

Research on the Amplitude Couplings in Ultrashort Pulses Using Amplitude Correlation Functions

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Abstract　A novel approach for analyzing spatiotemporal couplings in ultrashort pulses is presented using amplitude correlation functions. Amplitude correlation degrees and correlation bandwidth are defined, which can readily indicate the severity of spatiotemporal couplings. Intuitive pictures of pulses with different amounts of spatial chirp are given. With amplitude correlation functions, the amplitude couplings in ultrashort pulses, such as the first-order spatial chirp and angular dispersion caused by angular dispersion elements and the second-order spatial chirp caused by diffraction effects, are studied, and corresponding analytical expressions for the amplitude correlation degrees are given. It can be found that the refraction and diffraction of broadband optical pulses are frequency dependent, resulting in the decrease of amplitude correlation degree and leading to a coupling of spatial and temporal effects. Specifically the severity of the spatial chirp caused by an angular disperser increases with the increase of propagation distance and finally tends to be constant. Moreover, as the diffraction of ultrashort pulsed Gaussian beams is frequency dependent, the second-order spatial chirp of ultrashort pulsed Gaussian beams caused by diffraction effects will decrease during propagation. Finally, the severity of angular dispersion caused by an angular disperser does not vary as the beam propagates in the free space.

Key words　ultrafast optics; spatiotemporal coupling; ultrashort pulse; spatial chirp; angular dispersion

OCIS codes　320.5550; 320.7120

利用振幅关联函数研究超短脉冲中的时空耦合效应

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摘要　提出了一种利用振幅关联函数来分析超短脉冲中时空耦合效应的方法。定义了振幅关联度和关联带宽来衡量时空耦合的严重程度。给出了不同空间啁啾程度的脉冲光束的时空分布图。采用振幅关联函数研究了超短脉冲中的一些振幅耦合效应，如由角色散元件引起的一阶空间啁啾和角色散效应以及由衍射引起的二阶空间啁啾效应，并给出了相应的振幅关联度的解析表达式。结果表明，由于宽带激光脉冲的衍射和折射与频率密切相关，导致其振幅关联度下降并出现时空耦合。具体地，角色散引起的空间啁啾将会随着传输距离的增大而减弱直至趋于一定值。另外，由自由空间衍射引起的二阶空间啁啾效应也会随着传输距离的增大而减弱。最后，由角色散元件引起的角色散效应并不会随着传输距离的变化而变化。

关键词　超快光学；时空耦合；超短脉冲；空间啁啾；角色散

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1　Introduction

In the design of optical systems using broadband optical pulses, the manipulation of spatiotemporal couplings is

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one of the most important issues[1]. The diffraction and refraction of broadband optical pulses propagating through the systems are frequency dependent, leading to a coupling of spatial and temporal effects[2]. And for ultrashort pulses comprising only a few oscillations, the spectral bandwidth of which becomes comparable to their carrier frequencies, the spatial and temporal characteristics interact with each other even for propagation in a nondispersive medium[3-4]. Making use of spatiotemporal couplings of ultrashort pulses allows some applications, such as pulse compression[5] and shaping[6], which are not possible to be realized in their absence. However, spatiotemporal couplings can also be detrimental, for example, when focusing an ultrashort pulse[7]. For these reasons, much attention is paid to spatiotemporal couplings in recent years.

So far, numerous researchers have tried to develop a common quantity to describe or quantify spatiotemporal couplings. Unfortunately, such a general quantity does not currently exist. In Ref. [1], a degree of spatiotemporal uniformity was defined to quantify spatiotemporal couplings in both amplitude and phase. In Ref. [6], spatiotemporal couplings in femtosecond pulse shaping were quantified by the space-time coupling constant $\Delta x/\Delta t$ or the frequency-to-space mapping $\Delta\Omega/\Delta x$. Akturk *et al.*[8] presented a general theory of the first-order spatiotemporal couplings, and classified spatiotemporal couplings into amplitude couplings and phase couplings. Amplitude couplings include spatial chirp[9-10], angular dispersion[11-12], pulse-front tilt[13-14], etc. Phase couplings include wave-front rotation, wave-front-tilt dispersion, angular spectral chirp, etc.[8]. In Refs. [15-16], several correlation coefficients were defined to evaluate the severity of the first-order and the second-order amplitude couplings, respectively. In Ref. [17], three coupling coefficients were defined to describe spatiotemporal couplings in ultrashort pulses.

In this paper, we defineamplitude correlation functions and correlation degrees, which can be used to analyze arbitrary order amplitude couplings in ultrashort pulses. With amplitude correlation functions, the amplitude couplings in ultrashort pulses, such as the first-order spatial chirp and angular dispersion caused by angular dispersion elements and the second-order spatial chirp caused by diffraction effects, are studied.

2 Definitions of correlation function and correlation degree

The electric field of the pulse in the x-ω domain can be expressed in the form

$$\widetilde{E}(x,\omega)=|\widetilde{E}(x,\omega)|\exp[\mathrm{i}\Phi(x,\omega)],\tag{1}$$

where $|\widetilde{E}(x,\omega)|$ denotes the spectral amplitude, and $\Phi(x,\omega)$ is the spectral phase.

Dorrer *et al.*[1] proposed the spatiotemporal correlation function to quantify spatiotemporal couplings in both amplitude and phase, i.e.,

$$C(\omega_1,\omega_2)=\int\widetilde{E}(x,\omega_1)\widetilde{E}^*(x,\omega_2)\mathrm{d}x.\tag{2}$$

However, it is not suitable for describing amplitude couplings such as spatial chirp or angular dispersion, which contains couplings only in amplitude.

Considering that the amplitude couplings depend only on $|\widetilde{E}(x,\omega)|$, the amplitude correlation function can be defined as

$$C(\omega_1,\omega_2)_{\mathrm{a}}=\int|\widetilde{E}(x,\omega_1)||\widetilde{E}(x,\omega_2)|\mathrm{d}x.\tag{3}$$

It is useful to normalize the amplitude correlation function by setting

$$\mu(\omega_1,\omega_2)=\frac{C(\omega_1,\omega_2)_{\mathrm{a}}}{[C(\omega_1,\omega_1)_{\mathrm{a}}]^{1/2}[C(\omega_2,\omega_2)_{\mathrm{a}}]^{1/2}}.\tag{4}$$

With the Cauchy-Schwarz inequality[18], we have

$$0\leqslant\mu(\omega_1,\omega_2)\leqslant 1,\tag{5}$$

for all values of the arguments ω_1 and ω_2. We will refer to $\mu(\omega_1,\omega_2)$ as the amplitude correlation degree between angular frequency ω_1 and ω_2. An increased value of $\mu(\omega_1,\omega_2)$ indicates an increase in correlation degree between ω_1 and ω_2. Analogous quantities can be defined in the k-ω domain (see Section 4). In this paper, we mainly consider amplitude couplings in the x-ω and k-ω domains.

3 Spatial chirp

In the x-ω domain, the amplitude couplings are spatial chirp[8]. The schematic diagram of spatial chirp can be seen clearly in Fig. 1. As shown in Fig. 1, the refraction of broadband optical pulses propagating through the prism pair is frequency dependent, leading to a coupling in the x-ω domain, i.e., spatial chirp.

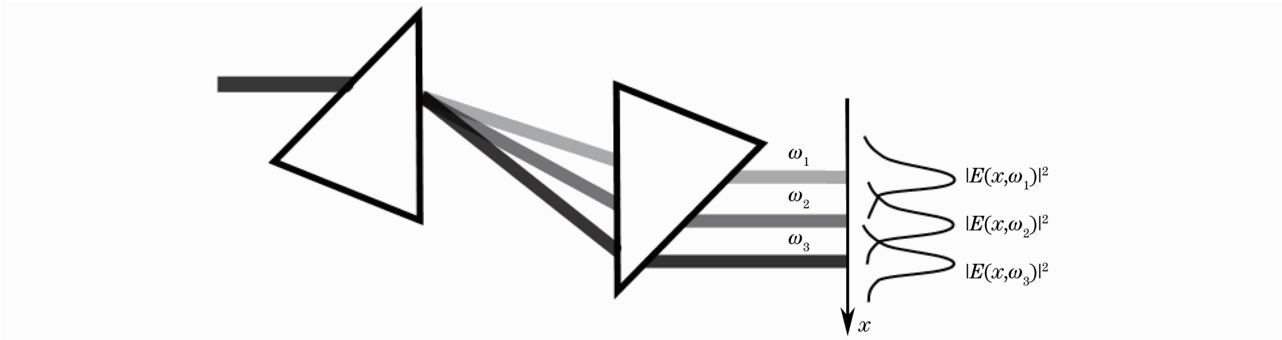

Fig. 1 Spatial chirp after a prism pair[19]

3.1 First-order spatial chirp caused by angular dispersion

Gaussian ultrashort pulses with Gaussian spatial profiles with the first-order spatiotemporal couplings in the x-ω domain can be expressed in the form[8]

$$E(x,\Omega) \propto \exp(R_{xx}x^2 + 2R_{x\omega}x\Omega - R_{\omega\omega}\Omega^2) = \exp(Ax^2 + 2Bx\Omega - C\Omega^2)\exp[\mathrm{i}(Ex^2 + 2Fx\Omega - G\Omega^2)], \quad (6)$$

where $A = R^{\mathrm{R}}_{xx}$, $B = R^{\mathrm{R}}_{x\omega}$, $C = R^{\mathrm{R}}_{\omega\omega}$, $E = R^{\mathrm{I}}_{xx}$, $F = R^{\mathrm{I}}_{x\omega}$, $G = R^{\mathrm{I}}_{\omega\omega}$, $\Omega = \omega - \omega_0$, and ω_0 is the central angular frequency. Superscripts "R" and "I" are used for the real and imaginary parts of the parameter, respectively. B and F are the coupling coefficients for amplitude couplings and phase couplings, respectively. Because A and C are related to beam spot size and bandwidth, respectively[8], A is always negative, whereas C is always positive. Introducing the expression (6) into Eq. (4), the amplitude correlation degree for an ultrashort pulse described by expression (6) can be deduced as

$$\mu(\Omega_1,\Omega_2) = \exp\left[\frac{B^2(\Omega_2-\Omega_1)^2}{2A}\right]. \quad (7)$$

Equation (7) implies that the amplitude correlation degree for the Gaussian pulses and beams with the first-order spatiotemporal couplings is also Gaussian function. Letting

$$\Delta\omega_{\mathrm{c}} = |\Omega_2 - \Omega_1| = \left(-\frac{2A}{B^2}\right)^{1/2}, \quad (8)$$

then we have $\mu(\Omega_1,\Omega_2) = 1/\mathrm{e}$. Consequently, we can define $\Delta\omega_{\mathrm{c}}$ as the correlation bandwidth. It can be seen from Eq. (8) that the larger the amplitude coupling coefficient B, the smaller the correlation bandwidth $\Delta\omega_{\mathrm{c}}$. As shown in

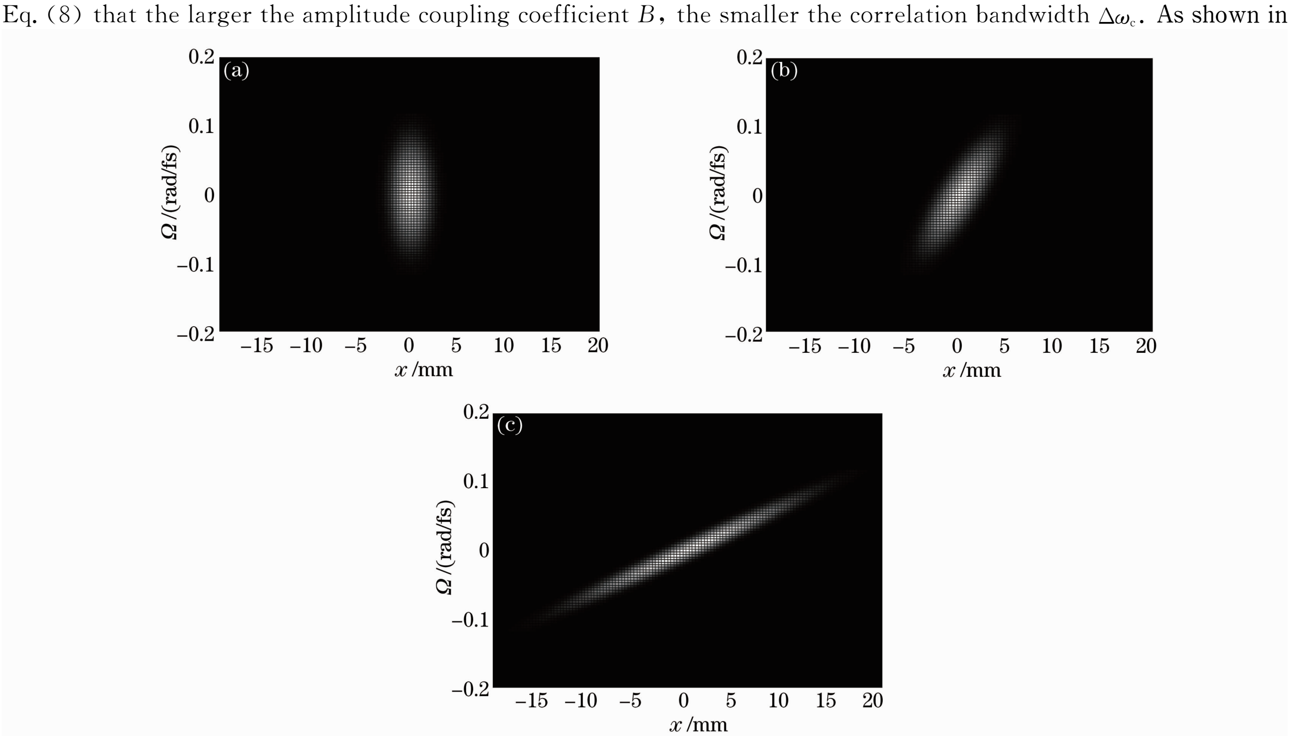

Fig. 2 Profiles of an ultrashort pulse with increasing amounts of spatial chirp caused by angular dispersion, and hence with decreasing values of $\Delta\omega_{\mathrm{c}}$[17]. (a) $\Delta\omega_{\mathrm{c}} = 12.9$ rad/fs; (b) $\Delta\omega_{\mathrm{c}} = 93.2$ rad/ps; (c) $\Delta\omega_{\mathrm{c}} = 25.4$ rad/ps

Fig. 2(a), the "spatial dispersion (SPD)[10]" is 0, i. e., $\partial x_0/\partial\omega=0$, where x_0 is the mean beam position for a given frequency ω; whereas SPD gets a non-zero value in Fig. 2(b) and a maximum value in Fig. 2(c). The larger the value of SPD, the larger the spatial chirp. So a decreased value of $\Delta\omega_c$ indicates an increase in the magnitude of spatial chirp.

The dispersive characteristic of an angular dispersion component can be described approximately by Martinez′s model[20]. By using the Kirchhoff-Fresnel integral, the expression for the electric field of Gaussian ultrashort pulses with Gaussian spatial profiles at an arbitrary distance from an angular disperser can be expressed in the frequency domain as[20-21]

$$\widetilde{E}(x,\Omega)=b_3\exp\left(-\frac{\tau^2\Omega^2}{4}\right)\exp\left(-\mathrm{i}\frac{k_c x^2}{2z}\right)\exp\left[\frac{\mathrm{i}k_c}{2z}\frac{\tilde{q}(d)}{\tilde{q}(d+\alpha^2 z)}(x+\beta\Omega z)^2\right], \tag{9}$$

where b_3 is an amplitude constant, τ is the Gaussian pulse parameter [full-width at half-maximum (FWHM) of intensity $\Delta t=\sqrt{2\ln 2}\tau$], k_c is the wave number of the central frequency, d is the distance from the Gaussian beam waist to the angular dispersion component, z is the propagation distance of the laser beam from angular dispersion component (see Fig. 1 in Ref. [22]), and $\tilde{q}$ is a complex parameter of the Gaussian beam. $\alpha=\partial\theta/\partial\gamma$ is the factor of angular magnification of the beam, where θ is the emerging angle and γ is the angle of incidence. $\beta=\partial\theta/\partial\omega$ is the first-order angular dispersion factor of the angular dispersion component.

It is easy toreduce Eq. (9) to be the form of expression (6), and corresponding parameters $A\sim G$ can be obtained, such as

$$A=-\frac{k_c z_{R0}\alpha^2}{2[(d+\alpha^2 z)^2+z_{R0}^2]}, \tag{10}$$

$$B=-\frac{k_c z_{R0}\alpha^2\beta z}{2[(d+\alpha^2 z)^2+z_{R0}^2]}, \tag{11}$$

where z_{R0} is the diffraction length at the central angular frequency ω_0. Substituting Eqs. (10) and (11) into Eq. (8), the correlation bandwidth for the spatial chirp caused by an angular disperser can be written as

$$\Delta\omega_c=\frac{2}{\alpha\beta z}\left[\frac{(d+\alpha^2 z)^2+z_{R0}^2}{k_c z_{R0}}\right]^{1/2}. \tag{12}$$

It can be seen from Eq. (12) that thecorrelation bandwidth for the spatial chirp caused by an angular disperser depends on the propagation distance z, the diffraction length z_{R0}, as well as the angular dispersion parameters (α, β). It can also be shown that the correlation bandwidth after an angular disperser decreases with propagation distance and finally tends to the asymptotic value $2\alpha/(k_c z_{R0}\beta^2)^{1/2}$.

In Ref. [15], a normalized spatial chirp parameter $\rho_{x\omega}$ was defined to estimate the severity of the first-order spatial chirp (see Eq. (1) in Ref. [15]), and an increased value of $\rho_{x\omega}$ indicated an increase in the magnitude of the first-order spatial chirp. Figure 3 shows the correlation bandwidth and normalized spatial chirp parameter of the first-order spatial chirp caused by angular dispersion. As shown in Fig. 3, the spatial chirp parameter $\rho_{x\omega}$ increases with the increase of propagation distance z, and finally tends to be 1; while the correlation bandwidth $\Delta\omega_c$ decreases with the increase of propagation distance z, and finally tends to be constant. It means that the severity of the spatial chirp caused by an angular disperser increases with the increase of propagation distance z and finally tends to be constant. It should be noted that the spatial chirp parameter defined in Ref. [15] can only be used for describing the first-order

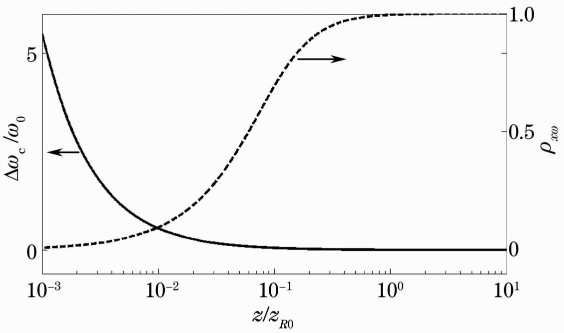

Fig. 3 Variation of correlation bandwidth $\Delta\omega_c$ and normalized spatial chirp parameter $\rho_{x\omega}$ with propagation distance z. Corresponding parameters are $\alpha=-0.27$, $\beta=-0.29$ fs, $\omega_0=2.35$ rad/fs, initral beam waist $s=0.5$ mm ($z_{R0}=0.98$ m), $d=z_{R0}$, and $\tau=22.7$ fs

spatial chirp, whereas the amplitude correlation functions and correlation degrees defined by Eqs. (3) and (4) can be used for describing arbitrary order spatial chirp.

3.2 Second-order spatial chirp caused by diffraction effects

Thespatial chirp of ultrashort pulsed Gaussian beams caused by diffraction effects is the second-order spatial chirp[16]. The spectral amplitude of ultrashort pulsed Gaussian beams in the x-ω domain can be expressed in the form

$$|\widetilde{E}(x,\omega)| = \hat{p}_\omega a(x,\omega), \tag{13}$$

where $\hat{p}_\omega$ is the frequency spectrum at $x=0$, and $a(x,\omega)$ is the amplitude factor. For Gaussian beams propagating in free space, the amplitude $a(x,\omega)$ and the beam waist $s_\omega(z)$ can be written as[23]

$$a(x,\omega) = \frac{s}{s_\omega(z)}\exp\left[-\frac{x^2}{s_\omega^2(z)}\right], \tag{14}$$

$$s_\omega(z) = s\left(1+\frac{z^2}{z_R^2}\right)^{1/2}, \tag{15}$$

where s is the initial beam waist, z is the propagation distance, $z_R = \omega s^2/2c$ is the diffraction length for each frequency, and c is the speed of light in vacuum. It can be seen from Eqs. (14) and (15) that the diffraction of ultrashort pulsed Gaussian beams depends on frequency.

Introducing Eqs. (13) ~ (15) into Eq. (4), the amplitude correlation degree for the spatial chirp caused by diffraction effects can be written as

$$\mu(\omega_1,\omega_2) = \frac{\int a(x,\omega_1)a(x,\omega_2)\mathrm{d}x}{\left[\int a(x,\omega_1)^2\mathrm{d}x\int a(x,\omega_2)^2\mathrm{d}x\right]^{1/2}} = \left[\frac{\omega_1\omega_2(\omega_1^2 s^4+4z^2c^2)^{1/2}(\omega_2^2 s^4+4z^2c^2)^{1/2}}{\omega_1^2\omega_2^2 s^4+2z^2c^2(\omega_1^2+\omega_2^2)}\right]^{1/2}. \tag{16}$$

It can be seen from Eq. (16) that the amplitude correlation degree for the spatial chirp caused by diffraction effects depends on the propagation distance z and the initial beam waist s, but does not depend on the frequency spectrum $\hat{p}_\omega$. Furthermore, Eq. (16) implies that $\mu(\omega_1,\omega_2)\equiv 1$ for the case of $z=0$, indicating that the spatial chirp does not exist in the initial ultrashort pulsed Gaussian beams. Figure 4 shows the correlation degree of the second-order spatial chirp caused by diffraction effects. As shown in Fig. 4, if $|\omega-\omega_0|\ll\omega_0$, the correlation degree is close to 1. It means that the spatial chirp caused by diffraction effects is relatively small even for few-cycle ultrashort pulses. Figure 4 also shows that at a large propagation distance ($z > 5z_R$), the correlation degree for a given frequency ω tends to be a constant. It can be explained by the fact that the frequency shift tends to be a constant at a large propagation distance (see Fig. 5 of Ref. [23]). In a word, as the diffraction of ultrashort pulsed Gaussian beams is frequency dependent, the amplitude correlation degree will decrease during propagation, leading to the spatial chirp.

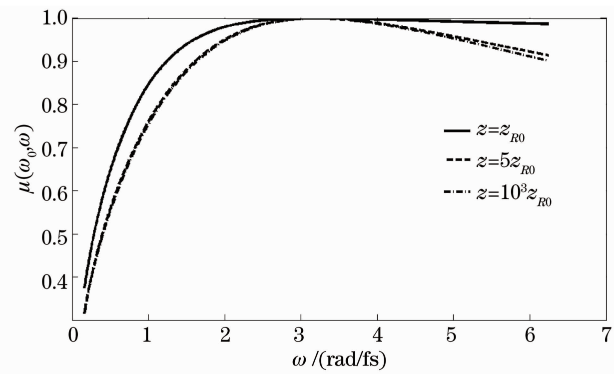

Fig. 4 Variation of the correlation degree of the second-order spatial chirp caused by diffraction with ω for different propagation distance z. $\omega_1=\omega_0=3.2$ rad/fs, $\omega_2=\omega$, and $s=20$ μm ($z_{R0}=2.1$ mm)

4 Angular dispersion

In the k-ω domain, the amplitude couplings are angular dispersion[8]. The schematic diagram of angular dispersion can be seen clearly in Fig. 1 of Ref. [12]. Similar to Eq. (3), the amplitude correlation function in the k-ω domain can be defined as

$$C(\omega_1,\omega_2)_a = \int|\widetilde{E}(k_x,\omega_1)||\widetilde{E}(k_x,\omega_2)|\mathrm{d}k_x. \tag{17}$$

Consequently, theamplitude correlation degree $\mu(\omega_1,\omega_2)$ in the k-ω domain can be readily obtained by Eqs. (4) and (17).

Gaussian ultrashort pulses with Gaussian spatial profiles with the first-order spatiotemporal couplings in the k-ω domain can be expressed in the form[8]

$$\widetilde{E}(k,\Omega) \propto \exp[S_{kk}k^2 + 2S_{k\omega}k\Omega - S_{\omega\omega}\Omega^2] = \exp(Hk^2 + 2Ik\Omega - J\Omega^2)\exp[\mathrm{i}(Kk^2 + 2Lk\Omega - M\Omega^2)]. \tag{18}$$

where $H = S_{kk}^{\mathrm{R}}$, $I = S_{k\omega}^{\mathrm{R}}$, $J = S_{\omega\omega}^{\mathrm{R}}$, $K = S_{kk}^{\mathrm{I}}$, $L = S_{k\omega}^{\mathrm{I}}$, $M = S_{\omega\omega}^{\mathrm{I}}$. I and L are the coupling coefficients for amplitude couplings and phase couplings, respectively. Because H and J are related to angular divergence and bandwidth, respectively[5], H is always negative, whereas J is always positive. By using Eqs. (4) and (17), the amplitude correlation degree for an ultrashort pulse described by the expression (18) can be deduced as

$$\mu(\Omega_1,\Omega_2) = \exp\left[\frac{I^2(\Omega_2 - \Omega_1)^2}{2H}\right]. \tag{19}$$

Similar to Eq. (8), the correlation bandwidth $\Delta\omega_{\mathrm{c}}$ for an ultrashort pulse described by expresson (18) can be defined as

$$\Delta\omega_{\mathrm{c}} = \left(-\frac{2H}{I^2}\right)^{1/2}. \tag{20}$$

By using Eq. (2.133) of Ref. [4], an expression for the electric field of pulsed Gaussian beam just beyond an angular disperser can be expressed in the k-ω domain as

$$\widetilde{E}(k_x,\Omega) = b_4\tilde{\varepsilon}(\Omega)\exp\left[\frac{\mathrm{i}\tilde{q}(d)}{2k_{\mathrm{c}}\alpha^2}(k_x - k_{\mathrm{c}}\beta\Omega)^2\right], \tag{21}$$

where b_4 is an amplitude constant, $\tilde{\varepsilon}(\Omega)$ is the pulse spectrum, and k_x is the spatial frequency. By using the transfer function [Eq. (1.6.13) in Ref. [24]], the electric field at an arbitrary distance from an angular disperser can be expressed as

$$\widetilde{E}(k_x,\Omega) = b_4\tilde{\varepsilon}(\Omega)\exp\left[\frac{\mathrm{i}\tilde{q}(d)}{2k_{\mathrm{c}}\alpha^2}(k_x - k_{\mathrm{c}}\beta\Omega)^2\right]\exp\left[-\mathrm{i}k_{\mathrm{c}}\left(1 - \frac{k_x^2}{2k_{\mathrm{c}}^2}\right)z\right]. \tag{22}$$

It is easy to reduce Eq. (22) to be the form of expression (18), and the corresponding amplitude correlation degree and correlation bandwidth can be obtained by using (19) and (20), i.e.,

$$\mu(\Omega_1,\Omega_2) = \exp\left[-\frac{\beta^2 z_{R0}k_{\mathrm{c}}(\Omega_2 - \Omega_1)^2}{4\alpha^2}\right], \tag{23}$$

and

$$\Delta\omega_{\mathrm{c}} = \frac{2\alpha}{\beta(z_{R0}k_{\mathrm{c}})^{1/2}}. \tag{24}$$

It can be seen from Eqs. (23) and (24) that the amplitude correlation degree and the correlation bandwidth of angular dispersion caused by an angular disperser depend on the angular dispersion parameters α and β intimately, but do not depend on the propagation distance z and the pulse spectrum $\tilde{\varepsilon}(\Omega)$. It means that the severity of angular dispersion caused by an angular disperser does not vary as the beam propagates in the free space. Also with Eqs. (23) and (24), for the case of $\beta = 0$, there is $\mu(\Omega_1,\Omega_2) \equiv 1$ and $\Delta\omega_{\mathrm{c}} \to \infty$. It means that angular dispersion does not exist if there is no angular disperser.

It should be noted that the amplitude correlation functions and correlation degrees defined in this paper are different from those of coherence theory[25], which is based on statistical theory.

Finally, to measure the amplitude correlation degree, one can use self-referencing technique[1,26], frequency-resolved optical gating (FROG) technique[9,27], or SEA TADPOLE (spatial encoded arrangement for temporal analysis by dispersing a pair of E-fields) technique[28], etc.

5 Conclusions

A novel approach for analyzing spatiotemporal couplings in ultrashort pulses by using amplitude correlation functions has been presented. The amplitude correlation degrees and correlation bandwidth have been defined to describe the severity of spatiotemporal couplings. The amplitude couplings in ultrashort pulses, such as the first-order spatial chirp and angular dispersion caused by angular dispersion elements, as well as the second-order spatial chirp caused by diffraction effects, have been studied by using amplitude correlation functions. We have found that the refraction and diffraction of broadband optical pulses is frequency dependent, resulting in the decrease of amplitude correlation degree and leading to a coupling of spatial and temporal effects. It can be shown that the amplitude correlation functions and correlation degrees defined in this paper can help a better understanding of amplitude couplings and their consequences. This approach may also be extended to analyze the phase couplings, such as wave-front-tilt dispersion, and angular spectral chirp[8].

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