Emission spectra of a Ξ-type three-level atom in a Kerr medium

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We investigate the emission spectra of a Ξ-type three-level atom interacting with a single-mode optical field in an ideal cavity filled with a Kerr medium and discuss the structure of emission spectrum when the optical field is initially in a pure number state and a coherent state, respectively. It is shown that the structure of emission spectrum depends not only on the photon number distribution, but also on the strength of incident field and the coupling of Kerr medium to the field.

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The theoretical work in optical spectra, particularly, varieties of phenomenological spectrum structures based on the Jaynes-Cummings model (JCM), was well-studied and had played a central role in quantum optics over the past two decades[1−6]. More recently, a lot of interest has been focused on the structure of emission spectra for a three-level atom, which was modeled as a generalized JCM[7−9]. Typically, Ashraf discussed the emission spectra for a Λ-type quantum-beat three-level atom, which interacts with a single-mode radiation field, and pointed out that they are generally four-peak structure and can reduce to three-peak one when the incident photon number is considerably large[9]. In this paper we will discuss the emission spectra for a Ξ-type three-level atom under the conditions with Kerr effect and without Kerr effect.

We consider the interacting system of a Ξ-type three-level atom (one upper level e_A, two lower levels e_B and e_C, and the energy eigenstate is $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively) and single-mode cavity field with a Kerr medium. It is also assumed that the separations between e_A and e_B, e_B and e_C are equal and the double carrier frequency of the mode field ω_0 is equal to the frequency ω_{ac} of an optically forbidden atomic transition $|a\rangle \to |c\rangle$, i.e., $\omega_{ab} = \omega_{bc} = \omega_0$. Moreover, the transitions $|a\rangle \to |b\rangle$ and $|b\rangle \to |c\rangle$ are optically allowed. The Hamiltonian for such a system can be written in the rotating wave approximation as (let $\hbar = 1$)

$$\begin{aligned} H = \omega_0(R_{aa} - R_{cc}) + \omega_0 a^+ a + \chi(a^+ a)^2 \\ + g(aR_{ab} + a^+ R_{ba} + aR_{bc} + a^+ R_{cb}) \end{aligned} \quad (1)$$

where a^+ and a are the creation and annihilation operators of the mode field, $R_{ij}(i, j = a, b, c)$ denotes the pseudo-spin operators of the three-level atom, g is the coupling strength of the field-atom interaction, and χ is the coupling constant of Kerr medium and cavity.

The solution of the above model can be obtained by using the so-called dress-state representation method. In this approach we find the eigenvalues and eigenfunctions of the Hamiltonian [Eq. (1)], which are given by

$$\begin{aligned} E_n^{(j)} = (n+1)\omega_0 + \varepsilon g[(n+1)^2 + 1] + g\delta_n^{(j)}, \\ (n \geq 0, j = 1,2,3; n = -1, j = 1,2), \end{aligned} \quad (2)$$

$$|\phi_n^{(j)}\rangle = \begin{cases} \alpha_n^{(j)}|n,a\rangle + \beta_n^{(j)}|n+1,b\rangle + \gamma_n^{(j)}|n+2,c\rangle, \\ \quad (n \geq 0) \\ \beta_n^{(j)}|n+1,b\rangle + \gamma_n^{(j)}|n+2,c\rangle, \\ \quad (n = -1) \end{cases} \quad (3)$$

respectively, where the parameters in Eqs. (2) and (3), can be defined as (let $\varepsilon = \chi/g$)
for $\varepsilon = 0$,

$$\begin{cases} \delta_n^{(1)} = \sqrt{2n+3} = C_n, \delta_n^{(2)} = -C_n, \delta_n^{(3)} = 0, \\ \quad (n \geq 0) \\ \delta_n^{(1)} = 1, \delta_n^{(2)} = -1, \\ \quad (n = -1) \end{cases} \quad (4)$$

$$\begin{cases} \alpha_n^{(1)} = \alpha_n^{(2)} = \frac{\sqrt{n+1}}{\sqrt{2}C_n}, \beta_n^{(1)} = -\beta_n^{(2)} = \frac{1}{\sqrt{2}C_n}, \\ \gamma_n^{(1)} = \gamma_n^{(2)} = \frac{\sqrt{n+2}}{\sqrt{2}C_n}, \\ \alpha_n^{(3)} = \frac{\sqrt{n+2}}{C_n}, \beta_n^{(3)} = 0, \gamma_n^{(3)} = -\frac{\sqrt{n+1}}{C_n}, \\ \quad (n \geq 0) \end{cases} \quad (5a)$$

$$\beta_n^{(1)} = \gamma_n^{(1)} = -\beta_n^{(2)} = \gamma_n^{(2)} = \frac{1}{\sqrt{2}}, \quad (n = -1); \quad (5b)$$

for $\varepsilon \neq 0$,

$$\begin{cases} \delta_n^{(j)} = \frac{2}{3}A_n \cos\left[\theta_n + (j-1)\frac{2\pi}{3}\right] - \frac{\varepsilon}{3}, \\ \alpha_n^{(j)} = \sqrt{n+1}[\delta_n^{(j)} - 2(n+1)\varepsilon](D_n^{(j)})^{-1}, \\ \beta_n^{(j)} = [(\delta_n^{(j)})^2 - 4(n+1)^2\varepsilon^2](D_n^{(j)})^{-1}, \\ \gamma_n^{(j)} = \sqrt{n+2}[\delta_n^{(j)} + 2(n+1)\varepsilon](D_n^{(j)})^{-1}, \\ D_n^{(j)} = \{(n+1)[\delta_n^{(j)} - 2(n+1)\varepsilon]^2 \\ \quad +[(\delta_n^{(j)})^2 - 4(n+1)^2\varepsilon^2]^2 \\ \quad +(n+2)[\delta_n^{(j)} + 2(n+1)\varepsilon]^2\}^{1/2}, \\ \theta_n = \frac{1}{3}\arccos\left[\frac{B_n}{2(A_n)^2}\right], \\ A_n = [9 + 6n + 12(n+1)^2\varepsilon^2 + \varepsilon^2]^{1/2}, \\ B_n = [27 + 36n + 72(n+1)^2\varepsilon^2 - 2\varepsilon^2]\varepsilon, \\ \quad (n \geq 0, j = 1,2,3) \end{cases} \quad (6)$$

$$\begin{cases} \delta_n^{(j)} = -\frac{1}{2}[\varepsilon + (-1)^j\sqrt{\varepsilon^2 + 4}], \\ \beta_n^{(j)} = \delta_n^{(j)}[1 + (\delta_n^{(j)})^2]^{-1/2}, \\ \gamma_n^{(j)} = [1 + (\delta_n^{(j)})^2]^{-1/2}. \\ \quad (n = -1, j = 1,2) \end{cases} \quad (7)$$

For further discussion, we will derive the atomic emission spectrum $S(\omega)$ according to the formula[10]

$$S(\omega) = 2\Gamma \int_0^T \mathrm{d}t_1 \int_0^T \mathrm{d}t_2 \exp[-(\Gamma - i\omega)(T - t_1) -(\Gamma + i\omega)(T - t_2)]\langle \xi, \eta | R^+(t_1) R(t_2) | \xi, \eta \rangle, \quad (8)$$

where T is the measurement time, Γ^{-1} is the response time of the filter, $|\xi, \eta\rangle$ is the initial state of the system, and $R(t)$ is defined as $R(t) = R_{ba}(t) + R_{cb}(t)$. Provided that the atom is initially in the eigenstate $|a\rangle$ of upper level e_A, and the field is prepared in arbitrary state, then the state vector of the atom-field system at $t = 0$ factorizes into a direct product, i.e. $|\xi, \eta\rangle = |\xi\rangle \otimes |\eta\rangle = \sum q_n |n, a\rangle$, where $q_n = \langle n|\xi\rangle$ denotes the probability amplitude of the photon number of optical field. And thus, the emission spectrum $S(\omega)$ can be written as a product of the photon number distribution $\rho_n = |q_n|^2$ and the function $S_n(\omega)$, which describes the spectrum of the photon number-state $|n\rangle$

$$S(\omega) = \sum_{n=0}^{\infty} \rho_n S_n(\omega). \quad (9)$$

Subject to the above-mentioned initial conditions, we can write the function $S_n(\omega)$ in the form

$$\begin{aligned} S_n(\omega) = & \frac{\Gamma}{(C_n)^4 (C_{n'})^2} \\ & \{\frac{n(n+1)}{2} |f(\omega, C_n, 0) + f(\omega, -C_n, 0)|^2 \\ & + |(n+2) f(\omega, 0, C_{n'}) + \frac{n+1}{2} f(\omega, -C_n, C_{n'}) \\ & + \frac{n+1}{2} f(\omega, C_n, C_{n'})|^2 \\ & + |(n+2) f(\omega, 0, -C_{n'}) + \frac{n+1}{2} f(\omega, -C_n, -C_{n'}) \\ & + \frac{n+1}{2} f(\omega, C_n, -C_{n'})|^2\}, \quad (\varepsilon = 0) \end{aligned} \quad (10a)$$

and

$$\begin{aligned} S_n(\omega) = & 2\Gamma \sum_k |\sum_{j=1}^{3} J_n^{(1)} [\alpha_n^{(j)} K_{n'}^{(2)} + \beta_n^{(j)} K_{n'}^{(3)}] \\ & \times \frac{\exp\{-ig[\Omega - \varepsilon(2n+1) - \delta_n^{(j)} + \delta_{n'}^{(k)}]T\} - \exp(-\Gamma T)}{\Gamma - ig[\Omega - \varepsilon(2n+1) - \delta_n^{(j)} + \delta_{n'}^{(k)}]}|^2, \\ & (\varepsilon \neq 0; n > 0, k = 1,2,3; n = 0, k = 1,2) \end{aligned} \quad (10b)$$

where $n' = n - 1, \Omega = (\omega - \omega_0)/g, C_n = \sqrt{2n+3}$, and $J(\mathrm{or}K) = \alpha, \beta, \gamma$, corresponding to $j(\mathrm{or}k) = 1,2,3$. The function $f(\omega, C_n, C_{n'})$ in Eq. (10a) can be defined as

$$\begin{aligned} f(\omega, C_n, C_{n'}) = & (C_n + C_{n'}) \\ & \times \frac{\exp[-ig(\Omega + C_n + C_{n'})T] - \exp(-\Gamma T)}{\Gamma - ig(\Omega + C_n + C_{n'})}. \end{aligned} \quad (11)$$

By using numerical simulations, we demonstrate in detail the characteristics of the spectra by Eqs. (9) and (10) in Figs. 1 and 2, according to the two typical cases: number state and coherent state. Moreover, for more heuristic discussions, we will begin with the simpler case when the field is initially prepared in pure number state. Obviously, it is shown from Eq. (10b) that the physical spectrum is of a nine-peak structure when Kerr effect is considered, one more peak than original eight-peak one in the absence of Kerr nonlinearity. All the number state spectra are depicted in Figs. 1(a), 1(b) and 1(c) in the cases of $\varepsilon = 0, \varepsilon = 0.1$ and $\varepsilon = 1.0$, respectively. For the convenience of narration, we define symbol $P(j, k; n)$ as the peak at $\omega = \omega_0 + g[(2n+1) + \delta_n^{(j)} - \delta_{n'}^{(k)}]$ for the case of $\varepsilon \neq 0$.

As a special case, the spectrum structure of the vacuum state, i.e., $n = 0$, is different from that of $n \neq 0$. We can see easily from Eqs. (10a) and (10b) that this spectral structure is theoretically of a six-peak structure and dependent on the value of ε, as stated in Fig. 1. For $\varepsilon = 0$, we can observe four spectral lines [see Fig. 1(a)]. Two of them are in fact made up of four peaks which locate at $\omega = \omega_0 \pm (\sqrt{3} - 1)g$ and $\omega = \omega_0 \pm g$, and form two inner spectral lines, whereas other two peaks occurring

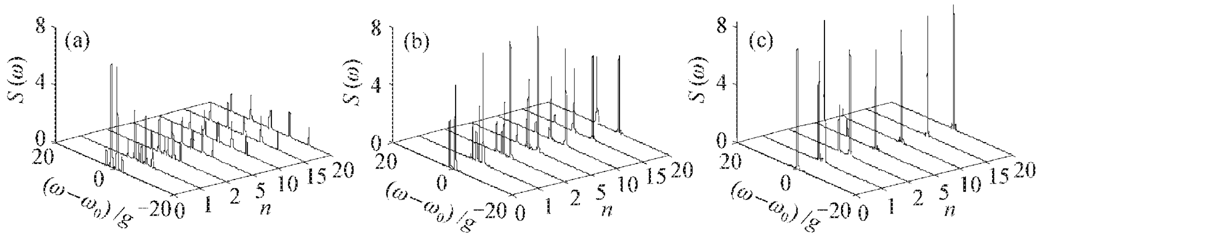

Fig. 1. The function $S_n(\omega)$ when the field is initially in a pure number state for (a) $\varepsilon = 0$, (b) $\varepsilon = 0.1$, (c) $\varepsilon = 1$. The interaction time and the spectral resolution of the spectrometer are assumed to be $T_g = 20, \Gamma = 0.02g$.

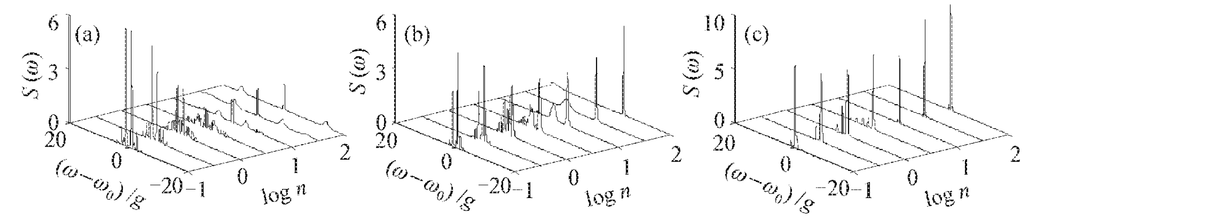

Fig. 2. The function $S_n(\omega)$ when the field is initially in a coherent state for (a) $\varepsilon = 0$, (b) $\varepsilon = 0.1$, (c) $\varepsilon = 1$. The interaction time and the spectral resolution of the spectrometer are assumed to be $T_g = 20$, $\Gamma = 0.02g$.

at $\omega = \omega_0 \pm (\sqrt{3}+1)g$ constitute two outer spectral lines. For small ε, two inner spectral lines $P(2,2;0) - P(3,1;0)$ and $P(1,1;0) - P(3,2;0)$ are still be observed in Fig. 1(b), but the outer spectral lines $P(1,2;0)$ and $P(2,1;0)$ disappear completely because of the relatively smaller peak values. When ε increases, the inner spectral lines are coming closer, as depicted in Fig. 1(c).

Let us consider further the case when the initial state is number state $|n\rangle (n > 0)$ and Kerr effect is not considered [see Fig. 1(a)]. If the coupling constant g is not too large and resolution of spectrograph not too low, we can observe eight spectral lines for small n. As n increases, the peaks locating at $\omega = \omega_0 \pm \sqrt{2n+3}g$ and $\omega = \omega_0 \pm \sqrt{2n+1}g$ tend to close up gradually, and the other two inner peaks occurring at $\omega = \omega_0 \pm (\sqrt{2n+3} - \sqrt{2n+1})g$ come closer and closer. Finally, there are totally six spectral lines to be observed in this case. This result is obviously different from that of standard JCM.

In the presence of Kerr nonlinearity, the number state spectrum, in principle, consists of nine peaks, and moreover, these peaks will shift to the left for different distances with respect to original positions. It is of interest to note that the frequency of the extra peak $P(3,3;n)$ becomes the resonant frequency as $\varepsilon \to 0$, and therefore we argue that the appearance of this peak results from the splitting of the resonant peak due to coupling of nonlinear medium to the field. For small $\varepsilon(\varepsilon = 0.1)$ [see Fig. 1(b)], we can observe five lines easily. More definitely, the peaks $P(2,2;n) - P(3,3;n)$ and $P(2,3;n) - P(3,1;n)$ form two spectral lines since they overlap partially, whereas the peaks $P(3,2;n), P(1,3;n)$ and $P(2,2;n)$ constitute three spectral lines, but the peaks $P(1,2;n)$ and $P(2,1;n)$ is too low to be observed. When n increases, the two inner lines approach to each other and the peak value of $P(1,3;n)$ tends to 0. Finally, there are in all three lines to be observed. For large $\varepsilon(\varepsilon = 1)$, the spectral line, resulting from the overlap of the peaks $P(2,3;n)$ and $P(3,1;n)$, shifts to the central position and forms the so-called resonant line. The overlap of the peaks $P(2,2;n)$ and $P(3,3;n)$ forms another line which moves away from the resonant frequency and whose intensity becomes weaker and weaker as n increases. Other peaks disappear completely for small peak values.

Now we focus our attention on the case when the field is prepared in the coherent state [see Fig. 2]. In this instance, the spectral structure $S(\omega)$ can be described by Eq. (9), and photon number statistics are subject to Poisson distribution ρ_n, this is

$$\rho_n = \exp(-\overline{n})\overline{n}^n/n!, \tag{12}$$

where $\overline{n}$ denotes the mean photon number of initial field. Obviously, Eq. (9) shows that the maximum of this distribution appears only when the condition $n = \overline{n}$ is satisfied, and the photon-number fluctuation $\langle \Delta n \rangle$ is directly proportional to $\overline{n}$. For definite $\overline{n}$, and with the increment of n, the left spectral line, whose position is far from resonant line, is modulated by distribution function Eq. (12) and more strongly than the right one does which nears resonant line. We can see from Fig. 2 that, for small $\overline{n}$, $2 - 6$ main lines near the center and some waves surrounding them due to the quantum-beat effect appear, whereas for larger $\overline{n}$, there exists only one line (i.e. resonant line) because the others as well as waves are modulated rapidly to zero as $\overline{n}$ increases. Also, from there we can see that quantum-beat effect for $\varepsilon = 1.0$, in which the middle line changes relatively slowly, is smaller than that for $\varepsilon = 0.1$.

In summary, we have investigated the emission spectra of a Ξ-type three-level atom interacting with a single-mode optical field in an ideal cavity filled with Kerr medium and discussed the spectrum structures when the optical field is initially in a pure number state and a coherent state, respectively. It is shown that: 1) the spectral structures depend on the photon-number distribution function. We can observe two spectral lines for $n = 0$ or five for small n or three for larger n. Due to the strongly coupling of the nonlinearity, one line or two are observed in general. On the other hand, when the field is initially in coherent state, there appear $1 - 6$ lines and some waves around, moreover these waves disappear slowly as ε increases; 2) the line intensity depends on the strength of initial field and the coupling of Kerr medium to the field, that is, the intensity of the left spectral line becomes lower and lower with the increasing of n, while the intensity of the right one, in general, becomes larger and larger.

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