# Resonant gradient force on atom interacting with laser field

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The gradient force, as a function of position and velocity, is derived for a two-level atom interacting with a standing-wave laser field. Basing on optical Bloch equations, the numerical solutions for the gradient force $f_{\perp,n}$ ($n = 0, 1, 2, 3, 4, \cdots$) pointing in the direction of the transverse of the laser beam are given. It is shown the higher order gradient force plays important role at strong intensity ($G = 64$), the contribution of them can not be neglected.

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During the past years, the theoretical and experimental investigations of laser cooling and trapping have become one of the major fields in atomic, molecular and optical physics. The development of laser cooling and trapping technology is important for the applications such as Bose-Einstein condensation, atom optics, atomic fountain, new generations of atomic clocks and atom lasers[1−8].

In investigations of the controllable force on the atoms in laser cooling and trapping, the model employed is either a two-level atom or a multi-level atom interacting with a laser field. However, in most of the previous analysis, only the light pressure force in the laser beam direction is calculated[9−11]. But the discussion of the gradient force is not sufficient. Actually, the gradient force is proportional to the intensity gradient and points in the direction of the intensity gradient[12−15].

In this paper, the numerical results for the harmonics of the gradient force are presented, and the analysis of the gradient force acting on the two-level atom moving across the laser field are given. The gradient force is derived through the optical Bloch equations and calculated. The exact numerical results of the force are discussed. The results show that the gradient force has a non-uniform transverse distribution. The behavior of the higher terms of the gradient force is investigated for different velocities and saturation parameters.

We consider a two-level atom system having transition frequency $\omega_0$ and interacting with a standing-wave laser field of angular frequency $\omega$. The frequency $\delta$ denotes the laser detuning with respect to the atomic transition frequency, $\delta = \omega - \omega_0$. The evolution of the system may be described by motion equations for the atomic density matrix. The theory we use here results in the optical Bloch equations. In the rotating-wave approximation, we express the density matrix in the optical Bloch equations[4]

$$\begin{aligned}
\frac{\mathrm{d}\rho_{\mathrm{gg}}}{\mathrm{d}t} &= +\gamma\rho_{\mathrm{ee}} + \frac{i}{2}\left(\Omega^*\tilde{\rho}_{\mathrm{eg}} - \Omega\tilde{\rho}_{\mathrm{ge}}\right), \\
\frac{\mathrm{d}\rho_{\mathrm{ee}}}{\mathrm{d}t} &= -\gamma\rho_{\mathrm{ee}} + \frac{i}{2}\left(\Omega\tilde{\rho}_{\mathrm{ge}} - \Omega^*\tilde{\rho}_{\mathrm{eg}}\right), \\
\frac{\mathrm{d}\tilde{\rho}_{\mathrm{ge}}}{\mathrm{d}t} &= -\left(\frac{\gamma}{2} + i\delta\right)\tilde{\rho}_{\mathrm{ge}} + \frac{i}{2}\Omega^*\left(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}\right), \\
\frac{\mathrm{d}\tilde{\rho}_{\mathrm{eg}}}{\mathrm{d}t} &= -\left(\frac{\gamma}{2} - i\delta\right)\tilde{\rho}_{\mathrm{eg}} + \frac{i}{2}\Omega\left(\rho_{\mathrm{gg}} - \rho_{\mathrm{ee}}\right),
\end{aligned} \tag{1}$$

where $\tilde{\rho}_{\mathrm{ge}}$ represents an element of the density matrix, the subscript g and e denote the ground state and excited state, respectively, and $2\gamma$ is the natural linewidth for the decay of the upper level to the lower one, $\Omega$ stands for the Rabi frequency defined by $\Omega = 2\Omega_0 \cos kz$. Here $\Omega_0 = \mu E_0/\hbar$, $\mu$ is the matrix element of the resonance transition dipole moment.

Equations (1) can be derived with the usual normalization and hermiticy condition

$$\begin{aligned}
&\rho_{\mathrm{gg}} + \rho_{\mathrm{ee}} = 1, \\
&\tilde{\rho}_{\mathrm{eg}} = \tilde{\rho}^*_{\mathrm{ge}} = e^{i\omega t}\rho_{\mathrm{eg}}.
\end{aligned} \tag{2}$$

Then introducing the real blochian variables $r$, $s$ and $c$, one has

$$\begin{aligned}
&r = \rho_{\mathrm{gg}} - \rho_{\mathrm{ee}}, \\
&is = \tilde{\rho}_{\mathrm{eg}} - \tilde{\rho}_{\mathrm{ge}} = 2i\mathrm{Im}\tilde{\rho}_{\mathrm{eg}}, \\
&c = \tilde{\rho}_{\mathrm{eg}} + \tilde{\rho}_{\mathrm{ge}} = 2\mathrm{Re}\tilde{\rho}_{\mathrm{eg}}.
\end{aligned} \tag{3}$$

Using these variables and conditions, we can transform Eqs. (1) into the following equations

$$\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}t} &= \gamma(1-r) + \Omega s, \\
\frac{\mathrm{d}s}{\mathrm{d}t} &= -\frac{1}{2}\gamma s + \delta c + \Omega r, \\
\frac{\mathrm{d}c}{\mathrm{d}t} &= -\frac{1}{2}\gamma c - \delta s.
\end{aligned} \tag{4}$$

In the adiabatic approximation, the time rate of the change of atomic velocity is slow comparing with that of internal atomic state. The exact expression can be obtained on the base of the Fourier expansion

$$\begin{aligned}
&u(z,\rho,v) = \sum_{n=-\infty}^{\infty} u_n(\rho,v)\mathrm{e}^{inkz}, \\
&u_{-n} = u_n^* \ (u = r, s, c), \quad z = vt.
\end{aligned} \tag{5}$$

where $z$ is along the direction of the light beam, $\rho$ is the radius of the coordinate, $v$ is the velocity of the atom and $k$ is the wave vector of the laser field. Substituting the Fourier expansion (5) into Eqs. (4) and using conditions (2) and (3), we can obtain the recursion relations for the Fourier coefficients

$$\begin{aligned}
&(\gamma + inkv)r_n = \gamma\delta_{n,0} - \frac{\Omega}{2}(s_{n+1} + s_{n-1}), \\
&\left(\frac{\gamma}{2} + inkv\right)s_n = \delta c_n + \frac{\Omega}{2}(r_{n+1} + r_{n-1}), \\
&\left(\frac{\gamma}{2} + inkv\right)c_n = -\delta s_n.
\end{aligned} \tag{6}$$

The light pressure force is given by[4]

$$F(\rho, v) = \mu \nabla \left(E_0(\rho) \cos kz\right) c = F_{//} + F_{\perp}. \quad (7)$$

Two orthogonal components: a so-called longitudinal light pressure force component $F_{//}$ pointing in the direction of the laser beam and a gradient component $F_{\perp}$ pointing in the transverse direction,

$$F_{//} = F_{//}\hat{e}_z = -\hbar k \Omega_0 \sin kz \hat{e}_z. \quad (8)$$

$$F_{\perp} = F_{\perp}\hat{e}_\rho = -\hbar(\rho/a^2)\Omega_0 \cos kz \hat{e}_\rho. \quad (9)$$

Here $a$ is the radius of the beam waist.

The exact expression for the light pressure force can be obtained on the base of the Fourier expansion (5).

$$F_{\perp} = F_{\perp}^0 + \sum_{n=1}^{\infty} \left(F_{\perp,n}^c \cos 2nkz + F_{\perp,n}^s \sin 2nkz\right), \quad (10)$$

where

$$F_{\perp}^0 = -\hbar k \frac{1}{ka} \frac{\rho}{a} \Omega_0 \mathrm{Re} c_1,$$

$$F_{\perp,n}^c = -\hbar k \frac{1}{ka} \frac{\rho}{a} \Omega_0 \mathrm{Re}\left(c_{2n+1} + c_{2n-1}\right),$$

$$F_{\perp,n}^s = \hbar k \frac{1}{ka} \frac{\rho}{a} \Omega_0 \mathrm{Im}\left(c_{2n+1} + c_{2n-1}\right). \quad (11)$$

We introduce the saturation parameter $G$

$$G = 2|\Omega_0|^2/\gamma^2 = 2\mu^2 E_0^2/\hbar^2\gamma^2. \quad (12)$$

For small saturation parameter $G$, keeping only the terms up to $n = 1$, the gradient force consists of two parts $F_{\perp} = F_{\perp}^{\mathrm{ind}} + F_{\perp}^{\mathrm{sp}}$. One of them is called the gradient induced force $F_{\perp}^{\mathrm{ind}} = F_{\perp}^0 + F_{\perp,1}^c \cos 2kz = F_{\perp}^0 2\cos^2 kz$, the other one is called the gradient spontaneous force$F_{\perp}^{\mathrm{sp}} = F_{\perp,1}^s \sin 2kz$, where $F_{\perp}^0 = F_{\perp,1}^c$. The gradient induced force $F_{\perp}^{\mathrm{ind}}$ and the gradient spontaneous force $F_{\perp}^{\mathrm{sp}}$ affect the moving atom differently. The gradient induced force $F_{\perp}^{\mathrm{ind}}$ is presented by both constant and oscillating terms. This force modulates the velocity and position of the atom. The gradient spontaneous force $F_{\perp}^{\mathrm{sp}}$ accelerates the atoms directly. The direction of atomic acceleration depends on the sign of the frequency detuning $\delta$. We have chosen $\delta < 0$, which leads a tendency to trapping of slow atoms in the transverse direction.

The first three gradient force components $F_{\perp}^0$, $F_{\perp,1}^c$ and $F_{\perp,1}^s$ as a function of velocity are presented in Figs. 1 and 2, respectively.

From Fig. 1, it is seen that $F_{\perp}^0$ is an even function of the velocity $v$ and the curves are symmetric about $v = 0$. The force $F_{\perp}^0$ is along the gradient of the optical field intensity and is dispersive in character, being in the direction of the gradient when below resonance, and in the opposite direction when above resonance. When the field intensity increases, the curves display a rich structure that is assigned to multi-photon resonances. There is a little ridge appears near zero velocity due to the gradient spontaneous force $F_{\perp,1}^s$, which is heating in a small region of velocities around $v = 0$ (refer to Fig. 2(b)).

Figure 2 shows the behaviors of the force $F_{\perp,1}^c$ and $F_{\perp,1}^s$ as a function of the velocity $v$ and the saturation parameter $G$ with the fixed detuning $\delta = -10\gamma$. Figure 2(a) shows that the force $F_{\perp,1}^c$ is similar as $F_{\perp}^0$. Figure 2(b) shows that the force $F_{\perp,1}^s$ has the Doppleron structures at low intensity. For high intensity, the curves become considerably sharp. It is very interesting to note the change of direction over a small velocity range, a kink around $v = 0$. This velocity range becomes large when the saturation parameter $G$ and the detuning increase. The positive slope around zero velocity gives rise to heating effect of the

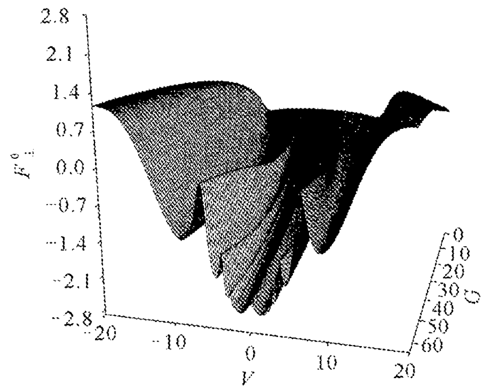


Fig. 1. The gradient induced force $F_{\perp}^0$ as a function of velocity $v$ and saturation parameter $G$. The detuning is $\delta = -10\gamma$. The force is in units of $\hbar k \gamma \frac{1}{ka}\frac{\rho}{a}$, velocity is in units of $\frac{\gamma}{k}$.

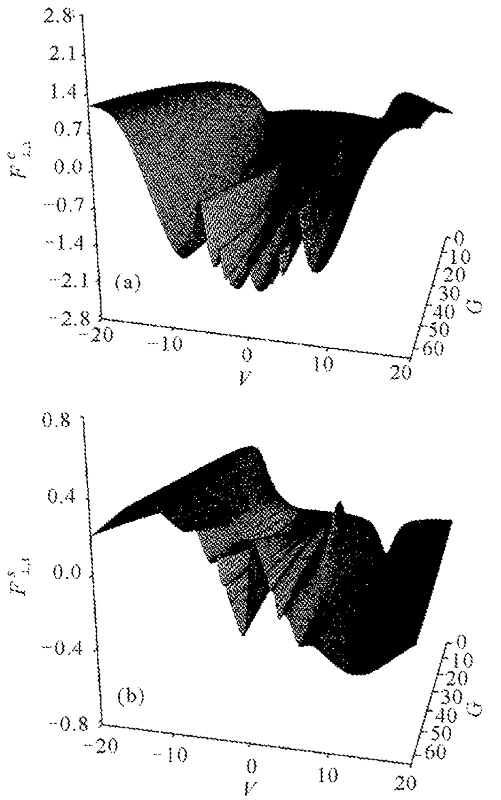


Fig. 2. (a) The gradient induced force $F_{\perp,1}^c$ and (b) the gradient spontaneous force $F_{\perp,1}^s$ ($n = 1$) as a function of velocity $v$ and saturation parameter $G$. The detuning is $\delta = -10\gamma$. The force is in units of $\hbar k \gamma \frac{1}{ka}\frac{\rho}{a}$, velocity is in units of $\frac{\gamma}{k}$.

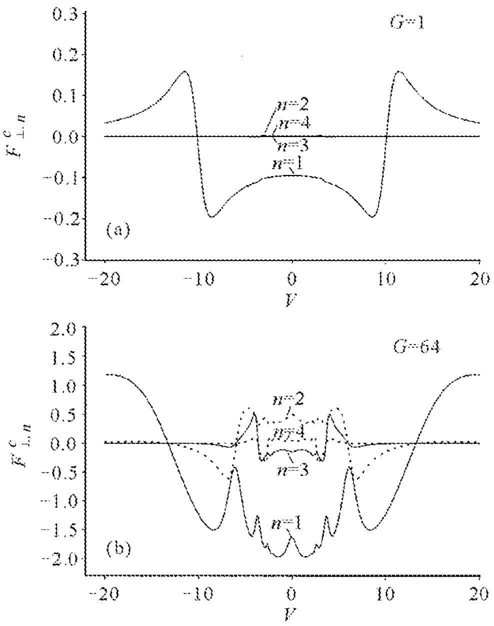


Fig. 3. The gradient induced force $F^c_{\perp,n}$ as a function of the velocity $v$ and the saturation parameter $G$ with $\delta = -10\gamma$. (a) $G = 1$; (b) $G = 64$.

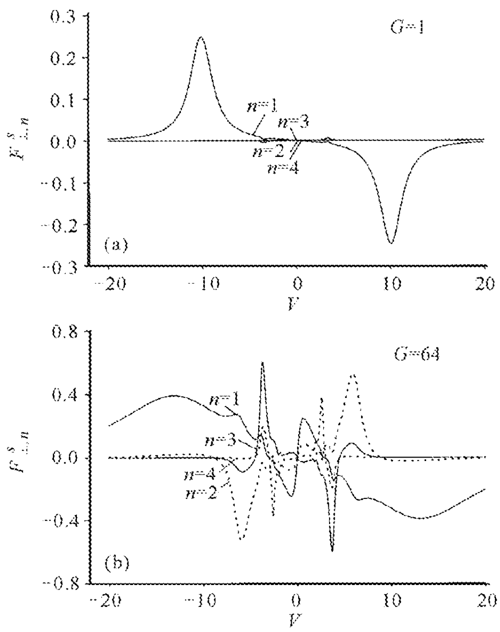


Fig. 4. The gradient spontaneous force $F^s_{\perp,n}$ as a function of the velocity $v$ and the saturation parameter $G$ with $\delta = -10\gamma$. (a) $G = 1$; (b) $G = 64$.

distribution. These correspond to higher-order processes and at higher intensities the multi-photon or Doppleron resonance starts to appear one by one. In a $(2n+1)$-multiphoton process, $n+1$ absorptions from one and $n$ stimulated emissions into the other traveling-wave component of the field is resonant at $kv = \pm|\delta|/(2n+1)$. When the laser intensity increases further, the Doppleron resonance is shifted towards higher velocities and is power broadened.

Figures 3 and 4 show the gradient induced force $F^c_{\perp,n}$ and the gradient spontaneous force $F^s_{\perp,n}$ (the detuning $\delta = -10\gamma$). It is seen that the gradient induced force $F^c_{\perp,n}$ is an even function of the velocity $v$, and the spontaneous one $F^s_{\perp,n}$ is an odd function of $v$.

For small saturation parameter with $G = 1$, $F^c_{\perp,n}$ and $F^s_{\perp,n}$ are plotted in Figs. 3(a) and 4(a), respectively. It is only the lower order gradient forces ($n = 0, 1$) have contributions. The higher order gradient forces ($n = 2, 3, 4, \cdots$) can be neglected. For high saturation parameter with $G = 64$, they are plotted in Figs. 3(b) and 4(b). It is seen that the high order terms of $n = 2, 3, 4, \cdots$ become more important and the contributes of them become large. At the same large saturation parameter $G$, the amplitudes of the force decrease with $n$. This means that the effects of the higher order forces become weak as the order $n$ increases.

The numerical results for the resonant gradient force have been presented. For weak laser intensity of small saturation parameter $G$, the higher order harmonic force can be neglected since their amplitudes are extremely small. However, for strong fields we must consider the higher order forces which will introduce scattering processes from one travelling wave to the other and increase the amplitudes. For large value of $G$, the amplitudes of the zero order and higher order gradient force are in the same order of magnitude. It is necessary to appear higher spatial harmonics in the expression for the gradient force.

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