What causes the superluminal propagation of light pulses

Shi-Yao Zhu (朱诗尧) and Li-Gang Wang (王立刚)

Department of Physics, Hong Kong Baptist University, Hong Kong

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In this paper, we discuss what causes the superluminal propagation of a pulse through dispersion by solving Maxwell's equations without any approximation. The coherence of the pulse plays an important role for superluminal propagation. When the pulse becomes partially coherent, the propagation changes from superluminal to subluminal. The energy velocity is always less than the vacuum velocity. The shape of the pulse is changed during the propagation.

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Superluminal propagation is a phenomenon that the group velocity of an optical pulse in a medium is greater than the light speed in vacuum[1]. This phenomenon has been discussed widely in many different media[2,3]. However, what causes the superluminal propagation remains much controversial. There are two different viewpoints on the experimental results. One is that the front and the back of the pulse undergo different gain or attenuation[4−6]. Another one contributes it to the interference between the different frequency components of the pulse, which undertake different phase shifts after passing through a medium of anomalous dispersion[7,8]. We ask ourselves what is the nature of the superluminal propagation, and use the partially coherent pulse to investigate this controversy.

In order to investigate the effects of coherence and interference on the superluminal propagation, we introduce a new kind of temporal partially coherent pulses. Light field from any real source is not fully coherent[9]. For stationary fields, the theory of coherence has been studied for a long time[9,10]. Recently, the theory of coherence for non-stationary fields is established[11,12]. The correlation function of a pulse in space-time domain is the key quantity for discussing partially coherent pulses.

The correlation function of a fully coherent plane-wave pulse[9] is defined by $\Gamma(z_1,t_1;z_2,t_2) = E^*(z_1,t_1)E(z_2,t_2)$. Decomposing the electric field into Fourier components, we can write the correlation function for a fully coherent pulse as

$$\Gamma(z_1,t_1;z_2,t_2) = \frac{1}{2\pi}\int\int W(0,\omega_1;0,\omega_2) \times \exp\{i[k(\omega_2)z_2 - k^*(\omega_1)z_1]\} \times \exp\{i(\omega_1 t_1 - \omega_2 t_2)\}\mathrm{d}\omega_1\mathrm{d}\omega_2, \quad (1)$$

where $k(\omega)$ is the complex wave vector and the generalized spectral density

$$W(0,\omega_1;0,\omega_2) = E^*(0,\omega_1)E(0,\omega_2) = \frac{1}{2\pi}\int\int \Gamma(0,t_1;0,t_2) \times \exp\left[-i(\omega_1 t_1 - \omega_2 t_2)\right]\mathrm{d}t_1\mathrm{d}t_2, \quad (2)$$

with $\Gamma(0,t_1;0,t_2)$ the initial correlation function of the pulse at $z=0$. By using Eqs. (1) and (2), we can obtain the evolution of the correlation function.

For a partially coherent pulse the correlation function is defined[9] by $\Gamma(z_1,t_1;z_2,t_1) = \langle E^*(z_1,t_1)E(z_2,t_2)\rangle$, where $\langle\ldots\rangle$ represents the statistical ensemble average. The evolution of the correlation function for partially coherent pulses is still controlled by Eqs. (1) and (2). In above and future discussion, we assume that the medium is stationary.

For the fully coherent plane-wave pulses, we have

$$\Gamma(0,t_1;0,t_2) = [I(0,t_1)I(0,t_2)]^{1/2}\exp[i\omega_0(t_1-t_2)]. \quad (3)$$

For the partial coherent pulses, the temporal correlation usually depends only on the time difference, and we assume the initial correlation function be Gaussian,

$$\Gamma(0,t_1;0,t_2) = [I(0,t_1)I(0,t_2)]^{1/2}\exp\left[-\frac{(t_1-t_2)^2}{4\sigma_{L0}^2}\right] \times \exp[i\omega_0(t_1-t_2)], \quad (4)$$

where σ_{L0} is the correlation time width, a measure of the correlations between two different space-time points. Note that $I(0,t_i) = \Gamma(0,t_i;0,t_i), (i=2)$ (the initial intensity of light field) is not dependent on σ_{L0}. That is to say, the space-time intensity profile of the pulse is the same for any value of σ_{L0}. We can call the pulse defined by Eq. (4) a kind of Schell-Model plane-wave pulse[12]. The completely coherent plane-wave light pulse is obtained at the limit $\sigma_{L0}\to\infty$. In the opposite limit $\sigma_{L0}\to 0$, all the space-time points become uncorrelated.

Now we consider the propagation of a partially coherent pulse in a gain medium from $z=0$ to L, and surrounded by the vacuum. For simplicity we neglect the reflection. The susceptibility of the gain medium is assumed a double Lorentz oscillator[2],

$$\chi(\omega) = \frac{M}{\omega-\omega_0-\Delta+i\gamma} + \frac{M}{\omega-\omega_0+\Delta+i\gamma}, \quad (5)$$

where M is related to the gain coefficient, and γ is the spectral width of two gain lines. The parameters will be used in this section are $\omega_0/2\pi = 3.5\times10^{14}$ Hz, $M/2\pi = 2.262$ Hz, $\gamma/2\pi = 0.46\times10^6$ Hz, $\Delta/2\pi = 1.35\times10^6$ Hz, which are fit to the experimental data reported in Ref.[2].

Let us consider the propagation of partially coherent Gaussian pulses. The initial intensity can be written as $I(0,t) = \exp\left(-t^2/\sigma_{\tau0}^2\right)$, with the pulse width

$\sigma_{\tau 0} = 1.2 \times 10^{-6}$ s. The initial correlation function of the temporally partially coherent Gaussian pulse is

$$\Gamma(0,t_1;0,t_2) = \exp\left(-\frac{t_1^2}{2\sigma_{\tau 0}^2}\right)\exp\left(-\frac{t_2^2}{2\sigma_{\tau 0}^2}\right) \times \exp\left[-\frac{(t_1-t_2)^2}{4\sigma_{L0}^2}\right] \times \exp[i\omega_0(t_1-t_2)]. \quad (6)$$

Substituting Eq. (6) into Eq. (2), we have

$$W(0,\omega_1;0,\omega_2) = 2\pi\sigma_{\tau 0}^2\sqrt{\frac{1}{1+(\sigma_{L0}/\sigma_{\tau 0})^2}} \times \exp[-\frac{(\omega_1-\omega_0)^2+(\omega_2-\omega_0)^2}{2(1/\sigma_{\tau 0}^2+1/\sigma_{L0}^2)} - \frac{(\omega_1-\omega_2)^2}{4(\sigma_{\tau 0}^2+\sigma_{L0}^2)/\sigma_{\tau 0}^4}]. \quad (7)$$

The generalized spectrum depends on both $\sigma_{\tau 0}$ and σ_{L0}. When $\sigma_{L0} \gg \sigma_{\tau 0}$, the light pulse is essentially fully temporal correlated (fully coherent), and the width of the generalized spectrum is determined by the temporal width $\sigma_{\tau 0}$. When $\sigma_{L0} \ll \sigma_{\tau 0}$, the light pulse is globally temporal un-correlated (incoherent), and the generalized spectral width is determined by the correlated time width σ_{L0}, and the generalized spectrum becomes very broad. The intensity of the pulse is determined by $I(0,t) = \Gamma(0,t;0,t)$, which is independent of σ_{L0}. Substituting Eq. (7) into Eq.(1), we can get the pulse evolution through the medium. In Fig. 1 we plot the peak delay time t_d as a function of the correlation time width σ_{L0}. It is very clear that the time delay increases as the pulse changes from almost fully coherent to almost incoherent. The transition from superluminal to subluminal propagation happens at $\sigma_{L0} \approx 200$ ns.

The spectrum width $\Delta\omega$ of a fully coherent pulse is related to its duration ΔT as $\Delta\omega \approx 1/\Delta T$. For a partially coherent pulse, we do not have its spectrum. However, we can use $W(\omega,\omega)$ as an equivalent power spectrum[12], which depends on the correlation time width σ_{L0} and its duration ΔT. For the partially coherent Gaussian pulse, the equivalent power spectrum width can be obtained from Eq. (7), $\Delta\omega = \sqrt{1/\sigma_{\tau 0}^2 + 1/\sigma_{L0}^2}$, which increases as the correlation time decreases. As the correlation time decreases, the peak delay time reaches a maximum value corresponding that the equivalent power spectrum of the pulse covers the whole normal dispersion region besides the central anomalous dispersion region, and then decreases to zero, because the equivalent power spectrum width reaches the almost zero (small anomalous) dispersion region outside the normal dispersion region.

In the above, we have shown that the coherence of the light pulses plays a very important role in superluminal propagations, and superluminal propagation is a wave interference phenomenon. Reducing the coherence, the superluminal phenomenon will disappear.

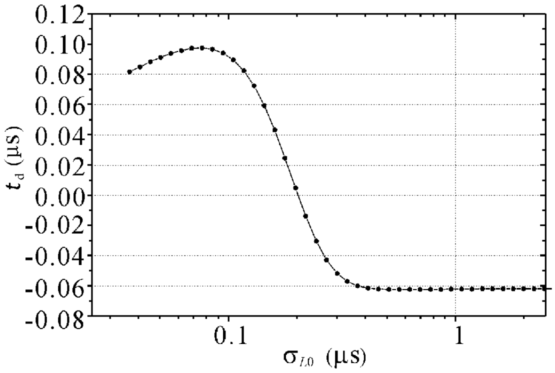

Fig. 1. Peak delay t_d as a function of the correlation time σ_{L0}. The related parameters are shown in text.

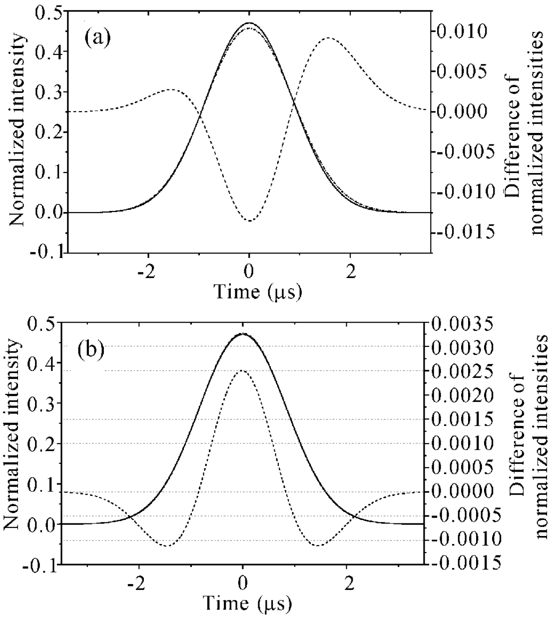

Fig. 2. Comparison of the shapes of the pulses after passing through the medium with the initial shape. (a) For partially coherent pulse (σ_{L0}=84.2 ns); (b) For fully coherent pulse.

Meanwhile, we note that the superluminal propagation is always companied by amplification (due to the gain of the medium). Now we examine what is the role of the amplification.

In Fig. 2, we compare the shape of the pulse after passing through the medium with the initial shape. The pulse passing through the medium is rescaled so that it contains the same energy as the initial one. Solid line is the initial pulse, dash-dot line is the pulse after passing through the medium, dash line is the difference. Figure 2(a) is for the partially coherent Gaussian (σ_{L0} = 84.2 ns). The pulse is broadened after the medium. For the fully coherent pulses ($\sigma_{L0} = \infty$), it is clear that the shape is compressed (see Fig. 2(b)).

It should be pointed out that the shape compressed or broadened is mainly due to the amplification and high order of dispersion. For the pure amplification (without the dispersion), the shape of the pulse through the medium is symmetrical for the front part and the back part of the

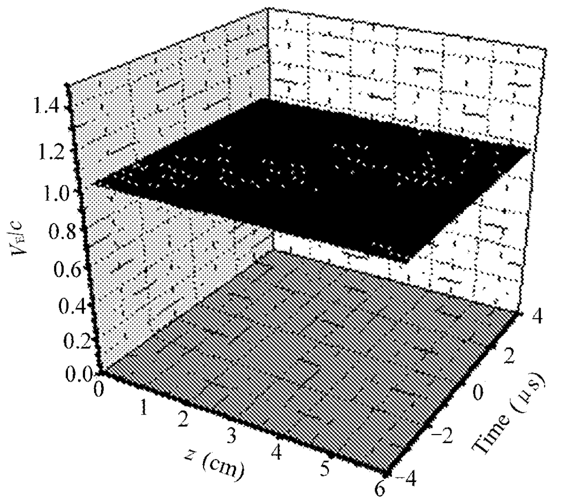

Fig. 3. The energy velocity for the fully coherent Gaussian pulse.

pulse, whatever the pulse is fully coherent or not. For the pure dispersion (without the amplification), the shape of the pulse through the medium is non-symmetrical due to the properties of the dispersion (mainly due to the higher-order dispersion), but the magnitude of distortion induced by the dispersion is much smaller than that induced by the amplification.

According to the definition of the energy velocity[10],

$$v_e = \frac{\mathbf{S}}{w_\mathrm{e} + w_\mathrm{m}}, \tag{8}$$

where w_e and w_m are the electric energy density and the magnetic energy density, respectively. For the fully coherent Gaussian pulse, we use Eq. (8) to calculate the energy velocity (see Fig. 3), and find that it is always approximately equal to c in the vacuum. No superluminality for the energy velocity has been found.

Although the intensity envelope is proportional to the energy density, the energy velocity is not equal to the group velocity. The energy density in the medium and at the exit end comes from two contributions, one from the income electromagnetic field and another from the energy preserved in the medium. The energy velocity determined by Eq. (8) is the propagating velocity of the electromagnetic field energy of the wave only. So the group and energy velocities are different.

In the linear media with dispersion and gain (or absorption), each Fourier component of the pulse propagates independently. Each Fourier component obtains a phase shift and is amplified (in gain media) or attenuated (in absorptive media). However, the phase shifts for different Fourier components are not the same, and the amplifications (or attenuations) are also not identical. Although each Fourier component propagates in a velocity is not faster than the c, the interference at the end of the medium during the re-superposition of all frequency components produces a new pulse which will appear in advance (for anomalous dispersion) or delay (for normal dispersion) compared with the pulse propagating in vacuum through the same distance.

S.-Y. Zhu's e-mail address is syzhu@hkbu.edu.hk.

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