Filter control polarization mode dispersion in dispersion managed soliton systems

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In this paper, the dispersion managed soliton (DMS) transmission equation is built on considering the effects of polarization mode dispersion (PMD) and filter control. The DMS transmission of filtering control in constant birefringence fibers is firstly analyzed by varitional method, from which the evolving rules of characteristical DMS parameters are obtained. Secondly, the stability of DMS transmission and its timing jitter are investigated in the random varying birefringence fibers with the conventional model of PMD. The results reveal that filter control DMS system has powerful robustness to PMD effects and DMS's timing jitter can be decreased considerably with the help of filters.

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Recently, the dispersion managed solitons (DMSs) are of great interest in soliton communication systems. They provide some prior advantages compared with conventional solitons, such as higher pulse energy and signal noise ratio, lower averaged dispersion line and timing jitter. However, in the dispersion managed systems, lower averaged dispersion line can be lead to polarization mode dispersion (PMD) becoming even more obvious at higher bit rates and longer transmission distance, which may induce to lots of dispersion waves and the differential group-delay (DGD) accumulating. In the conventional soliton systems, the inline filters have been proved favorable effect in constraining several perturbations of system. A very important issue is that what it would happen if filters are used to control PMD in DMS links.

In Ref. [1], the method of numerical integrating has been used to evaluate the PMD in conventional soliton filtering system. In Ref. [2], the perturbational approach of adiabatic was used to study filtering DMS system, but not considering the PMD effect. In Ref. [3], there had been proved experimentally that conventional soliton and DMS robust to PMD, but not putting forward how to further restraining the PMD in high speed rates and long distance transmit systems. In this paper, we will study the effects of the PMD and filter control in DMS system theoretically.

We firstly consider an erbium-doped fiber amplifier (EDFA) transmission line, the filters are inserted after every amplifier, and then the normalized DMS transmission equation incorporating filter effects can be written as[4]

$$
\begin{aligned}
&i\frac{\partial U}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 U}{\partial T^2}+Q(Z)|U|^2U \\
&=i\left[\delta-k_f''(i\frac{\partial}{\partial T}-\omega_f)^2\right]U, \qquad (1)
\end{aligned}
$$

where U, Z, T and $Q(Z)$ represent normalized amplitude, distance, time and factors of nonlinear and amplifier, respectively, $d(Z)$ is the normalized group-velocity dispersion parameter, and δ is an excess gain that compensates the loss caused by the transmission control, k_f'' is the strength of the filter, $\omega_f=\rho Z\neq 0$ is the difference of center frequency of the filter and soliton, where ρ is the normalized sliding rate if ω_f slides linearly with distance Z. while the filter center frequency is constant and equated to the soliton center frequency, $\omega_f=0$.

Now we assume

$$
R1=i\left[\delta-k_f''(i\frac{\partial}{\partial T}-\omega_f)^2\right]U, \qquad (2)
$$

when $k_f''\ll\bar{D}$[5] ($\bar{D}$ is real value of path averaged of group-velocity dispersion parameter). Then Eq. (1) can be written formally as DMS equation of perturbation

$$
i\frac{\partial U}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 U}{\partial T^2}+Q(Z)|U|^2U=R_1. \qquad (3)
$$

At the same time, propagation of orthogonal polarized optical pulse components U and V in a dispersion managed line is described by the so called coupled nonlinear Schrödinger equation (CNSE) of the form[1]. Assuming that the beat length is much smaller than the dispersion distance or period of soliton, then the CNSE can be transformed into

$$
\begin{aligned}
&i\frac{\partial U}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 U}{\partial T^2}+Q|U|^2U \\
&=-i\delta_g\frac{\partial U}{\partial T}-Q\gamma|V|^2U=R_2, \qquad (4a)
\end{aligned}
$$

$$
\begin{aligned}
&i\frac{\partial V}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 V}{\partial T^2}+Q|V|^2V \\
&=-\left[-i\delta_g\frac{\partial V}{\partial T}+Q\gamma|U|^2V\right]=R_2', \qquad (4b)
\end{aligned}
$$

where γ represents the coefficient of nonlinear across phase modulation (XPM) between U and V, $\delta_g=[(\mathrm{d}\beta_x/\mathrm{d}\omega)-(\mathrm{d}\beta_y/\mathrm{d}\omega)]L_D/(2t_0)=\Delta\beta'/t_0$ is the DGD, $\Delta\beta=L_D(\beta_x-\beta_y)/2$ ($\beta_x=n_x2\pi/\lambda_x$). Combining Eqs. (3) and (4), we now can get the DMS transmission equation that incorporates the effects of PMD and filter control at same time

$$i\frac{\partial U}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 U}{\partial T^2}+Q|U|^2U$$
$$=i[\delta+k_f''\frac{\partial^2}{\partial T^2}]U-[i\delta_g\frac{\partial U}{\partial T}+Q\gamma|V|^2U]=R, \quad (5a)$$

$$i\frac{\partial V}{\partial Z}+\frac{1}{2}d(Z)\frac{\partial^2 V}{\partial T^2}+Q|V|^2V$$
$$=i[\delta+k_f''\frac{\partial^2}{\partial T^2}]V-[-i\delta_g\frac{\partial V}{\partial T}+Q\gamma|U|^2V]=R'. \quad (5b)$$

In Eqs. (5), the terms between two equals signs are perturbations of system, in which the first terms are effects of filter including the excess gain and the filter strength, the second terms are effects of PMD including DGD and XPM, which can lead to DMS timing jitter and nonlinear coupling between polarized amplitude components.

Now assuming stable solution of Eq. (5) is

$$U(Z,T)=A_U(Z)f_U(\tau)\exp(i\phi_U),$$
$$V(Z,T)=A_V(Z)f_V(\tau)\exp(i\phi_V),$$
$$\begin{cases} f(\tau)=\exp(-\tau^2/2), \quad \tau=B_i(Z)T-T_{0i}, \\ \phi_i=\frac{C_i(Z)B_i^2(Z)}{2}(T-T_{0i}(Z))^2 \\ \quad -k_i(Z)(T-T_{0i}(Z))+\theta_i, \end{cases}$$
$$i=U \text{ or } V, \quad (6)$$

where A, B, C, k, T_0, θ represent soliton amplitude, width, chirp, frequency, position and phase, respectively, we can solve it by varitional method[6]

$$\dot{A}_i=-\frac{1}{2}A_iB_i^2d(Z)C_i+A_i\delta-A_ik_f''B_i, \quad (7a)$$

$$\dot{B}_i=-B_i^3d(Z)C_i-k_f''B_i^2, \quad (7b)$$

$$\dot{C}_i=d(Z)B_i^2(1+C_i^2)-Q(\sqrt{2}/2)A_i^2$$
$$-k_f''(2B_i^2C_i-B_iC_i/2)$$
$$-2Q\gamma\left(\frac{E_j}{\sqrt{\pi}B_i^2}\right)P^3(2P^2\Delta T^2-1)$$
$$\exp(-P^2\Delta T^2), \quad (7c)$$

$$\Delta\dot{k}=-2k_f''B_i^2C_i\Delta k$$
$$+2Q\gamma P^3E_{\rm SOL}\Delta T\exp(-P^2\Delta T^2)/\sqrt{\pi}, \quad (7d)$$

$$\Delta\dot{T}=-\Delta kd(Z)\pm\delta_g+2k_f''C_i\Delta k, \quad (7e)$$

where $1/P^2=1/(B_i^2+B_j^2)$, $E_{\rm SOL}=\sqrt{\pi}(A_i^2/B_i+A_j^2/B_j)$ represents the pulse energy, "+" or "−" before δ_g correspond to U, V respectively.

The above equations are the evolvement of soliton parameters in DMS system incorporating the effects of constant birefringence and filter. From Eqs. (7), we can know that the effect of birefringence fiber (δ_g and γ) on the pulse of polarized mode leads the frequency excursion, and makes pulse width broadening Eq. (7e). But the filter (shown by k_f'' in Eqs. (7)) has an ability of control in the parameters amplitude A, width B, excursion of the position ΔT and the frequency Δk.

Equations (7) represent the PMD effects in constant birefringence fiber such as PANDA fiber, but in fact, the usually practical fiber line is random varying birefringence fiber, and mode coupling between the polarized components varies with wavelength and its transmission distance. Here we use the conventional model of PMD in fibers, which is modelled as a cascade of many small segments with constant birefringence. We assume that all the segments have the identical length z_h, which is the mode-coupling length. The orientation of the birefringence and DGD δ_g varies randomly without correlation between adjacent segments, and δ_g satisfies with Gaussian distribution. When in condition of periods of dispersion managed fiber longer than the mode-coupling length (usually 100 m), δ_g obeys Gaussian distribution

$$\langle\delta_g\rangle=0,$$
$$\langle\delta_g(Z)\delta_g(Z')\rangle=\sigma^2\delta(Z-Z'),$$
$$\sigma^2=\langle\Delta\beta'^2\rangle z_h. \quad (8)$$

Now Eqs. (7) regard as randomly varying differential equations in whole fiber line, and effects of PMD express as $\sqrt{\langle\Delta\beta'^2\rangle z_h}$[7]. Through the Eqs. (7d) and (7e), we can study the stability and the timing jitter of the system with the adiabatic or conservation perturbation method[4].

Let $x=\Delta k$, $y=\Delta T$, and terms of ΔT in equations expand linearly around $\Delta T=0$, we obtain the ordinary differential equations for the difference of frequency and time position in the form

$$\frac{{\rm d}x}{{\rm d}Z}=-2k_f''B^2Cx+2Q\gamma P^3E_{\rm SOL}y/\sqrt{\pi}, \quad (9a)$$

$$\frac{{\rm d}y}{{\rm d}Z}=(2k_f''C-d)x\pm\delta_g. \quad (9b)$$

Assuming $a=-2k_f''B^2C$, $b=2Q\gamma P^3E_{\rm SOL}/\sqrt{\pi}$, $c=2k_f''C-d$, the linear equation groups have two values $\lambda_{1,2}=\frac{a\pm i\sqrt{-(a^2+4bc)}}{2}$, and have two sectors $\begin{pmatrix}1\\ -\frac{a-\lambda_1}{b}\end{pmatrix}$, $\begin{pmatrix}1\\ -\frac{a-\lambda_2}{b}\end{pmatrix}$.

When $a^2+4bc<0$, or

$$k_f''<\frac{2d\gamma P^3E_{\rm SOL}}{\sqrt{\pi}B^4C^2+2\gamma P^3E_{\rm SOL}C}, \quad (10)$$

Eqs. (9) have a formally stable solution

$$x(Z)=X_1(Z){\rm e}^{\lambda_1Z}+X_2(Z){\rm e}^{\lambda_2Z}, \quad (11a)$$

$$y(Z)=X_1(Z)\left[-\frac{a-\lambda_1}{b}\right]{\rm e}^{\lambda_1Z}$$
$$+X_2(Z)\left[-\frac{a-\lambda_2}{b}\right]{\rm e}^{\lambda_2Z}. \quad (11b)$$

And the filters have a ability of making the DMS systems into stable state. From Eqs. (8) and (11), we can get the excursions of center position and frequency of DMS in effects of filter control in random birefringence fiber

$$x(Z) = \mp\frac{b}{\lambda_2-\lambda_1}\int_0^Z \left(e^{-\lambda_1(Z-Z')} - e^{-\lambda_2(Z-Z')}\right)\delta_g(Z')dZ', \quad (12a)$$

$$y(Z) = \mp\frac{1}{\lambda_2-\lambda_1}\int_0^Z \left((a-\lambda_1)e^{-\lambda_1(Z'-Z)} - (a-\lambda_2)e^{-\lambda_2(Z'-Z)}\right)\delta_g(Z')dZ'. \quad (12b)$$

The mean square of DMS timing jitter is

$$\begin{aligned}\langle\Delta T^2\rangle &= \langle\Delta y^2(Z)\rangle = \langle y^2(Z)\rangle - \langle y(Z)\rangle^2 \\ &= \frac{\sigma^2}{2(\lambda_2-\lambda_1)^2}\times\left\{\frac{(a-\lambda_1)^2}{\lambda_1}\left[e^{2\lambda_1 Z}-1\right] + \frac{(a-\lambda_2)^2}{\lambda_2}\left[e^{2\lambda_2 Z}-1\right] + \frac{4(a-\lambda_1)(a-\lambda_2)}{\lambda_1+\lambda_2}\left[1-e^{(\lambda_1+\lambda_2)Z}\right]\right\}. \quad (13)\end{aligned}$$

If no filter is inserted in fiber line, $k_f'' = 0$, Eq. (13) would be simply expressed as

$$\langle\Delta T^2\rangle_{Nf} = \frac{\sigma^2 Z}{2}\left[1+\frac{\sin(\sqrt{4bd_0}Z)}{\sqrt{4bd_0}Z}\right], \quad (14)$$

where $d_0 = -\langle\beta_2\rangle z_0/t_0^2$ is path averaged group-velocity dispersion. The result is similar to that of Ref. [9]. When $b = 0$, we can simplify Eq. (13) to situation of linear transmission

$$\sqrt{\langle\Delta T^2\rangle_L} = \sqrt{\sigma^2 Z}. \quad (15)$$

The result of Eq. (15) consists with other published papers[7,8].

Equations (11) can apply to analyze the characteristics of DMS transmission with Fig. 1. We consider a DM link made of an arrangement of $l_1 = 35$ km anomalous dispersion fiber $D_2 = 17$ ps/(nm·km) and $l_2 = 5$ km of normal-dispersion fiber $D_2 = -118.2$ ps/(nm·km), yielding an averaged dispersion of $\bar{D} = 0.1$ ps/(nm·km), and assuming they have same loss and Kerr coefficients, we now investigate the propagation of Gaussian pulse of $t_0 = 5$ ps, bit duration at 40 Gb/s in the system mentioned above.

Figure 2 shows the evolution of timing jitter along the distance numerical computing from Eqs. (13), (14) and (15), respectively. It can be seen from Fig. 2 that, without in-line filtering, the DMS is rapidly affected by PMD, the time jitter fluctuates larger and larger along with the distance, it is about 3 ps at distance of $z = 3000$ km smaller than the linear system situation 5.5 ps. These indicate that DMS scheme can reduce the PMD effects considerably. However, the timing jitter induced by PMD would effectively decrease to 0.5 ps at $z = 3000$ km if applying in line filters.

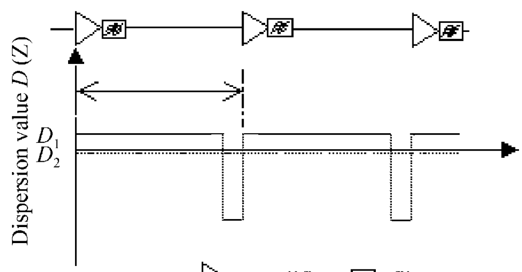

Fig. 1. Dispersion managed fiber line sketch map.

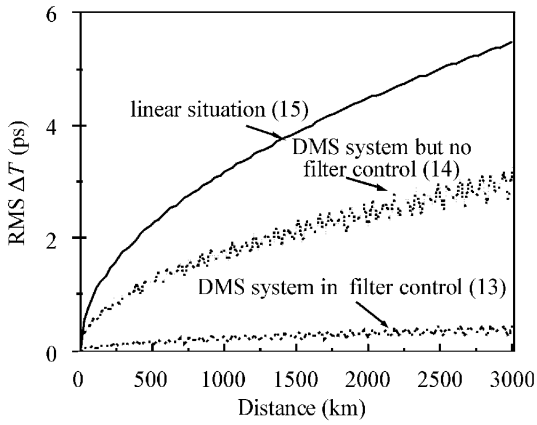

Fig. 2. Evolution of root-mean-square of $\sqrt{\langle\Delta T^2\rangle}$ in nonlinear and linear systems. $\sqrt{\langle\Delta\beta'^2\rangle z_h} = 0.1$ ps/km$^{\frac{1}{2}}$, $z_h = 0.1$ km. The filter strength $k_f'' = 0.015$ and excess gain $\delta = 0.02$.

We have presented an analytical theory of combining the varitional method and conservation perturbation method to show that filter control DMS system is powerful in robustness to PMD. Proper filter strength can strongly reduce the timing jitter of PMD induced. The filter-induced stability of DMS will be more important in high speed and long distance communications.

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