Application of wavelet transform to 3D shape measurement

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A new method for analyzing the phase distributions of deformed grating images on the surface of three-dimensional (3D) object to obtain its shape information has been presented. In the conventional technique, Fourier transform profilometry (FTP), there is an intrinsic problem of extracting the fundamental frequency component if the deformation of the grating pattern is either considerable or complicated, which will definitely bring bad influence to the analysis' accuracy. That means FTP is not appropriate to deal with the complex surfaces of 3D objects. The approach that we here introduce to solve this problem is to utilize Gabor wavelet transform (GWT), a tool excelling for its multiresolution in time-frequency domain, to analyze the phase distributions.

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Among those techniques of optical three-dimensional (3D) shape measurement that play important roles in various engineering areas, Fourier transform profilometry (FTP), proposed by Takeda *et al.*[1], is the most popular one[2,3]. In this method, the grating pattern projected on the object surface is Fourier-transformed to obtain the spatial frequency spectra. Afterwards we merely extract the fundamental spectrum component to calculate its inverse Fourier transform values and then derive a complex signal, which should correspond exactly to the intensity distribution of the grating. The phase of the complex signal contains the information of the displacement of the grating stripes and can be converted to the object's 3D shape. FTP has proved more accurate than the classic method of Moiré interferometry because it analyzes the phase values at all pixel positions of the grating pattern rather than the central positions of the grating stripes. Furthermore, comparing to those improved Moiré methods, FTP is still more suitable for automatic measurement through computer processing. However, if the single spectrum of the fundamental frequency component is extracted incompletely, usually owing to its being superposed by other higher components, the accuracy of FTP would reduce. This problem is inevitable for FTP when the deformation of the grating pattern is either considerable or complicated.

This paper presents a novel approach to the analysis of the phase distribution by applying Gabor wavelet transform (GWT). With the advantage of multiresolution and efficiency in local analysis of nonstationary and transient signals, GWT can overcome the difficulty for FTP mentioned above.

The wavelet transform (WT) has enjoyed a tremendous popularity and notable development in the last decade. Far beyond its original purpose in the geophysical signal analysis, WT has been recognized to be a versatile tool with very rich mathematical contents and great potential for the investigation of a multitude of diverse phenomena[4,5].

As we know, windowed Fourier transform (WFT) has the shortcoming of inaccuracy in time-frequency localization, which arises from the aliasing of neighboring frequency spectral components[4]. Because WFT chooses only one window function for the entire frequency domain, the resolution of the analysis is fixed at all locations in the time-frequency plane according to the Gabor-Heisenberg-Wegl Uncertainty Principle. So we can not expect WFT to be an appropriate tool for the analysis of the signals that cover a wide range of frequency and contain nonstationary parts. Taking the advantage of varying window size, WT is capable of dealing with WFT's dilemma of resolution to a certain extent and that is why WT comes into being. WT has short window functions and long ones for high- and low-frequency components, respectively, which can achieve good resolutions in both frequency domain and time domain.

To analyze the signal, we firstly build up a family of basis functions, namely "daughter wavelets", as

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\psi\left(\frac{x-b}{a}\right), \tag{1}$$

which leads to the WT of a given function or data signal, $S(x)$

$$W(a,b) = \int_{-\infty}^{\infty} S(x)\frac{1}{\sqrt{a}}\psi^*\left(\frac{x-b}{a}\right)\mathrm{d}x, \tag{2}$$

where $a > 0$, $-\infty < b < \infty$ and the asterisk superscript "*" indicates the complex conjugate. The daughter wavelets $\psi_{a,b}(x)$ in Eq. (1) are obtained from dilation and translation with the factors a and b, respectively and a fixed basis function $\psi(x)$ is known as the "mother wavelet". The normalization factor $\frac{1}{\sqrt{a}}$ in Eq. (1) ensures that $\psi_{a,b}(x)$ has a constant norm

$$\int_{-\infty}^{\infty} |\psi_{a,b}(x)|^2 \mathrm{d}x = 1 \tag{3}$$

in the space of square integrable functions.

The invertability of a WT requires

$$C_\Psi = \int_{-\infty}^{\infty} \frac{|\Psi(\mu)|^2}{\mu}\mathrm{d}\mu < \infty, \tag{4}$$

where $\Psi(\mu)$ is the Fourier transform of $\psi(x)$. Any function $\psi(x)$ satisfying this "admissibility condition" can serve as a "mother wavelet". This means that WTs do not only have a uniform bank of basis functions like the Fourier transform, which employs just the sine and cosine functions. We here adopt Gabor function as the "mother wavelet"

$$\psi(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) \exp(j\mu x), \quad (5)$$

where σ is a spread constant. It is a complex exponential enveloped by a Gaussian, which can achieve the lowest possible conjoint uncertainty (i.e. minimal dispersion) in both the space domain and frequency domain. The set of "daughter wavelets" generated from this "mother wavelet" can then be called Gabor wavelets.

Phase analysis is a key step during the whole procedure of the grating image data processing. When a signal has been Gabor Wavelet transformed, its phase is to be defined as the inverse tangent of the quotient of the imaginary part by dividing the real part

$$\phi = \arctan\left[\frac{\text{imag}(W(a,b))}{\text{real}(W(a,b))}\right], \quad (6)$$

and its amplitude is to be defined as

$$A = \sqrt{[\text{imag}(W(a,b))]^2 + [\text{real}(W(a,b))]^2}. \quad (7)$$

In this case, both the phase ϕ and the amplitude A are two-dimensional data, as they are the functions of the two variants a and b. What we need to do is to find out the most appropriate scale a at every position b from the figures of phase and amplitude of the GWT result.

We here introduce this step by using a simulation example. Figure 1 shows a simulating deformed one dimensional (1D) sinusoidal grating image. The displacement disp(x) at every position ($x = 0, 1, \cdots 511$) of the grating is given by

$$\text{disp}(x) = 16\left[\exp\left(1 - \cos\frac{2\pi}{L}x\right) - 1\right], \quad (8)$$

where L ($L = 512$) is the length of the 1D grating.

The amplitude and phase of WT to the deformed grating are shown in Figs. 2 and 3, respectively. In the two figures, Dilation factor a signifies the wavelength scale while translation factor b corresponds to the center position of the shifting window. As we know, the first spectrum component that we need consists of the maximum amplitudes at every position b and constitutes the entire spectra together with other components. Therefore we can directly obtain the local information about the fundamental spectrum from Fig. 2.

Next we look up the corresponding scales to the maximum amplitudes at every position b and extract the phase of WT from Fig. 3 according to the scales derived previously. Figure 4 presents the calculating result which is a wrapped phase distribution.

We then unwrap the phase result to verify its accuracy. By comparing the analyzed displacement with the actual one (Fig. 5), we can find them in good agreement.

We also test this analysis method through experiment. Figure 6 shows the original grating image projected on a plaster model of a boy head. The width of the grating pitch is set to be 8 pixels by the computer. The analyzed phase of Fig. 6 is shown in Fig. 7, which procedure and the simulation are alike. Figure 8 displays the reconstruction of the head model's 3D shape calculated through Gabor wavelet.

Fig. 1. Intensity of a deformed 1D grating.

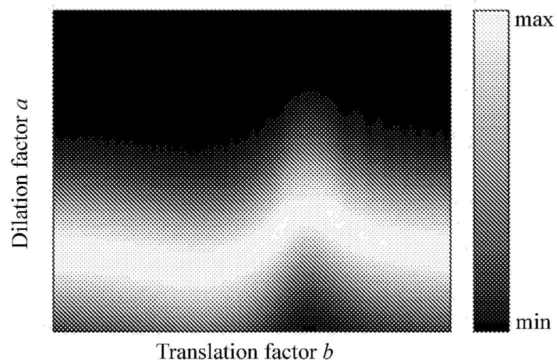

Fig. 2. Amplitude of GWT to deformed grating.

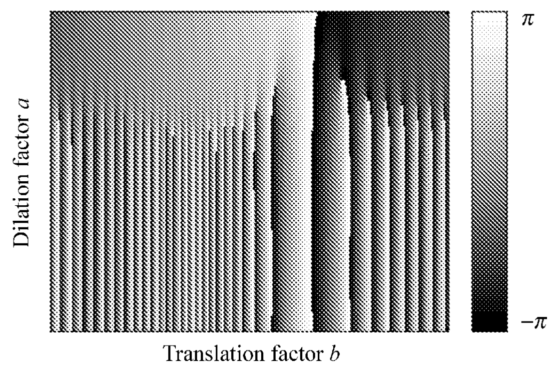

Fig. 3. Phase of GWT to deformed grating.

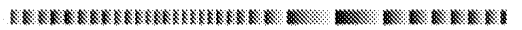

Fig. 4. Wrapped phase distribution of deformed grating.

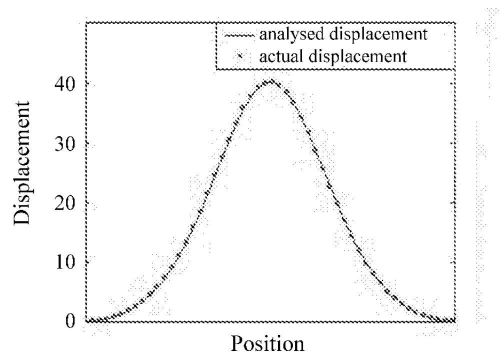

Fig. 5. Comparison between analyzed displacement with actual displacement.

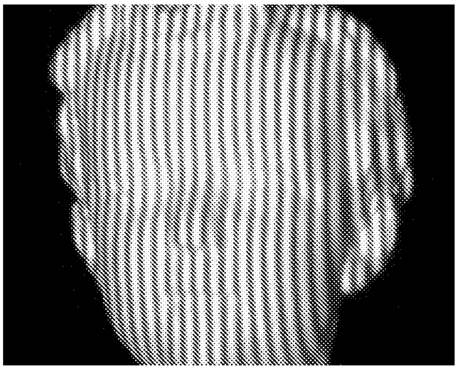

Fig. 6. Original grating image.

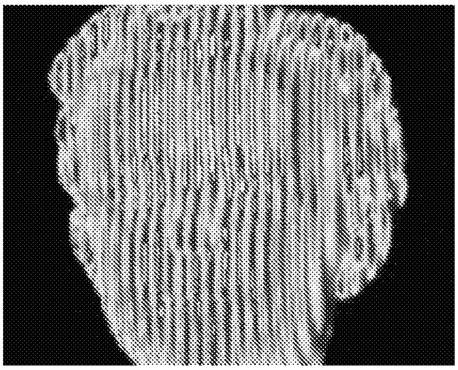

Fig. 7. Analyzed phase.

In summary, we have brought forward and tested a novel method for analyzing the phase distributions of deformed grating images on the surface of 3D object. The new method uses GWT, which makes it better than FTP in applications. The simulation results and the

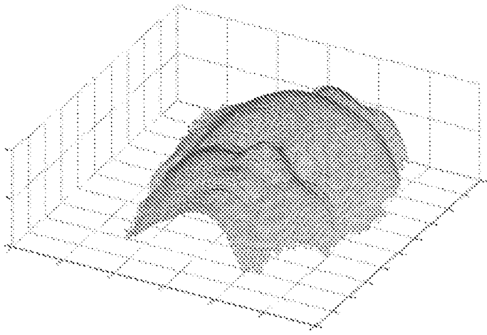

Fig. 8. 3D shape reconstruction.

experimental demonstrations show its effectiveness. We can expect some more promising outcomes in the further research.

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