along with the development of the binary optical technology. In the present paper, we analyze theoretically and calculate numerically the properties of the resonator composed of diffraction optical elements in terms of matrix eigenvalue method.

A DMSM laser resonator is consist of the plane output mirror M_1 and DMSM M_2 with a distance L between M_1 and M_2 .The radii for M_1 and M_2 are a and b, respectively (see Fig. 1).

The principle of a DMSM cavity[2] is as follows. First, we should set $U_1(x_1, y_1)$ as the output light field of output mirror M_1. When this field propagates from M_1 to M_2, the field $U_2(x_2, y_2)$ of M_2 is formed. If the reflectivity of DMSM for U_2 is $R_2(x_2, y_2) = U_2^*(x_2, y_2)/U_2(x_2, y_2)$, the self-reappearance condition of U_1's resonance can also be satisfied and the loss of fundamental mode on the mirror is 0. The DMSM has merely phase conjugation to $U_1(x_1, y_1)$, but it does not have phase conjugation to other modes.

Pare and Belanger have used Prony computing method to calculate output beam characteristics for a one-dimensional strip resonator[3]. The results showed that DMSM cavity has effective mode discrimination and preferable fundamental mode output. They reported their experimental results with a CW CO_2 laser using a "custom phase-conjugate resonator" designed for a super-Gaussian output beam profile[4].

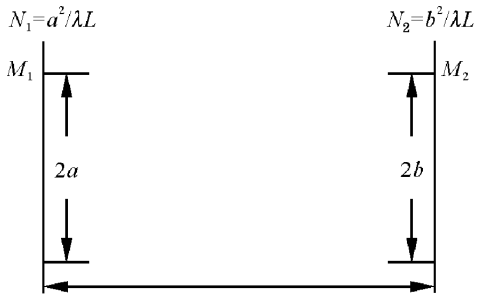

Fig. 1. The resonator with DMSM.

In this paper, we consider a two- dimensional resonator with a circular reflector.

Suppose that the light field on M_1 is $\psi(r_1) = \exp\left[-\left(\frac{r_1}{w_0}\right)^6\right]$, where w_0 is the radius of the beam. After a transition of $\psi(r_1)$, the reflectivity of DMSM is $R(r_2) = \exp[-2I\theta(r_2)]$, where I expresses a phase conjugation factor and $\theta(r_2)$ is phase distribution function. It satisfies $R_2(x_2, y_2) = U_2^*(x_2, y_2)/U_2(x_2, y_2)$.

The diffractive integral equation for the resonator with circular plane reflector for a transition from M_1 to M_2 may be expressed as[5]

$$U(r_2, \varphi_2) = \int_0^a \int_0^{2\pi} K(r_2, \varphi_2; r_1, \varphi_1) U(r_1, \varphi_1) r_1 \mathrm{d}\varphi_1 \mathrm{d}r_1, \quad (1)$$

where

$$K(r_2, \varphi_2; r_1, \varphi_1) = \frac{j}{\lambda L} \times \exp\{-jk[(r_1^2 + r_2^2)/2L - (r_1 r_2/L)\cos(\varphi_1 - \varphi_2)]\}. \quad (2)$$

The self-reappearance diffractive integral equation for moving forth and back between M_1 and M_2 is

$$U(r_1, \varphi_1) = \gamma \int_0^b \int_0^{2\pi} K(r_1, \varphi_1; r_2, \varphi_2) R(r_2) \times \int_0^a \int_0^{2\pi} K(r_2, \varphi_2; r_1, \varphi_1) U(r_1, \varphi_1) r_1 \mathrm{d}\varphi_1 \mathrm{d}r_1 r_2 \mathrm{d}\varphi_2 \mathrm{d}r_2, \quad (3)$$

where

$$K(r_2,\varphi_2;r_1,\varphi_1)=K(r_1,\varphi_1;r_2,\varphi_2)=$$
$$\frac{j}{\lambda L}\exp\{-jk[(r_1^2+r_2^2)/2L-(r_1r_2/L)\cos(\varphi_1-\varphi_2)]\},\qquad(4)$$

and

$$\exp\{jn[(\pi/2)-\varphi_2]\}J_n\left(k\frac{r_1r_2}{L}\right)=$$
$$\frac{1}{2\pi}\int_0^{2\pi}\exp[jk(r_1r_2/L)\cos(\varphi_1-\varphi_2)-jn\varphi_1]\mathrm{d}\varphi_1,\quad(5)$$

where J_n is n-order Bessel function. If

$$U(r_1,\varphi_1)=V_n(r_1)\exp(-in\varphi_1)\quad(n=0,1,2,\cdots),\qquad(6)$$

where

$$V_n(r_1)=\gamma_n\int_0^b K_n(r_1,r_2)R(r_2)r_2\mathrm{d}r_2$$
$$\times\int_0^a K_n(r_2,r_1)V_n(r_1)r_1\mathrm{d}r_1,\qquad(7)$$

where

$$K_n(r_1,r_2)=K_n(r_2,r_1)=$$
$$\frac{j^{n+1}k}{L}J_n\left(k\frac{r_1r_2}{L}\right)\exp[-jk(r_1^2+r_2^2)/2L].\qquad(8)$$

It can satisfy the integral Eq. (3).

In Eq. (7), integral kernel is

$$K_n'(r_1,r_2)=\frac{j^{n+1}k}{L}J_n\left(k\frac{r_1r_2}{L}\right)$$
$$\times\exp[-jk(r_1^2+r_2^2)/2L]/R(r_2).\qquad(9)$$

Then the circular DMSM cavity has the eigen integral equation relating to r. It is

$$V_n(r_1)=\gamma_n\int_0^b K_n'(r_1,r_2)r_2\mathrm{d}r_2$$
$$\times\int_0^a K_n(r_2,r_1)V_n(r_1)r_1\mathrm{d}r_1.\qquad(10)$$

Eq. (10) may also be used to calculate $R(r_2)$, if $V_n(r_1)$ is equal to $\psi(r_1)$.

For all modes except fundamental mode we can only get the numerical solution in virtue of non-uniform reflectivity. The traditional numerical calculation for resonators is Fox-Li iterative method[5]. It can be calculated eigenvalue, but its astringency is bad under the condition of enormous Fresnel number. By the way, it is effective for low order modes but ineffective for most high order modes. Afterward Siegman proposed Prony method. It is a matrix algebraic method. Its advantages are to get field distribution, phase shift and loss for several different modes in the meantime. But the correctness of eigen mode separating requires to be verified in the process of calculation. In this paper we use the matrix eigenvalue method.

The complex integral formula may be written as[7]

$$\int_0^b f(x)\mathrm{d}x.\qquad(11)$$

In order to solve formula (11) between $[a,b]$, we can divide $[a,b]$ into n parts. The divided point is $x_k=a+kh(k=1,2,\cdots,n)$, where $h=\frac{b-a}{n}$, so

$$\int_0^b f(x)\mathrm{d}x\approx T_n=\frac{h}{2}[f(a)+2\sum_{k=1}^{n-1}f(x_k)+f(b)].\qquad(12)$$

The method of numerical integral was applied to the resonators with DMSM, when the radius of M_1's circular aperture is a, the radius r_2 of M_2 is b, Eq. (10) can be rewritten as

$$\varepsilon(x_n')=\gamma\frac{h}{2}\cdot\frac{h'}{2}\sum_{k=1}^{N}K'(x_n',x_k)D_{nn}$$
$$\sum_{n=1}^{N}K(x_k,x_n')D_{nn}\varepsilon(x_n').\qquad(13)$$

If we set n points in the integral area, $h=a/n,h'=b/n$, the elements K'_{mn} and K_{mn} can be gotten by Eq. (9) multiplied by r_2 and Eq. (8) multiplied by r_1, respectively. D_{nn} is a diagonal matrix, the elements in the diagonal are $1,2,2,2,\cdots,2,2,2,1$, and ε indicates n dimension vectors corresponded to a possible radial distribution of eigen mode. Therefore, the solution of the resonator's eigen equation becomes an eigenvalues solution of matrix. Mathematics can be applied to evaluate the eigenvalues of γ_{mn} and corresponding vectors. The loss of a round trip in the resonator is $1-|\gamma_{mn}|^2$, so the corresponding vectors are the numerical solution of the corresponding modes.

We assume that the output field of output mirror is third order super-Gaussian beams, i.e.

$$\psi(r_1)=\exp\left[-\left(\frac{r_1}{w_0}\right)^6\right].$$

Both output aperture and DMSM aperture are 1 mm and the wavelength is 1μm. In the different length of the cavity we calculated the varieties of the modes loss, as shown in Fig. 2.

Figure 2 shows that the diffraction loss of DMSM cavity decreases slowly with shortening cavity length. But when Fresnel number is larger than 5, DMSM cavity loss of low order mode becomes 0. Therefore it is helpful for output fundamental mode.

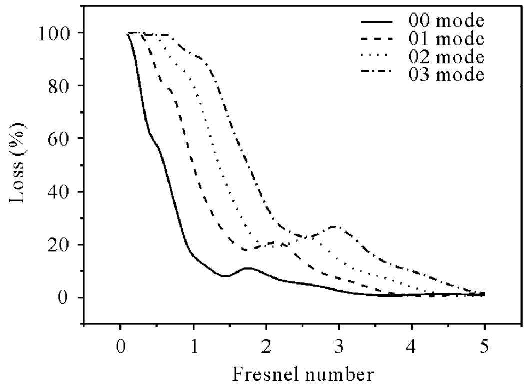

Fig. 2. Loss of the cavity versus Fresnel number of the cavity.

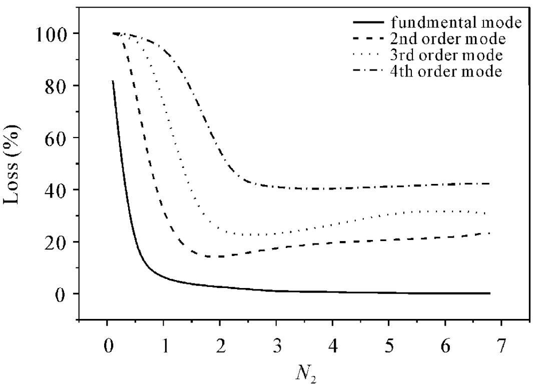

Fig. 3. Loss curve of the resonator with DMSM, in which the reflector is infinited in one dimension.

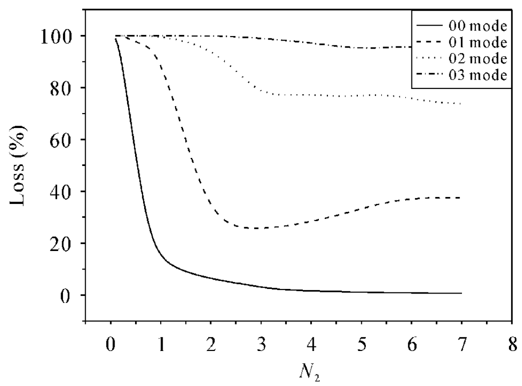

Fig. 4. Loss curve of cavity with circular DMSM.

In the second case, we changed the radius of the DMSM, and fixed output mirror aperture (a = 1 mm) and the length of cavity (1 m). We calculated the diffraction loss in the case that reflector is infinite in one dimension and circular, as shown in Fig. 3 and Fig. 4, respectively. For the first case, our results coincide with Belanger's[3].

The above figures show that DMSM can separate modes efficiently, but modes discrimination of the circular mirror is not better than the case of one dimension. One dimension reflector is an ideal case, and two-dimensional reflector is real case. When DMSM Fresnel number is 2, the difference of loss between fundamental mode and second order mode in one dimension is 12%,

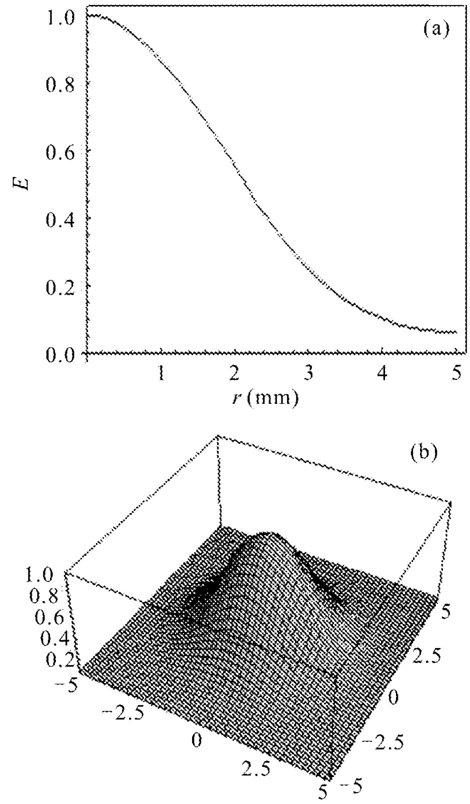

Fig. 5. The amplitude distribution for fundamental mode for N_2 = 1. (a) Radial amplitude distribution. (b) Amplitude distribution on 3-D.

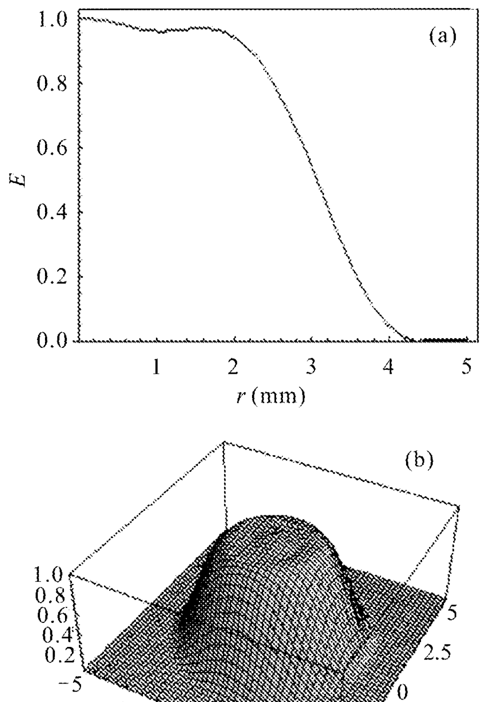

Fig. 6. The amplitude distribution for fundamental mode for N_2 = 8. (a) Radial amplitude distribution. (b) Amplitude distribution on 3-D.

the difference of loss between TEM_{00} mode and TEM_{10} mode is 9%, and the difference of loss between TEM_{00} and TEM_{01} is 29%. These show the fundamental mode occupies the primary parts in output field. The other modes are restrained because of enormous loss. Although we cannot simulate the full distribution of output light field, we analyzed the amplitude distribution of fundamental mode to express the main part of output field. Figures 5 and 6 show the amplitude distribution of output fundamental mode when circular reflector cavity in different cases of aperture radius.

Figures 5 and 6 show that fundamental mode beams are different from fundamental Gaussian mode beams when DMSM's Fresnel number is different. The amplitude distribution becomes Gaussian-like when DMSM's Fresnel number is 1. The amplitude distribution begins to become flatness when DMSM's Fresnel number is 3. It shows the characteristics of high order mode in circular parallel plane cavity. It will be close to the third order mode output when Fresnel number is 8. An improved output beams distribution of fundamental mode shows the super Gauss characteristic. We can conclude that the output beam amplitude distribution is Gaussian-like in small Fresnel numbers and trends toward super-Gaussian distribution as Fresnel numbers increasing.

The analysis and calculations show that DMSM can improve the uniformity of fundamental mode in the cavity. It is a preferable element of mode selection. We can design and customize the DMSM mirror according to different requests. In practice, we should enlarge the DMSM radius as possible as we can. It can improve the characteristics of output beams because of the large mode discrimination between fundamental mode and high order modes. It can also increase the mode cubage of beams so as to increase output power.

This project is supported by the National Natural Science Foundation of China under the Grant No. 19970438.
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