The bright and dark photon number states in Young's interference experiment

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Received August 29, 2002

In this paper, we derive the bright and dark photon number states for spatial interference of two or more light beams and succeed in the explanation of Young's interference experiment, and also achieve a better comprehension of the well known comment of Dirac "each photon only interferences with itself". From the fully quantum point of view, the origin of the interference fringes consists in the mode transformation and the detection of double-slit states.

OCIS codes: 030.0030, 030.4070, 040.5570, 270.0270, 270.1670, 270.5290.

During past years, the coherent state, due to its properties similar to those of a classical electromagnetic wave in the limit of large amplitudes, has been studied early by the authors[1−4]. The coherent state is important also because the laser generates a coherent-state excitation of a cavity mode when operated well above threshold[5]. As for the light beams interference, the classical electromagnetic wave superposition and the first-order correlation $\langle E^*(r_1,t_1)E(r_2,t_2)\rangle$ analysis is enough to give a satisfying explanation of the fringes observed in experiment [6,7], regardless of the radiation statistical property[8]. However, from the fully quantum mechanical point of view, we first derive the bright and dark photon number states for spatial interference of two light beam and succeed in the explanation of fringes observed in Young's experiment, then generalize the discussion to the spatial interference of more light beams, and finally achieve a better comprehension of the well known Dirac's comment "Each photon only interferes with itself. Interference between two different photons never occurs"[9].

Figure 1 shows a simplified version of Young's experiment. The light from a point source S is rendered parallel by a lens L_1, falls on a screen S_1 which contains two slits and then focuses on S_2 by L_2. Interference fringes are sought on second screen S_2 to the right of the first screen. The Hamiltonian of the photon on S_1 can be written as the sum of $a_1^+a_1$ on slit 1 and $a_2^+a_2$ on slit 2, where a_1^+, a_1; a_2^+, a_2 denote the creation and annihilation operators for photon on slit 1 and slit 2 respectively. Now considering a detector localized at a point P on screen S_2 detects the photon when passing through either slit 1 or slit 2. Thus the Hamiltonian of the photon on S_2 should be $\frac{a_1^+\pm a_2^+}{\sqrt{2}}\frac{a_1\pm a_2}{\sqrt{2}}$, the conservation of Hamiltonian is observed,

$$\begin{aligned} H &= \hbar\omega\left(a_1^+a_1 + a_2^+a_2\right) \\ &= \hbar\omega\left(\frac{a_1^+ + a_2^+}{\sqrt{2}}\frac{a_1+a_2}{\sqrt{2}} + \frac{a_1^+ - a_2^+}{\sqrt{2}}\frac{a_1-a_2}{\sqrt{2}}\right) \\ &= \hbar\omega\left(s^+s + t^+t\right), \end{aligned} \quad (1)$$

where $s = \frac{a_1+a_2}{\sqrt{2}}$, $t = \frac{a_1-a_2}{\sqrt{2}}$. It is easy to prove the following relation

$$\begin{aligned} &st^+ - t^+s = s^+t - ts^+ = 0, \\ &ss^+ - s^+s = tt^+ - t^+t = 1, \\ &ss^{+n} - s^{+n}s = ns^{+(n-1)}, \\ &s^+s^n - s^ns^+ = -ns^{(n-1)}, \\ &tt^{+n} - t^{+n}t = nt^{+(n-1)}, \\ &t^+t^n - t^nt^+ = -nt^{(n-1)}. \end{aligned} \quad (2)$$

Assuming the vacuum state for the Hamiltonian H being $\Psi_{0,0} = |0,0\rangle$, i.e. $s\Psi_{0,0} = t\Psi_{0,0} = 0$, then the Hamiltonian H has the eigenvalue $n\hbar\omega$ and eigenstates $\Psi_{n-m,m}$,

$$\Psi_{n-m,m} = \frac{1}{\sqrt{(n-m)!m!}}s^{+(n-m)}t^{+m}\Psi_{0,0}, \quad m = 0,1,\cdots,n, \quad (3)$$

$$\begin{aligned} H\Psi_{n-m,m} &= \hbar\omega(s^+s + t^+t)\Psi_{n-m,m} \\ &= \hbar\omega(n-m+m)\Psi_{n-m,m} \\ &= n\hbar\omega\Psi_{n-m,m}. \end{aligned} \quad (4)$$

The number operators $N_s = s^+s$, $N_t = t^+t$ have the eigenvalues $n-m$ and m respectively. In the following we will demonstrate that the operators N_s, N_t can be used to describe the bright and dark fringes detected on S_2, thus we call N_s, N_t bright and dark operators. The corresponding eigenvalues $n-m$ and m being the number of brighton (photon in bright state) and darkon (photon in dark state), the total number is n. According to

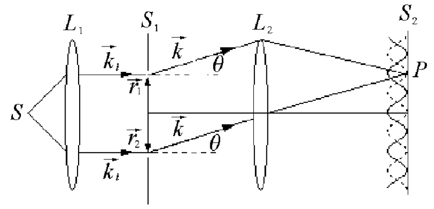

Fig. 1. Two slits interference experiment. $|\vec{r}_1| = |\vec{r}_2| = d/2$.

Eq. (3), we write out the eigenfunctions for $n = 0, 1, 2$ immediately,

$$\begin{aligned} n=0 \quad & \Psi_{0,0} = |0,0\rangle, \\ n=1 \quad & \Psi_{1,0} = s^{+}|0,0\rangle = \frac{|1,0\rangle + |0,1\rangle}{\sqrt{2}}, \\ & \Psi_{0,1} = t^{+}|0,0\rangle = \frac{|1,0\rangle - |0,1\rangle}{\sqrt{2}}, \\ n=2 \quad & \Psi_{2,0} = \frac{s^{+2}}{\sqrt{2}}|0,0\rangle = \frac{|2,0\rangle + \sqrt{2}|1,1\rangle + |0,2\rangle}{2}, \\ & \Psi_{1,1} = s^{+}t^{+}|0,0\rangle = \frac{|2,0\rangle - |0,2\rangle}{\sqrt{2}}, \\ & \Psi_{0,2} = \frac{t^{+2}}{\sqrt{2}}|0,0\rangle = \frac{|2,0\rangle - \sqrt{2}|1,1\rangle + |0,2\rangle}{2}. \end{aligned} \quad (5)$$

The photon number operators on S_2 are $N_s = s^{+}s$, $N_t = t^{+}t$, imply that the direction of observation $\vec{k}$ coincide with that of incidence $\vec{k}_i$ (for simplification we assume $\vec{k}_i$ being perpendicular to S_1). In the case of the observation, direction $\vec{k}$ deviates from the incidence direction $\vec{k}_i$ by an angle θ as shown in Fig. 1, according to Dirac[9], "For a photon to be in a definite state of motion it need not be associated with one single beam of light, but may be associated with two or more beams of light which are the components into which one original beam has been split". We may associate a plane wave $\vec{\varepsilon}_{ks}\mathrm{e}^{i\vec{k}\cdot\vec{r}_1 - i\omega t}$ ($\vec{\varepsilon}_{ks}\mathrm{e}^{i\vec{k}\cdot\vec{r}_2 - i\omega t}$) with the photon passing through slit 1 (slit 2), where $|\vec{r}_1| = |\vec{r}_2| = d/2$, $\vec{\varepsilon}_{ks}$ is the unit polarization vector, and it will be omitted for brevity. We express the operator $N_s(\theta)$ in the form

$$\begin{aligned} N_s(\theta) &= \frac{a_1^{+}\mathrm{e}^{i\varphi_1} + a_2^{+}\mathrm{e}^{i\varphi_2}}{\sqrt{2}} \frac{a_1^{+}\mathrm{e}^{-i\varphi_1} + a_2^{+}\mathrm{e}^{-i\varphi_2}}{\sqrt{2}} \\ &= \left(\cos\frac{\varphi}{2}s^{+} + i\sin\frac{\varphi}{2}t^{+}\right)\left(\cos\frac{\varphi}{2}s - i\sin\frac{\varphi}{2}t\right) \\ &= \cos^2\frac{\varphi}{2}N_s + \sin^2\frac{\varphi}{2}N_t - i\sin\varphi\frac{s^{+}t - t^{+}s}{2}, \end{aligned} \quad (6)$$

where $\varphi_1 = \vec{k}\cdot\vec{r}_1 - \omega t$, $\varphi_2 = \vec{k}\cdot\vec{r}_2 - \omega t$, $\varphi = \varphi_1 - \varphi_2 = kd\sin\theta$. For one photon, $n = 1$, the expectation value of $N_s(\theta)$ in states $\Psi_{1,0}$, $\Psi_{0,1}$ are

$$\begin{aligned} \langle\Psi_{1,0}^{+}N_s(\theta)\Psi_{1,0}\rangle &= \cos^2\frac{\varphi}{2}\langle\Psi_{1,0}^{+}N_s\Psi_{1,0}\rangle = \cos^2\frac{\varphi}{2}, \\ \langle\Psi_{0,1}^{+}N_s(\theta)\Psi_{0,1}\rangle &= \sin^2\frac{\varphi}{2}\langle\Psi_{0,1}^{+}N_t\Psi_{0,1}\rangle = \sin^2\frac{\varphi}{2}. \end{aligned} \quad (7)$$

The variation of $\langle\Psi_{1,0}^{+}N_s(\theta)\Psi_{1,0}\rangle$ versus $\varphi = kd\sin\theta$ is depicted by the solid curve on S_2 in Fig. 1 which corresponds to the bright fringes observed in Young's experiment, while $\langle\Psi_{0,1}^{+}N_s(\theta)\Psi_{0,1}\rangle$ versus φ is depicted by the dotted curve which corresponds to the dark fringes, no response by the detector. Therefore the photon on S_1 can always be detected, no matter what its position of incidence on slits 1 or 2. But after the transformation

$$\begin{pmatrix} \Psi_{1,0} \\ \Psi_{0,1} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} |1,0\rangle \\ |0,1\rangle \end{pmatrix}, \quad (8)$$

only the photon in bright state $\Psi_{1,0}$, $\langle\Psi_{1,0}^{+}N_s\Psi_{1,0}\rangle = 1$ can be detected, the detector gives a response. While photon in dark state $\Psi_{0,1}$, $\langle\Psi_{0,1}^{+}N_s\Psi_{0,1}\rangle = 1$ can not be detected, the detector gives no response (although not be detected, it is in existence). The probability distribution among the "bright" and "dark" states given by Eq. (7) is very similar to the polarization experiment. "The photon with its polarization at angle α to the crystal optic axis, has a probability $\sin^2\alpha$ passing through the tourmaline and appearing on the back side polarized perpendicular to the axis and a probability $\cos^2\alpha$ of being absorbed"[9]. Moreover, the model of detection, responses only to "bright" state and no response to "dark" state, i.e. "selection response detection" (SRD), is just one of the possibilities. Of course, one may envisage the other possibility besides SRD, for instance, the photon emitted by the point source S, usually in single mode as required in the interference experiment, jumps into the "bright" state $\Psi_{1,0}$ rather the "dark" state $\Psi_{0,1}$, i.e. "selection jumping detection" (SJD). Hence the photon in "dark" state is empty. The observation of dark fringes in Young's experiment is a natural consequence. A further discussion will be given later.

Now we generalize the above discussion to multi-slit interference. Let $b_0, b_1, \cdots, b_{N-1}$ be the annihilation operators on slits $0, 1, \cdots, N-1$ on S_1, satisfying the relation

$$b_l b_m^{+} - b_m^{+} b_l = \delta_{l,m}, \quad (9)$$

and $s_0, s_1, \cdots, s_{N-1}$ be the annihilation operators on S_2, the state (mode) transformation is

$$\begin{pmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-1} \end{pmatrix} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & \omega^{N-1} & \cdots & \omega^{(N-1)^2} \end{pmatrix} \cdot \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{pmatrix}, \quad (10)$$

where $\omega = \mathrm{e}^{i\frac{2\pi}{N}}$, and

$$\begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{N-1} \end{pmatrix} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega^{-1} & \cdots & \omega^{-(N-1)} \\ \vdots & \vdots & & \vdots \\ 1 & \omega^{-(N-1)} & \cdots & \omega^{-(N-1)^2} \end{pmatrix} \cdot \begin{pmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-1} \end{pmatrix}. \quad (11)$$

From Eqs. (10) and (11), it is easy to prove

$$\begin{aligned} s_i s_j^{+} - s_j^{+} s_i &= \frac{1}{N}\sum_{l,m}\omega^{li-mj}(b_l b_m^{+} - b_m^{+} b_l) \\ &= \frac{1}{N}\sum_{l}\omega^{(i-j)l} = \delta_{i,j}. \end{aligned} \quad (12)$$

Refer to the two slits results, we have

$$\Psi_{0,0,\cdots,0} = |0,0,\cdots,0\rangle,$$
$$\Psi_{m_0,m_1,\cdots,m_{N-1}} = \frac{1}{\sqrt{m_0!m_1!\cdots m_{N-1}!}} \cdot s_0^{+m_0} s_1^{+m_1} \cdots s_{N-1}^{+m_{N-1}} \Psi_{0,0,\cdots,0}. \quad (13)$$

The Hamiltonian H for the system can be expressed as

$$H = \sum_{i=0}^{N-1} b_i^+ b_i = \sum_{i=0}^{N-1} s_i^+ s_i,$$
$$H\Psi_{m_0,m_1,\cdots,m_{N-1}} = (m_0 + m_1 + \cdots + m_{N-1})\Psi_{m_0,m_1,\cdots,m_{N-1}}, \quad (14)$$

where m_0 is the number of brighton, the others $m_1, \cdots, m_{N-1}$ are the number of darkon. The "bright" state operators N_s and $N_s(\theta)$ read as

$$N_s = \frac{b_0^+ + b_1^+ + \cdots + b_{N-1}^+}{\sqrt{N}} \frac{b_0 + b_1 + \cdots + b_N}{\sqrt{N}} = s_0^+ s_0,$$
$$N_s(\theta) = \frac{b_0^+ + b_1^+ e^{i\varphi} + \cdots + b_{N-1}^+ e^{i(N-1)\varphi}}{\sqrt{N}} \times \frac{b_0^+ + b_1 e^{-i\varphi} + \cdots + b_{N-1} e^{-i(N-1)\varphi}}{\sqrt{N}}$$
$$= \sum_{j,k=0}^{N-1} A_j^+ A_k s_j^+ s_k, \quad (15)$$

where

$$A_j^+ = \frac{1}{N}\left(1 + \omega^j e^{i\varphi} + \cdots + \omega^{j(N-1)} e^{i(N-1)\varphi}\right) = \frac{1}{N} \frac{\sin\frac{N}{2}\left(\varphi + j\frac{2\pi}{N}\right)}{\sin\frac{1}{2}\left(\varphi + j\frac{2\pi}{N}\right)} e^{i\frac{N-1}{2}\left(\varphi + j\frac{2\pi}{N}\right)},$$
$$A_k = \frac{1}{N}\left(1 + \omega^{-k} e^{-i\varphi} + \cdots + \omega^{-k(N-1)} e^{-i(N-1)\varphi}\right) = \frac{1}{N} \frac{\sin\frac{N}{2}\left(\varphi + k\frac{2\pi}{N}\right)}{\sin\frac{1}{2}\left(\varphi + k\frac{2\pi}{N}\right)} e^{-i\frac{N-1}{2}\left(\varphi + k\frac{2\pi}{N}\right)}. \quad (16)$$

We note

$$\langle \Psi^+_{m_0,m_1,\cdots,m_{N-1}} s_j^+ s_k \Psi_{m_0,m_1,\cdots,m_{N-1}} \rangle = m_k \delta_{jk}. \quad (17)$$

From Eqs. (15) and (17), the expectation value for $N_s(\theta)$ is

$$\langle \Psi^+_{m_0,m_1,\cdots,m_{N-1}} N_s(\theta) \Psi_{m_0,m_1,\cdots,m_{N-1}} \rangle = m_0 A_0^2 + \sum_{i=1}^{N-1} m_i A_i^2. \quad (18)$$

After detection, only the first "bright" term $m_0 A_0^2$ is kept, and the bright fringes observed in multi-slit interference experiment is

$$m_0 A_0^2 = \langle \Psi^+_{m_0,m_1,\cdots,m_{N-1}} N_s \Psi_{m_0,m_1,\cdots,m_{N-1}} \rangle \cdot \left(\frac{1}{N} \sin N\frac{\varphi}{2} / \sin\frac{\varphi}{2}\right)^2, \quad (19)$$

while the other "dark" term

$$\sum_{i=1}^{N-1} m_i A_i^2$$

is deleted. For $N = 2$, $m_0 = 1$, $m_1 = 0$, Eq. (19) reduces to the first part of Eq. (7), $\langle \Psi^+_{1,0} N_s(\theta) \Psi_{1,0} \rangle = \cos^2\frac{\varphi}{2}$.

Finally we give a brief discussion about the following points.

1. The double-slit states and beam-split states.

According to Ref. [8], the Mach-Zehnder temporal interference experiment is shown in Fig. 2. The photon counts in outputs MZ1 and MZ2, as a function of path difference Δ, are proportional to $\cos^2\frac{\Delta}{2}$ and $\sin^2\frac{\Delta}{2}$ respectively. Using $\Phi_{1,0}$ ($\Phi_{0,1}$) to denote the single photon state transmitted through (reflected from) the beam-split BS2 to MZ1 (MZ2), we have $\Phi_{1,0} = \frac{a_1^+ + a_2^+}{\sqrt{2}} \Phi_{0,0} = s^+ \Phi_{0,0}$, $\Phi_{0,1} = \frac{a_1^+ - a_2^+}{\sqrt{2}} \Phi_{0,0} = t^+ \Phi_{0,0}$, where a_1^+, a_2^+ denote creation operators through different path marked on Fig. 2. We call $\Phi_{1,0}$, $\Phi_{0,1}$ as the beam-split states, whereas $\Psi_{1,0}$, $\Psi_{0,1}$ in Young's double-slit spatial interference experiment as the double-slit states. Mathematically the beam-split states $\Phi_{1,0}$, $\Phi_{0,1}$ are similarly to the double-slit states $\Psi_{1,0}$, $\Psi_{0,1}$, yet physically they are quite different. Because the states $\Phi_{1,0}$, $\Phi_{0,1}$ are separable in space, which can be detected on MZ1 and MZ2 respectively. Referring to Fig. 2, the two beams transmitted and reflected to MZ2 have an additional phase difference π in comparison with that to MZ1, thus the state $\Phi_{0,1}$ arrived at MZ2 is changed from "dark" to "bright", so finally all of $\Phi_{1,0}$, $\Phi_{0,1}$ are bright states, the direct photon numbers detections on MZ1, MZ2 provide a check of the energy conservation $\cos^2\frac{\Delta}{2} + \sin^2\frac{\Delta}{2} = 1$. Whereas the double-slit states $\Psi_{1,0}$, $\Psi_{0,1}$ are inseparable in space. Only the bright state $\Psi_{1,0}$ can be detected, the dark state $\Psi_{0,1}$ cannot be detected. Using the bright photon detection $\propto \cos^2\frac{\varphi}{2}$ and the conservation of energy, we can deduce indirectly the dark photon distribution $\sin^2\frac{\varphi}{2} = 1 - \cos^2\frac{\varphi}{2}$, but a direct detection of dark photon distribution is impossible. However, we cannot do the same thing for double-slit state $\Psi_{0,1}$, changed from "dark" to "bright" by adding a π phase plate in front of slit 2, because at the same time, state $\Psi_{1,0}$ is changed from "bright" to "dark". In one word, we cannot prepare all of $\Psi_{1,0}$, $\Psi_{0,1}$ in the bright states at the same time. In any case, there must be one in bright state, and the other in dark state.

2. The detection of double-slit states on S_2.

From the fully quantum mechanics analysis given above, after mode transformation, the eigenmode double-slit states $\Psi_{1,0}$, $\Psi_{0,1}$ appear. These just provide the Bose

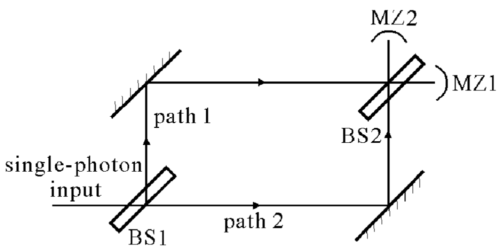

Fig. 2. Mach-Zehnder interference experiment.

cells. One or more photons passing through the slits can jump into one of the cells obeying Bose-Einstein statistics. The next is the detection on S_2. The SRD implies that the detection on S_2 gives response only to the "bright" state $\Psi_{1,0}$ rather than the "dark" state $\Psi_{0,1}$. On the other hand, the SJD asserts the single mode photon jumps into "bright" state rather than "dark" state. Thus for a stationary point light source S (that is the long-time average intensity, $\bar{I}$ is fixed and independent of the particular long period of time chosen to be measured), the single photon counts of N_1 plus N_2 detected at slits 1, 2 on S_1 should be equal to that of $N_{\rm B}$ in "bright" state on S_2, i.e. $N_1 + N_2 = N_{\rm B}$, because the photon in "dark" state is empty. If the single photon counts experiments confirm this equality, SJD is the correct model. However, if the experimental data shows that the equality is violated, namely $N_{\rm D} = N_1 + N_2 - N_{\rm B} > 0$ by conservation of energy, the fact suggests that the photon in "dark" state really exists, but cannot be detected. Now no matter what the real detection mechanism being, the essential point is that the eigenmodes $\Psi_{1,0}$, $\Psi_{0,1}$, therefore the spatial distribution $\cos^2\frac{\varphi}{2}$, $\sin^2\frac{\varphi}{2}$ associated, and the detection (SRD or SJD) depends only on the detection system (including two slits, focusing lens and detector position), has nothing to do with the photon number and the light source S being classical or non-classical (single-photon states)[8].

3. On the fringes observed in Young's experiment.

As mentioned above, the classical electromagnetic theory provides the "wave superposition and interference" picture for the light beam interference. Up to Dirac, "interference between different photons" is excluded, however "each photon only interferes with itself" is still preserved. This suggests that the fringes observed are due to the "photon interferes with itself". The question is that the spatial distribution $\cos^2\frac{\varphi}{2}$ of observed fringes is completely determined before the arrival of the photon. So we think a better comprehension of Dirac's comments may be interpreted as "photon jumps into an eigenmode with fringes pattern determined by the detection system".

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