# Design of Optimum Beam Divergence Angle for Intersatellite Optical Communication Systems with Pointing Errors

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**Abstract** Pointing errors can reduce the number of received signals. The influence of pointing errors on intersatellite optical communication systems is decreased by selecting an optimum laser beam divergence angle for optimizing link reliability in the presence of pointing errors. Given that bias errors are part of pointing errors, an analytic expression of link reliability is obtained in the absence of bias errors. By maximizing link reliability, an analytic expression of the optimum beam divergence angle can be obtained. Bias errors are considered in maximizing link reliability. The numerical results and analytic approximation of the optimum ratio of beam divergence angle to pointing errors for a desired bit rate are obtained. With the application of the designed optimum divergence angle for intersatellite optical communication systems, good system performance can be achieved even in the presence of pointing errors.

**Key words** free space optical communication; optimum beam divergence angle; link reliability optimization; pointing error

**OCIS codes** 060.2605; 060.4510; 350.6090

# 星间光通信中跟瞄误差下最优光束发散角设计

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**摘要** 跟瞄误差会引起接收光信号的衰减。为了减小跟瞄误差对星间光通信系统的影响，在跟瞄误差存在的条件下，通过选取最优光束发散角来优化链路可靠性。偏置误差是跟瞄误差的一部分，在没有偏置误差存在的情况下获得了链路可靠性的解析表达式，通过将链路可靠性进行最大化设计，获得了最优光束发散角的解析表达式。将偏置误差也考虑进去再对链路可靠性进行最大化设计，对于给定的通信速率获得了光束发散角与跟瞄误差的最佳比值的数值解及其近似解析表达式。在星间光通信中采用所设计的最优光束发散角，可在存在跟瞄误差的情况下获得良好的系统性能。

**关键词** 自由空间光通信; 最优光束发散角; 链路可靠性优化; 跟瞄误差

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## 1 Introduction

Laser satellite communication has become especially attractive in recent years[1-2] and is perceived to be an important element of future satellite communication infrastructure. In maintaining laser communication links, the pointing acquisition and tracking (PAT) system is essential[3-6]. However,

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factors such as vibration of satellite platforms, relative motion of satellites, electrical noise, and mechanical noise in the tracking loop limit the pointing accuracy of PAT systems. Thus, optical terminals must operate in the presence of pointing errors, which can cause optical intensity fluctuations at receivers and deteriorate link performance. A communication system may experience data loss when instantaneous mispoint losses exceed the value assigned in the link budget. In such cases, link parameter optimization becomes essential in mitigating the effects of pointing errors.

In many existing studies, performance parameters are optimized through optimum beam divergence angles or beam width selection in the presence of pointing errors. In Ref.[7], link reliability, rate, and range tradeoff are optimized through beam width selection in the presence of atmospheric turbulence and misalignment fading. In Ref.[8], an optimum beam width that minimizes outage performance is obtained in the presence of strong turbulence and misalignment fading channels. Meanwhile, in Ref.[9], an optimization procedure is established by finding the optimum beam width that yields the minimum bit error rate (BER) in turbulence channels with pointing errors. In Ref.[10], joint beam width and spatial coherence length optimization is proposed to maximize average capacity under the combined effects of atmospheric turbulence and pointing errors. In the aforementioned works, the bias errors are assumed to be zero, and the pointing errors follow a Rayleigh distribution. However, given that perfect calibration is not always achieved, bias errors could emerge. Thus, pointing errors follow a Rician distribution[11-12].

To achieve the highest possible link reliability performance, the optimum beam divergence angle in the presence of pointing errors without bias errors and the optimum beam divergence angle with consideration of bias errors are investigated. Given BER and bit rate, link reliability is maximized through beam divergence angle selection in the presence of pointing errors.

## 2 System model and definition

### 2.1 Received signal model

To investigate the influence of pointing errors on intersatellite optical communication systems, we herein introduce a received signal model. We consider an intersatellite optical link (IOL) that employs on-off keying (OOK) modulation with intensity modulation/direct detection (IM/DD), which is widely used in practical optical communication systems. The transmitted data are modulated onto the instantaneous intensity of an optical beam, which is then transmitted with misalignment to the receiver; the received optical power is then converted to an electrical signal through a photodiode with responsivity $R_v$. The received signal y at the detector can then be modeled as[7,12-13]

$$y = R_v A_r \eta h_p x + n , \tag{1}$$

where $x \geqslant 0$, is the transmitted intensity signal with an average optical power $E\{x\} \leqslant P_t$, $P_t$ is the average transmitted optical power, $A_r = \pi D_r^2/4$ is the receiver antenna area, $D_r$ is the receiver diameter, $\eta$ represents the optical efficiency of the transmitter and receiver, $n$ is a signal-independent zero-mean white Gaussian noise with variance $\sigma_n^2$, and $h_p$ denotes a random fluctuation of the optical intensity resulting from the pointing error of the intersatellite optical channels.

For a Gaussian beam of half-width divergence angle $\theta_b$, the channel fading factor $h_p$ resulting from the geometric spread with a pointing error angle $\theta_p$ at communication distance $L$ from the transmitter can be expressed as[12-13]

$$h_p = \frac{2}{\pi L^2 \theta_b^2} \exp\left(-\frac{2\theta_p^2}{\theta_b^2}\right), \quad 0 \leqslant h_p \leqslant \frac{2}{\pi L^2 \theta_b^2} . \tag{2}$$

### 2.2 Link reliability

System performance is conventionally described by BER, link reliability, and bit rate. In accordance with the aforementioned system model, the BER of the OOK modulation with IM/DD is given by[7,14]

$$Q(h_{\mathrm{p}}) = \frac{1}{2}\mathrm{erfc}\left(\frac{R_{\mathrm{v}} A_{\mathrm{r}} \eta h_{\mathrm{p}} P_{\mathrm{t}}}{\sigma\sqrt{2R_{\mathrm{b}}}}\right), \tag{3}$$

where erfc(·) is the complementary error function, $R_b$ is the bit rate, and s is a coefficient related to noise standard deviation $\sigma_n$,

$$\sigma_{\mathrm{n}} = \sigma\sqrt{R_{\mathrm{b}}}. \tag{4}$$

If $Q_T$ is the target BER of a communication link and if

$$Q(h_{\mathrm{pT}}) = Q_{\mathrm{T}}, \tag{5}$$

where $h_{pT}$ is the threshold of the channel fading factor, then link reliability $\varphi$ is the probability that the system BER is less than $Q_T$ [7]. According to Eqs.(3) and (5),

$$\varphi = \mathrm{prob}[Q(h_{\mathrm{p}}) < Q_{\mathrm{T}}] = \mathrm{prob}[h_{\mathrm{p}} > h_{\mathrm{pT}}], \tag{6}$$

where prob(·) denotes the probability of an event.

### 2.3 Pointing error model

The received optical intensity fluctuation, which is caused by pointing errors, has a significant impact on link reliability and bit rate. The probability density function (PDF) of radial pointing error angle $\theta_p$ can be modeled as a Rician distribution given by[11-12]

$$f_{\theta_{\mathrm{p}}}(\theta_{\mathrm{p}}) = \frac{\theta_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2}\exp\left(-\frac{\xi^2+\theta_{\mathrm{p}}^2}{2\sigma_{\mathrm{p}}^2}\right) I_0\left(\frac{\xi\theta_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2}\right), \tag{7}$$

where $\sigma_p$ is the standard deviation of the pointing error, $\xi$ is the bias error angle, and $I_0$(·) is the modified 0th- order Bessel function of the first kind. The bias error is slowly time- varying and corresponds to a static portion while the other part of the pointing error is frequently time-varying and corresponds to a dynamic component known as a dynamic error. When the bias error is zero (i.e., $\xi$=0), the PDF of the radial pointing error angle $\theta_p$ becomes a Rayleigh distribution given by

$$f_{\theta_{\mathrm{p}}}(\theta_{\mathrm{p}}) = \frac{\theta_{\mathrm{p}}}{\sigma_{\mathrm{p}}^2}\exp\left(-\frac{\theta_{\mathrm{p}}^2}{2\sigma_{\mathrm{p}}^2}\right). \tag{8}$$

## 3 Link reliability maximization

### 3.1 Link reliability maximization with zero bias error

To improve link performance, we first investigate the link reliability and bit rate of the IOL in the presence of pointing errors without bias errors. With consideration of the PDF of $\theta_p$ without bias errors, as given by Eq.(8), and the relation between $h_p$ and $\theta_p$, as given by Eq.(2), the PDF of $h_p$ can be expressed as

$$f_{h_{\mathrm{p}}}(h_{\mathrm{p}}) = \frac{\theta_{\mathrm{b}}^2}{4\sigma_{\mathrm{p}}^2}\cdot\left(\frac{\pi L^2\theta_{\mathrm{b}}^2}{2}\right)^{\frac{\theta_{\mathrm{b}}^2}{4\sigma_{\mathrm{p}}^2}}\cdot h_{\mathrm{p}}^{\frac{\theta_{\mathrm{b}}^2}{4\sigma_{\mathrm{p}}^2}-1}. \tag{9}$$

Combining Eqs. (6) and (9), we analytically obtain link reliability $\varphi$ as

$$\varphi = \int_{h_{\mathrm{pT}}}^{\infty} f_{h_{\mathrm{p}}}(h_{\mathrm{p}})\mathrm{d}h_{\mathrm{p}} = 1 - \left(\frac{\pi L^2\theta_{\mathrm{b}}^2}{2}h_{\mathrm{pT}}\right)^{\frac{\theta_{\mathrm{b}}^2}{4\sigma_{\mathrm{p}}^2}}, \tag{10}$$

where $h_{pT}$ can be obtained by combining Eqs. (3) and (5), as in

$$h_{\mathrm{pT}} = \frac{\sigma\sqrt{2R_{\mathrm{b}}}}{R_{\mathrm{v}} A_{\mathrm{r}} \eta P_{\mathrm{t}}}\mathrm{erfc}^{-1}(2Q_{\mathrm{T}}). \tag{11}$$

According to Eqs. (10) and (11), the transmitted power $P_t$, distance $L$, and random pointing error $\sigma_p$ are assumed to be constant. The variation of link reliability $\varphi$ as a function of beam divergence angle $\theta_b$ at various desired bit rates ($R_b$) is shown in Fig.1. The parameters used in these numerical examples are shown in Table 1[7,15]. In Fig.1, the existence of the maximum link reliability corresponding to the optimum beam divergence half angle $\theta_b^{opt}$ can be clearly observed in all four curves.

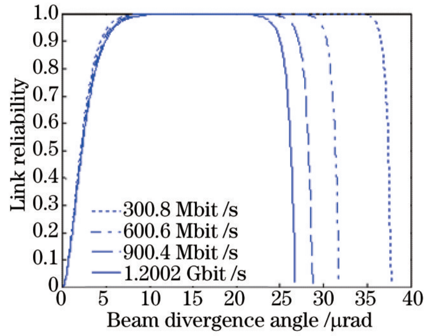


Fig.1 Link reliability versus half-width divergence angle $\theta_b$ for various desired bit rates when bias error is zero

Therefore, the optimum beam divergence angle $\theta_b^{opt}$ can be selected to maximize link reliability $\varphi$ given target BER $Q_T$, distance $L$, and bit rate $R_b$. According to Eq.(10), $\theta_b^{opt}$ can be obtained theoretically and analytically using the following condition:

$$\frac{\partial \varphi}{\partial \theta_b} = 0 , \tag{12}$$

which yields

$$\theta_b^{opt} = \sqrt{\frac{2}{\exp(1)\cdot \pi L^2 h_{pT}}} . \tag{13}$$

This expression is an analytic expression of optimum beam divergence angle $\theta_b^{opt}$ in the presence of pointing errors without bias errors.

Table 1 System parameters

| Parameter | Symbol | Value |
|---|---|---|
| Detector responsivity | $R_v$ | 0.5 |
| Optical efficiency | $\eta$ | 0.8 |
| Transmitted optical power | $P_t$ | 3 W |
| Receiver diameter | $D_r$ | 250 mm |
| Noise standard deviation (at 1 Gbit/s) | $\sigma_n$ | $5\times10^{-7}$ A |
| Distance | $L$ | 4000 km |
| Required BER | $Q_T$ | $1\times10^{-99}$ |
| Pointing error standard deviation | $\sigma_p$ | 3 μrad |

### 3.2 Link reliability maximization with nonzero bias error

In practical systems, perfect calibration cannot always be achieved. This condition leads to the emergence of nonzero bias errors. Therefore, without loss of generality, we investigate the link reliability and bit rate of the IOL with consideration of bias errors. Combining Eq. (2) and the PDF of $\theta_p$ with the bias error given by Eq.(7), we can rewrite the PDF of $h_p$ as[12]

$$f_{h_p}(h_p) = \frac{\beta}{\alpha^{\beta}} \exp\left(-\frac{\xi^2}{2\sigma_p^2}\right)\cdot h_p^{\beta-1}\cdot I_0\left[\frac{\xi}{\sigma_p^2}\sqrt{\frac{-\theta_b^2 \ln\left(h_p/\alpha\right)}{2}}\right], \tag{14}$$

where $\alpha = 2/(\pi L^2 \theta_b^2), \beta = \theta_b^2/(4\sigma_p^2)$, and $0 \leqslant h_p \leqslant \alpha$. Thus, according to Eqs.(6) and (14), we can obtain link reliability $\varphi$ in the presence of pointing jitter with bias error as

$$\varphi=\frac{\beta}{\alpha^{\beta}}\exp\left(-\frac{\xi^{2}}{2\sigma_{p}^{2}}\right)\int_{h_{pT}}^{\alpha}h_{p}^{\beta-1}\cdot I_{0}\left[\frac{\xi}{\sigma_{p}^{2}}\sqrt{\frac{-\theta_{b}^{2}\ln\left(h_{p}/\alpha\right)}{2}}\right]\mathrm{d}h_{p}\,, \tag{15}$$

Using a series representation of $I_0(\cdot)$[16] and letting $x=h_p/\alpha$, we obtain

$$\varphi=\beta\exp\left(\frac{-\xi^{2}}{2\sigma_{p}^{2}}\right)\sum_{m=0}^{\infty}\left[\frac{1}{(m!)^{2}}\left(-\frac{\xi^{2}\theta_{b}^{2}}{8\sigma_{p}^{4}}\right)^{m}\int_{h_{pT}/\alpha}^{1}x^{\beta-1}(\ln x)^{m}\mathrm{d}x\right]. \tag{16}$$

Applying an integral identity [16] to Eq.(16), we obtain

$$\varphi=\beta\exp\left(-\frac{\xi^{2}}{2\sigma_{p}^{2}}\right)\sum_{m=0}^{\infty}\left\{\frac{1}{(m!)^{2}}\left(-\beta\frac{\xi^{2}}{2\sigma_{p}^{2}}\right)^{m}\times\left[\frac{(-1)^{m}m!}{\beta^{m+1}}-\frac{1}{m+1}\left(\frac{h_{pT}}{\alpha}\right)^{\beta}\sum_{n=0}^{m}\frac{(-1)^{n}(m+1)!}{\beta^{n+1}(m-n)!}\left(\ln\frac{h_{pT}}{\alpha}\right)^{m-n}\right]\right\}. \tag{17}$$

For obtaining the maximum link reliability, the preceding equation cannot be solved analytically using the condition $\mathrm{d}\varphi/\mathrm{d}\theta_b=0$; nevertheless, numerical results can be obtained. Such results are shown in Fig.2. Link reliabilities $\varphi$ are plotted as a function of $\theta_b/\sigma_p$ and desired bit rate $R_b$ for the fixed transmitted power $P_t$, distance $L$, bias error $\xi$, and random pointing error $\sigma_p$. The parameters used in these numerical results are also shown in Table 1. Bias error $\xi$ is assumed to be 10 μrad. The numerical calculation is performed with a relative tolerance of $10^{-12}$. As shown in Fig.2, an optimum value of $\theta_b/\sigma_p$ maximizes link reliability, and for every $R_b$, the numerical results of the optimum ratio $(\theta_b/\sigma_p)^{opt}$ can be obtained. By applying the least squares fit to these calculated values, an analytic approximation for $(\theta_b/\sigma_p)^{opt}$ as a function of the desired bit rate $R_b$ is given by

$$(\theta_b/\sigma_p)^{opt}=a_{10}R_b^{10}+a_9R_b^9+a_8R_b^8+a_7R_b^7+a_6R_b^6+a_5R_b^5+a_4R_b^4+a_3R_b^3+a_2R_b^2+a_1R_b^1+a_0\,, \tag{18}$$

where the coefficients $a_i$ ($i$=0,1,2⋯,10) are as presented in Table 2. The accurate numerical results and the estimated values from the analytic approximation of Eq.(18) are plotted in Fig.3. The difference between the numerical and estimated results is less than 0.12% when 60.96 Mbit/s$<R_b<$1.5 Gbit/s.

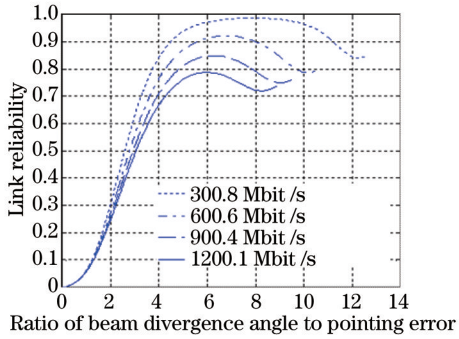


Fig.2 Link reliability versus ratio of beam divergence angle to pointing error for various desired bit rates when bias error is nonzero

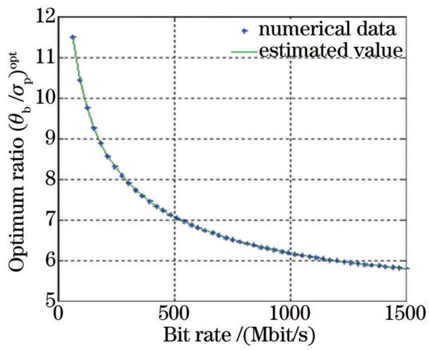


Fig.3 Comparison of the numerical results of optimum ratio $(\theta_b/\sigma_p)^{opt}$ and its estimated values from analytic approximation

Table 2 Coefficients for approximate expression describing the optimum beam divergence angle for a desired bit rate

| Coefficient | Value | Coefficient | Value |
|---|---|---|---|
| $a_0$ | 15.2914 | $a_6$ | $1.2912\times10^{-50}$ |
| $a_1$ | $-8.9748\times10^{-8}$ | $a_7$ | $-9.8651\times10^{-60}$ |
| $a_2$ | $5.7629\times10^{-16}$ | $a_8$ | $4.7466\times10^{-69}$ |
| $a_3$ | $-2.3743\times10^{-24}$ | $a_9$ | $-1.3032\times10^{-78}$ |
| $a_4$ | $6.3206\times10^{-33}$ | $a_{10}$ | $1.5557\times10^{-88}$ |
| $a_5$ | $-1.1088\times10^{-41}$ | | |

## 4 Conclusion

A link design method for intersatellite optical communication systems with misalignment fading is presented. Link reliability is optimized through beam divergence angle selection with a target BER and bit rate. The analytic expression of the optimum beam divergence angle $\theta_b^{opt}$ in the presence of pointing errors with zero bias errors is obtained. The numerical results and analytic approximation of $(\theta_b/\sigma_p)^{opt}$ in the presence of misalignment fading with nonzero bias errors are obtained. In some cases, such as when the bias error is much smaller than the dynamic error, the influence of the former can be ignored. In general, however, bias errors must be considered because they greatly affect system performance. The results of this work and the proposed link design method may be useful in the parametric estimation optimization of intersatellite optical communication systems.

## References

1 Liu Yun, Zhao Shanghong, Yang Shengsheng, *et al.*. Reliability evaluation of space irradiated Llser diode in laser satellite communication system[J]. Laser & Optoelectronics Progress, 2014, 51(5): 051401.
刘　韵, 赵尚弘, 杨生胜, 等. 空间辐射下卫星光通信系统半导体激光器的可靠性估计[J]. 激光与光电子学进展, 2014, 51(5): 051401.

2 Ma Xiaoping, Sun Jianfeng, Zhi Yanan, *et al.*. Research of DPSK modulation and self-differential homodyne coherent detection technology to overcome atmospheric turbulence effect in the satellite-to-ground laser communication[J]. Acta Optica Sinica, 2013, 33(7): 0706017.
马小平, 孙建锋, 职亚楠, 等. DPSK调制/自差动零差相干探测技术克服星地激光通信中大气湍流效应的研究[J]. 光学学报, 2013, 33(7): 0706017.

3 Yu Xiaonan, Tong Shoufeng, Zhao Xin, *et al.*. Simulation system of the beacon pointing and acquisition process in the satellite laser communication[J]. Chinese J Lasers, 2014, 41(8): 0805007.
于笑楠, 佟首峰, 赵　馨, 等. 基于合作信标的卫星激光通信瞄准捕获仿真系统[J]. 中国激光, 2014, 41(8): 0805007.

4 Qian Feng, Jia Jianjun, Zhang Liang, *et al.*. Defective pixel correction of spot-detecting camera in satellite-to-ground laser communication ATP system[J]. Chinese J Lasers, 2014, 41(5): 0505007.
钱　锋, 贾建军, 张　亮, 等. 星地激光通信ATP系统探测相机的坏点校正[J]. 中国激光, 2014, 41(5): 0505007.

5 Qin Hao, Fu Chengyu, Huang Yongmei. Adaptive composite control for jitter rejection of satellite in optical communications [J]. Laser & Optoelectronics Progress, 2014, 51(4): 040606.
覃　浩, 付承毓, 黄永梅. 光通信中卫星振动抑制的自适应复合控制[J]. 激光与光电子学进展, 2014, 51(4): 040606.

6 Dong Ran, Ai Yong, Xiong Zhun, *et al.*. Experiment and analysis of the effect of fine tracking system on the unstable platform in laser communication[J]. Optoelectronics Letters, 2013, 9(4): 301-304.

7 A A Farid, S Hranilovic. Link reliability, range and rate optimization for free-space optical channels[C]. 2009 10th International Conference on Telecommunications (ConTEL), 2009: 19-23.

8 A Garcia-Zambrana, C Castillo-Vazquez, B Castillo-Vazquez. Outage performance of MIMO FSO links over strong turbulence and misalignment fading channels[J]. Optics Express, 2011, 19(14): 13480-13496.

9 A Garcia-Zambrana, B Castillo-Vazquez, C Castillo-Vazquez. Asymptotic error-rate analysis of FSO links using transmit laser selection over gamma-gamma atmospheric turbulence channels with pointing errors[J]. Optics Express, 2012, 20(3): 2096-2109.

10 It Ee Lee, Zabih Ghassemlooy, Wai Pang Ng, *et al.*. Joint optimization of a partially coherent Gaussian beam for free-space optical communication over turbulent channels with pointing errors[J]. Optics Letters, 2013, 38(3): 350-352.

11 Shlomi Arnon. Minimization of outage probability of WiMAX link supported by laser link between a high-altitude platform and a satellite[J]. Journal of Optical Society of America A, 2009, 26(7): 1545-1552.

12 Yang Fan, Cheng Julian, Theodoros A Tsiftsis. Free-space optical communication with nonzero boresight pointing errors[J]. IEEE Transactions on Communications, 2014, 62(2): 713-725.

13 Yang Fan, Cheng Julian, Theodoros A Tsiftsis. Free-space optical communications with generalized pointing errors[C]. IEEE International Conference on Communications (ICC), 2013: 3943-3947.

14 S Hranilovic. Wireless Optical Communication Systems[M]. New York: Springer, 2005: 52-53.

15 I S Gradshteyn, I M Rzyzhik. Table of Integrals, Series, and Products, 7th ed[M]. New York: Academic Press, 2007: 919, 238.

16 Guo Yongfu, Wang Humei. Survey on SILEX and subsequent development of space laser communication in Europe[J]. Spacecraft Engineering, 2013, 22(2): 88-93.
郭永富, 王虎妹. 欧洲SILEX计划及后续空间激光通信技术发展[J]. 航天器工程, 2013, 22(2): 88-93.

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