

Characteristics of the transmission spectrum of the long period fiber gratings based on the coupling of core mode to the higher order cladding modes

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The characteristics of the transmission spectrum of the Long-period fiber gratings (LPFGs) based on the coupling of core mode to a higher order cladding mode (HE mode) are investigated using the coupled mode theory. This kind of LPFGs is different from that based on the coupling of core mode to a lower order cladding mode because of the effect of the coupling of core mode to EH cladding mode. When the cladding mode order is higher, the coupling coefficients of core mode to HE and EH cladding modes are comparable and both of the propagation constants of HE and EH cladding modes approach, so the spectrum has an additional loss peak. The bandwidth of LPFG based on the coupling of core mode to different cladding mode differs greatly. With the change of the mode orders from lower to higher, the transmission spectrum changes from narrow to wide and more narrow.

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Long period fiber gratings (LPFGs) possess the special properties of wavelength selectivity, low insertion losses and low back reflection, etc. LPFGs can selectively couple the core mode to a forward-propagating cladding modes. A single-mode fiber has a multitude of cladding modes more than several hundreds. It has a more complicated coupling relation due to the multitude of cladding modes. In many fiber grating devices, the structure of the spectrum plays an important role, such as the center wavelength, bandwidth and the maximum loss, etc.

LPFGs widely used and studied are those based on the coupling of the core mode to lower order HE cladding modes^[1,2,4]. But the theoretical investigations^[3,5] show that the core mode can also be coupled to EH and higher-order HE cladding modes. Of course, the coupling coefficients for core mode to them are relatively less than that to HE cladding modes, particularly to the lower-order HE cladding modes.

When mode order is higher, HE and EH cladding modes appear in pairs. Their propagation constants are near and the coefficients for core mode to them are approximately equal^[5]. So according to the phase matching condition, a certain LPFG designed for core mode to couple with the HE cladding mode will couple the core mode to the EH cladding mode.

In this paper, we investigate the characteristics of the transmission spectrum of LPFGs based on the coupling of the core mode to the higher order cladding modes by using the coupled-mode theory. Our analysis reveals that the EH cladding modes have some effects on the spectral structure and LPFGs based on lower-, middle- and higher-order cladding modes possess a narrow, wide and more narrow loss peak respectively.

Consider a naked single-fiber with the core and cladding radius $a_1 = 4.15 \mu\text{m}$, $a_2 = 62.5 \mu\text{m}$ respectively, and the refractive indexes of the core, cladding, surrounding $n_1 = 1.4681$, $n_2 = 1.4628$, and $n_3 = 1$, respectively.

The period Λ_ν of LPFG based on ν -th cladding mode must satisfy the phase-matching condition $\beta^{\text{co}} - \beta_\nu^{\text{cl}} =$

$2\pi/\Lambda_\nu$, where β^{co} and β_ν^{cl} denote the propagation constants of the core mode and cladding modes respectively. To the fiber parameters given above, for the wavelength near 1550 nm, there are more than 180 cladding modes. So we can obtain 180 LPFGs with different periods. The values of the period range from several to 700 μm approximately. In Fig. 1, we show the curves of $\Lambda_\nu - \lambda$. The numbers in the figures are the mode orders ν , which are the root orders of the eigenvalue equation of the cladding modes. The odd numbers denote the HE cladding modes, and the even numbers denote the EH cladding modes. For example, 1 is $\text{HE}_{11}^{\text{cl}}$ mode and 2 is $\text{EH}_{11}^{\text{cl}}$ mode, etc. In the figure, we can see that the curves are almost linear, and every two curves are very close when the mode orders are higher. Both of the neighboring curves denote a pair of HE and EH cladding modes. With the increase of the mode orders, they become closer. The mode order of the upper curves is odd and the nether one is even, Here β^{co} and β_ν^{cl} are calculated from the eigenvalue equations of two- and three-layered optical fiber, respectively.

The figure demonstrates that both of neighboring HE

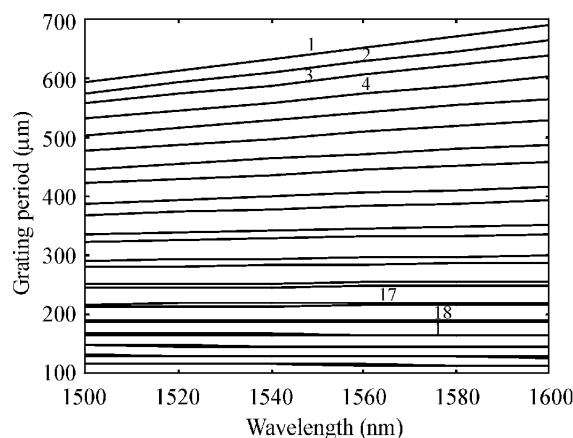


Fig. 1. Relationship between periods and wavelength of LPFGs.

and EH cladding modes coexist, and because their propagation constants are almost equal, the period of LPFGs based on them is very close. Therefore LPFG can enable core mode to couple with HE and EH cladding modes simultaneously.

According to above analysis and the general coupled-mode equations, we obtain the following coupled-mode equations

$$\begin{aligned} \frac{dA^{\text{co}}}{dz} &= -ik_{\nu}^{\text{co-cl}} A_{\nu}^{\text{cl}} \exp(i\Delta\beta_1 z) \\ &\quad - ik_{\nu+1}^{\text{co-cl}} A_{\nu+1}^{\text{cl}} \exp(i\Delta\beta_2 z), \end{aligned} \quad (1)$$

$$\frac{dA_{\nu}^{\text{cl}}}{dz} = -ik_{\nu}^{\text{co-cl}} A^{\text{co}} \exp(-i\Delta\beta_1 z), \quad (2)$$

$$\frac{dA_{\nu+1}^{\text{cl}}}{dz} = -ik_{\nu+1}^{\text{co-cl}} A^{\text{co}} \exp(-i\Delta\beta_2 z), \quad (3)$$

where $\nu = 1, 3, 5, \dots$ denotes $\text{HE}_{1,1}^{\text{cl}}, \text{HE}_{1,2}^{\text{cl}}, \text{HE}_{1,3}^{\text{cl}}, \dots$ cladding modes, respectively. $\nu + 1 = 2, 4, 6, \dots$ denotes $\text{EH}_{1,1}^{\text{cl}}, \text{EH}_{1,2}^{\text{cl}}, \text{EH}_{1,3}^{\text{cl}}, \dots$ cladding modes respectively. ν and $\nu + 1$ are the orders of the neighboring HE and EH cladding modes respectively. $A^{\text{co}} = A^{\text{co}'} \exp(-ik_{\nu}^{\text{co-co}} z)$, $A_{\nu}^{\text{cl}} = A_{\nu}^{\text{cl}'} \exp(-ik_{\nu}^{\text{cl-cl}} z)$, $A_{\nu+1}^{\text{cl}} = A_{\nu+1}^{\text{cl}'} \exp(-ik_{\nu+1}^{\text{cl-cl}} z)$. $k^{\text{co-co}}, k_{\nu}^{\text{co-cl}}$, and $k_{\nu+1}^{\text{co-cl}}$ denote the coupling coefficients of the core mode to core modes, core mode to ν -th and $\nu + 1$ -th cladding modes respectively. $k_{\nu-\nu}^{\text{cl-cl}}$ and $k_{\nu+1-\nu+1}^{\text{cl-cl}}$ denote the self-coefficients of ν -th and $\nu + 1$ -th cladding modes respectively. $A^{\text{co}'}, A_{\nu}^{\text{cl}'}$ and $A_{\nu+1}^{\text{cl}'}$ denote the amplitudes of the core mode, HE and EH cladding modes respectively. $A^{\text{co}}, A_{\nu}^{\text{cl}}$ and $A_{\nu+1}^{\text{cl}}$ are the parameters introduced for convenience. $\Delta\beta_1 = \beta^{\text{co}} - \beta_{\nu}^{\text{cl}} - 2\pi/\Lambda_{\nu}$, $\Delta\beta_2 = \beta^{\text{co}} - \beta_{\nu+1}^{\text{cl}} - 2\pi/\Lambda_{\nu}$, Λ_{ν} is the period of LPFG based on ν -th cladding mode(HE mode). Equation (1) denotes the coupling between core mode and both of the neighboring HE and EH cladding modes.

When the core mode is coupled to one HE cladding mode only, that is $k_{\nu+1}^{\text{co-cl}} = 0$ and $k_{\nu+1-\nu+1}^{\text{cl-cl}} = 0$, the above equations have the analytic solutions. We can obtain that when $\Delta\beta_1 = 0$ and $|k_{\nu}^{\text{co-cl}} L_0| = \pi/2$, that is, $L_0 = \pi/(2|k_{\nu}^{\text{co-cl}}|)$, the transmission is zero. Otherwise we cannot obtain the analytic solution. To solve them, we have to employ the numerical method. Here L_0 is an important parameter for LPFG. When the length L of a LPFG equals L_0 , the maximum transmission can be zero, otherwise it is not.

For a fiber grating applied no strain, the phase mismatch constants $\Delta\beta_1$ and $\Delta\beta_2$ are all the functions of wavelength. If the period Λ_{ν} is designed from the cou-

pling of core mode to ν -th cladding mode for the wavelength λ_0 , generally, to this wavelength, we have $\Delta\beta_1 = 0$ and $\Delta\beta_2 \neq 0$. With the variation of wavelength λ , we will have $\Delta\beta_1 \neq 0$ and $\Delta\beta_2 \neq 0$. So we have

$$\Delta\beta_1(\lambda) = \beta^{\text{co}}(\lambda) - \beta_{\nu}^{\text{cl}}(\lambda) - 2\pi/\Lambda_{\nu}, \quad (4)$$

$$\Delta\beta_2(\lambda) = \beta^{\text{co}}(\lambda) - \beta_{\nu+1}^{\text{cl}}(\lambda) - 2\pi/\Lambda_{\nu}. \quad (5)$$

Our analysis reveals that $\Delta\beta_1$ and $\Delta\beta_2$ have excellent linear relation with λ within the range of wavelength from 1300 to 1700 nm. Accordingly we can obtain the analytic linear equations of $\Delta\beta_1(\lambda)$ and $\Delta\beta_2(\lambda)$. When substitute the $\Delta\beta_1$ and $\Delta\beta_2$ in Eqs. (4) and (5) for $\Delta\beta_1$ and $\Delta\beta_2$ in Eqs. (1), (2), and (3) and solve the Eqs. (1), (2), and (3) by numerical method. One can obtain the transmission spectrum.

To solve the coupled-mode equations, we must know the value of the coefficients $k_{\nu}^{\text{co-cl}}$ and $k_{\nu+1}^{\text{co-cl}}$, which depend on induced-index change, the mode order and wavelength. For a certain LPFG and wavelength λ , the value of $k_{\nu}^{\text{co-cl}}$ is known and the ratio of $k_{\nu}^{\text{co-cl}}/k_{\nu+1}^{\text{co-cl}}$ is invariable. In Ref. [5], the formula of $k_{\nu}^{\text{co-cl}}$ and $k_{\nu+1}^{\text{co-cl}}$ are given, so we can obtain the ratio of $k_{\nu}^{\text{co-cl}}/k_{\nu+1}^{\text{co-cl}}$ and the value of $k_{\nu}^{\text{co-cl}}$. The values of $k_{\nu}^{\text{co-cl}}$ and $k_{\nu+1}^{\text{co-cl}}$ also have a linear relation to wavelength, but their change with the wavelength is not notable and we do not consider it. The value of $k_{\nu}^{\text{co-cl}}/k_{\nu+1}^{\text{co-cl}}$ is employed at the wavelength $\lambda_0 = 1550$ nm. Reference [5] demonstrates that for lower order cladding mode $k_{\nu+1}^{\text{co-cl}} \ll k_{\nu}^{\text{co-cl}}$, and for higher-order cladding mode, $k_{\nu+1}^{\text{co-cl}}$ and $k_{\nu}^{\text{co-cl}}$ are comparable.

Table 1 shows some of the expressions of $\Delta\beta_1(\lambda)$ and $\Delta\beta_2(\lambda)$ and the values of k , λ'_0 and Λ_{ν} , where $k = k_{\nu}^{\text{co-cl}}/k_{\nu+1}^{\text{co-cl}}$ and λ'_0 satisfied $\Delta\beta_2(\lambda) = 0$, at which an additional loss peak will appear. The value of λ'_0 can be greater or less than λ . In Table 1, when $\nu = 9/10$ and $15/16$, because the value $k_{\nu}^{\text{co-cl}}$ is much greater than $k_{\nu+1}^{\text{co-cl}}$, i.e., $k \gg 1$, the coupling of core mode to EH cladding modes is very little, so λ'_0 do not exist practically and is not given in the table.

When $\nu > 19$ approximately, both of the coefficients are comparable, the effect is marked. It produces an accompanying loss peak at λ'_0 besides the main loss peak at $\lambda = 1550$ nm. The value of the loss of the accompanying peak depends on the value of k .

Figure 2 shows some of the transmission spectra. When calculating, let $k_{\nu}^{\text{co-cl}} = 30 \text{ m}^{-1}$, $L = L_0$, where L is the length of the LPFG concerned. In the figure, the numbers written beside the curves denote the mode orders. For $\nu = 9$ and 15 , the coupling of core mode to EH cladding modes is very little. So EH cladding modes

Table 1. Some of the Typical Values or Expressions of $\Delta\beta_1(\lambda)$, $\Delta\beta_2(\lambda)$, k , λ'_0 (the Unit of λ and λ'_0 is nm)

ν	$\Delta\beta_1(\lambda) (\text{m}^{-1})$	$\Delta\beta_2(\lambda) (\text{m}^{-1})$	k	λ'_0
1	$\Delta\beta_1(\lambda) = -14.88\lambda + 23056$			
9/10	$\Delta\beta_1(\lambda) = -10.72\lambda + 16616$	$\Delta\beta_2(\lambda) = -10.55\lambda + 14618$	52.9	
15/16	$\Delta\beta_1(\lambda) = -4.49\lambda + 6955$	$\Delta\beta_2(\lambda) = -4.28\lambda + 3422$	23.5	
51/52	$\Delta\beta_1(\lambda) = 86.30\lambda - 133857$	$\Delta\beta_2(\lambda) = 87.14\lambda - 146248$	1.11	1678.2
79/80	$\Delta\beta_1(\lambda) = 232.5\lambda - 360329$	$\Delta\beta_2(\lambda) = 234.3\lambda - 380390$	1.05	1622.8

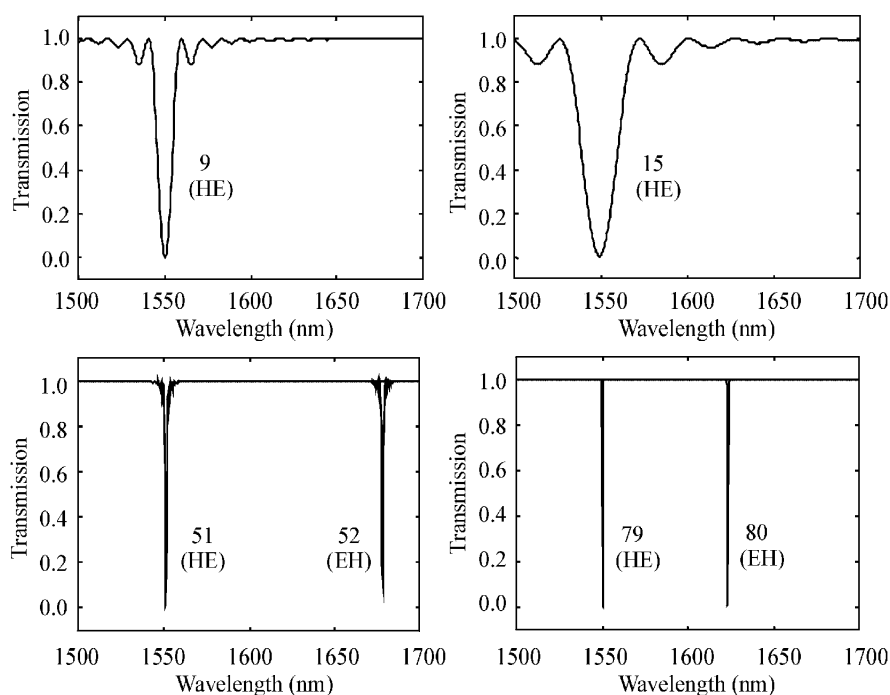


Fig. 2. The transmission spectrum of core mode to different cladding modes ($\nu = 9, 15, 51/52, 79/80$).

are neglected and there are no 10 and 16 in the figure. For ν is higher such as 51/52, 79/80, there are two loss peaks. One is produced by HE cladding mode and another is produced by EH cladding mode.

Figure 2 demonstrates that firstly, the spectral structure of LPFG based on different cladding modes has great difference. For the lower-order cladding modes it is narrow. With the increase of the mode orders, it becomes wider. At $\nu = 19$ approximately, the spectrum reaches to a maximum width. Then when the mode order increases sequentially, it becomes narrow. With the further increase of the mode order, it becomes more narrow. The theoretical result is consistent with the experiment^[2,4]. Secondly, for the lower-order cladding mode, the effect of EH cladding modes on the spectrum is very little, with the increase of the mode order, the effect becomes greater and makes another loss peak.

Analysis also shows that the slopes of the straight line of $\Delta\beta_1(\lambda)$ or $\Delta\beta_2(\lambda)$ are very different to each other. This property can give a great effect on the spectral structure. When the absolute value of the slope is greater, it means that $\Delta\beta_1(\lambda)$ or $\Delta\beta_2(\lambda)$ changes sharply with the variation of the wavelength and the width of the transmission spectra is smaller, and contrarily it is greater.

The slope of the line $\Delta\beta_1(\lambda)$ or $\Delta\beta_2(\lambda)$ can be written as

$$s = \frac{d}{d\lambda} \Delta\beta(\lambda) = \frac{d}{d\lambda} \beta^{co} - \frac{d}{d\lambda} \beta_{\nu}^{cl} = s^{co} - s_{\nu}^{cl}. \quad (6)$$

Because s^{co} is invariable for different cladding modes and s^{cl} changes, so the slope s is different greatly for different cladding mode. For example, for the parameter values given above, s changes from -14.88 to 1.06 , then to 246.0 when the cladding mode order changes from 1

to 81. From the view of geometrical optics, when the ray propagates in different angle with the axis of the fiber, the propagation constant β_{ν}^{cl} is also different and $d\beta_{\nu}^{cl}/d\lambda$ is not same. So the width of the spectrum is different greatly.

In conclusion, for HE and EH cladding modes with higher mode order, because their propagation constants approach and the coupling coefficients of core mode being coupled to them are comparable, they can strongly couple with the core mode. Therefore EH cladding mode can give a different impact on the transmission spectrum of LPFG based on the HE cladding mode. When the mode order is lower, EH cladding mode produces a very little loss peak that can be neglected. When the mode order is higher, a double-loss peak appears. In addition, the width of the spectrum is also different. The spectra of the LPFGs based on lower, middle and higher order cladding modes change from narrow to wide, then to more narrow. The characteristics are useful for one to design LPFG with different bandwidths.

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