Theoretical Study on Beam Steering Efficiency of Liquid Crystal Optical Phased Array

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Abstract The overall steering efficiency (OSE) of the liquid crystal optical phased array (LCOPA) is defined as the ratio of the first order diffraction's intensity and the incident intensity. It is affected by many factors. A numerical analysis model is established to analyze the OSE. The model regards a LCOPA as the combination of a phase grating and an amplitude grating, and thus, model parameters include not only the geometric structural parameters but also the parameters of the phase profile. The general relations between the OSE and three kinds of factors, i.e. steering angles, the parameters of the LCOPA and errors between ideal and practical phase profiles, are studied. Ways to improve the OSE are proposed as well. Results show that increase of steering angles reduces the OSE while raising fill factors and reducing the pixel size can improve the OSE. Proportional spacing pixels are not the optimal distribution for improving the OSE, for which the method of finite-difference time-domain (FDTD) should be used to design the optimal pixel distribution. Reducing the space of two adjacent phase shifters and raising the phase reset value can improve the OSE obviously.

Key words liquid crystal optics; optical phased array; beam steering efficiency; phase; pixel

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液晶光学相控阵光束偏转效率研究

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摘要 液晶光学相控阵(LCOPA)的光束偏转效率(OSE)定义为一级衍射光强与总入射光强的比值，它受很多因素的影响，因此建立一个数值分析的模型来分析光束偏转效率。这个模型将液晶光学相控阵器件当作一个相位光栅和一个振幅光栅的组合体，可控变量既包括几何结构参数又包括相位分布的参数。理论研究了光束偏转效率和三类因素即偏转角、器件参数及理想与实际相位分布误差之间的关系，提出了改善光束偏转光效率的方法。结果表明随着偏转角的增大光束偏转效率将会减小，而增大填充因子、减小像素大小会提高光束偏转效率。均匀排布像素并不能获得最佳偏转效率，需要使用如时域有限差分等方法来得到最佳的像素分布。减小相邻相移器之间的间隔、提高相位回置值可以明显提高光束偏转效率。

关键词 液晶光学；光学相控阵；光束偏转效率；相位；像素

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1 Introduction

The beam steering technology is widely used in many electric optics (EO) systems such as laser-associated sensing, laser radar, adaptive optics and laser communication systems[1~6]. However, the conventional mechanical beam steering technology becomes helpless for today's integrated EO equipments gradually because of its inevitable limits[2], which bring many problems such as a huge amount of driving power, slow response speed and low resolving power. The liquid crystal optical phased array (LCOPA) shows a promising prospect to be an alternate

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technology, benefitting from its characteristics, which are compact, lightweight, low power, agile, random-access and high resolving power[1~6]. However, the overall steering efficiency (OSE) is affected by many factors such as steering angles, the parameters of the LCOPA and errors between ideal and practical phase profiles. Here, OSE is defined as the ratio of the first order diffraction's intensity and the incident intensity, and it is a significant performance index to decide the application range of a LCOPA[2]. This issue has drawn wide attention, but the relations between the OSE and the affecting factors have not been studied to the best by our knowledge.

In this paper, we establish a numerical analysis model to study the relations between the OSE and the affecting factors, and then propose the ways to improve the OSE.

2 Model for efficiency study

According to Huygens principle, the propagating direction of the light is perpendicular to the wavefront. That means that as long as we change the distribution of the wavefront, in fact, which is the phase profile of the wavefront, the propagating direction of the light will be changed. This provides us an innovative approach to realize beam steering, that is to change the phase profile of the wavefront. Liquid crystal (LC) can fully display its capabilities in this area because of its characteristic of electro-birefringence. The LCOPA is made according to this principle. Figure 1(a) illustrates the basic structure of a LCOPA. It is composed of numerous phase shifters which are realized by the way of discrete transparent electrodes patterned above the LC layer. The rod-like LC molecules between every discrete electrode and the common ground, which will rotate when a voltage is applied, form an independent phase shifter, and then we can get whatever phase profile we want by controlling voltages added to every discrete electrode[2].

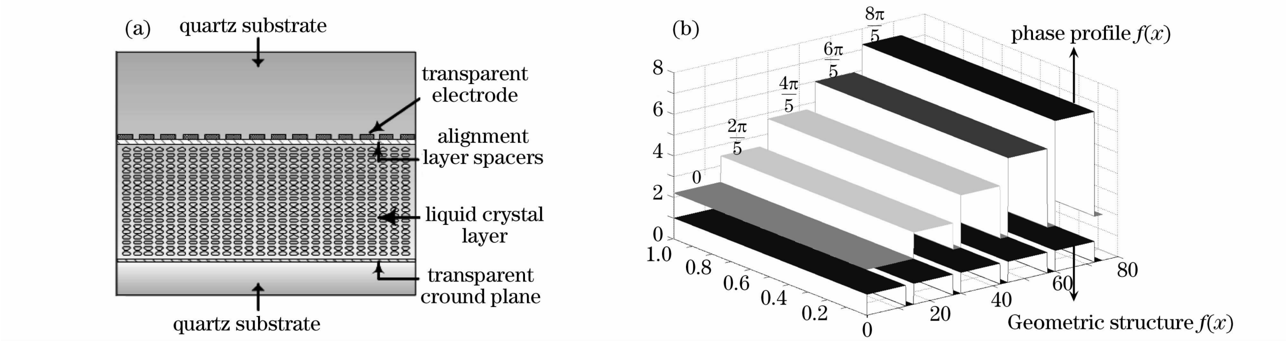

Fig. 1 LCOPA. (a) Structure of LCOPA, (b) ideal phase profile of LCOPA with $N=5$

A LCOPA is generally regarded as a blazed grating[2], while we regard it as a combination of a phase grating and an amplitude grating according to its working principle. The complex amplitude transmittance function of the LCOPA is given by

$$t(x) = \exp[\mathrm{j}\Phi(x)] \times f(x), \tag{1}$$

where $\Phi(x)$ is the contributes of the phase grating, which is also the additional phase shift generated by LC phase shifters, and $f(x)$ is the amplitude transfer function of the amplitude, which is also the contribution of the geometric structure. At the spectrum plane, the distribution function is given by the Fourier transform of the complex amplitude transmittance function[7~9], that is,

$$U(f_x) = \mathscr{F}[t(x)]. \tag{2}$$

For a normal incident unit planar wave, the far field pattern is decided by[7~9]

$$I(f_x) = \left(\frac{1}{\lambda f}\right)^2 \times |U(f_x)|^2, \tag{3}$$

where f is the focal length of the lens, and λ is the designed wavelength. Leaving the material absorption out of account, the OSE of the LCOPA can be expressed by the following equation

$$\eta = I_1 / I_{\text{incident}}, \tag{4}$$

where $I_{\text{incident}} = 1$, I_1 is the first order diffraction's intensity, which is also the intensity of the main lobe in the pattern. Thus, the overall steering efficiency $\eta = I_1$. In order to get the overall steering efficience, we only need to calculate the intensity of the first order diffraction.

In view of the complex relation between the distribution of the LC directors and the electric field applied on the discrete electrodes, we simplify the model for convenient deduction. We assume that there are no passing light in the gap between two adjacent pixels. Cai used this assumption and proved its validity for qualitative analysis[7]. Here, we introduce the differences between the ideal and real phase distributions, and then discuss the effect on the OSE. According to the above assumption, the model can be simplified as follows.

One period of a one-dimensional LCOPA grating, including N phase shifters, is showed in Fig. 1(b). The $f(x)$ is given by

$$f(x)=\sum_{m=0}^{N-1}\mathrm{rect}\left(\frac{x-md}{a}\right),\tag{5}$$

where d is the center-to-center spacing between phase shifters, and a is the width of the electrode pixel. $\Phi(x)$, the additional phase shift of the phase shifters, can be expressed as follows:

$$\Phi(x)=\sum_{m=0}^{N-1}\left[\mathrm{rect}\left(\frac{x-md}{a}\right)\times\frac{2m\pi}{N}\right].\tag{6}$$

According to Eq. (1), the transmittance function of one period of the LCOPA can be expressed as

$$t'(x)=f(x)\times\exp[\mathrm{j}\Phi(x)]=\sum_{m=0}^{N-1}\left[\mathrm{rect}\left(\frac{x-md}{a}\right)\times\exp\left(\mathrm{j}m\frac{2\pi}{N}\right)\right].\tag{7}$$

The transmittance function of the LCOPA, which contains M periods, is expressed by the following equation:

$$T(x)=t'(x)\otimes\sum_{m=0}^{M-1}\delta(x-mNd)=\sum_{m=0}^{N-1}\left[\mathrm{rect}\left(\frac{x-md}{a}\right)\times\exp\left(\mathrm{j}m\frac{2\pi}{N}\right)\right]\otimes\sum_{m=0}^{M-1}\delta(x-mNd).\tag{8}$$

According to Eq. (2), the distribution function at the spectrum plane is given by

$$U(f_x)=\mathscr{F}[t(x)]=a\,\mathrm{sinc}(af_x)\times\sum_{m=0}^{N-1}\exp\left[\mathrm{j}2\pi m\left(df_x+\frac{1}{N}\right)\right]\times\sum_{m=0}^{M-1}\exp(-\mathrm{j}2\pi mNdf_x).\tag{9}$$

The far-field pattern is decided by the following equation according to Eq. (3):

$$I(f_x)=\left(\frac{a}{\lambda f}\right)^2\mathrm{sinc}^2(af_x)\times\frac{\sin^2[N\pi(df_x+1/N)]}{\sin^2[\pi(df_x+1/N)]}\times\frac{\sin^2(M\pi Ndf_x)}{\sin^2(\pi Ndf_x)},\tag{10}$$

where $\mathrm{sinc}^2(af_x)$ gives the intensity distribution of one slit's Fraunhofer diffraction, whose width is a. The second factor shows the light distribution of a multi-beam interference while the third one reflects the interferences between periods of the whole LCOPA.

We choose the P-512 spatial light modulator (SLM) of BNS company as an example of LCOPA. According to Eq. (10), the far-field light intensity distribution of different steering angles [which are decided by $\theta=\lambda/(Nd)$[5~7]] can be got easily by means of Matlab, as shown in Fig. 2(a). Parameters of the SLM are given in Fig. 2(a), which will also be used in the following study, and we will not declare them repeatedly. F represents the fill factor which will be defined in Section 3.2. Since $\theta=\lambda/(Nd)$ is obvious, we will use the values of N instead of the steering angles in following sections to have succinct expressions. Using the far-field patterns shown in Fig. 2(a), we can calculate the corresponding OSE of every single steering angle according to Eq. (4). Figure 2(b) shows the corresponding OSEs of different steering angles.

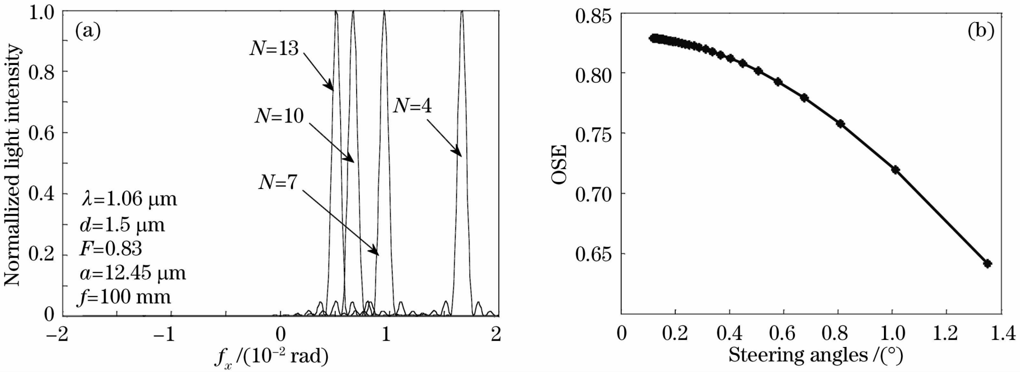

Fig. 2 Normalized far-field patterns and OSEs of different steering angles. (a) Comparison between different steering angles, (b) relationship curve between OSE and steering angles

3 Factors affecting OSE

We have established the model for calculating the far field pattern, using which we can get the OSE easily. Factors affecting the OSE include steering angles, parameters of the device and errors between ideal and practical phase profiles. We will study them respectively. The study steps can be summarized as follows. 1) Adding the variable such as steering angles, pixel sizes, fill factors and so on, which reflects the affecting factors, to Eq. (5) or (6), then we can get the corresponding far-field patterns like what Eq. (9) expresses. 2) Making use of the far-field pattern got in step 1), we can sum the whole intensity in the main lobe. 3) Dividing the value got in step 2) into the summation of the whole pattern, then we get the corresponding OSE. 4) Repeating these three steps, then we can get different OSEs in different conditions, according to which the relation curve is drawn.

3.1 Relation between steering angles and the OSE

Figure 2(b) shows the relations between the OSE and steering angles. With the increase of steering angles, the OSE decreases rapidly. Especially, when the steering angle reaches the maximum value, the OSE falls down to only about 60%.

Figure 3 shows the far-field patterns of two different steering angles, using the same parameters as those in Fig. 2(a). Comparing Fig. 3(a) with (b), we can find that with the increase of steering angle, the intensity in sidelobes and undesired diffraction orders increases too. This is the immediate cause of the decrease of the OSE. In fact, with the increase of steering angle, the grating period decreases abruptly while the phase difference between two adjacent phase shifters increases obviously[3], which leads to larger errors between the practical staircase phase profile and the ideal linear phase profile, and thus, more intensity is diffracted to sidelobes and undesired orders, causing an efficiency reduction.

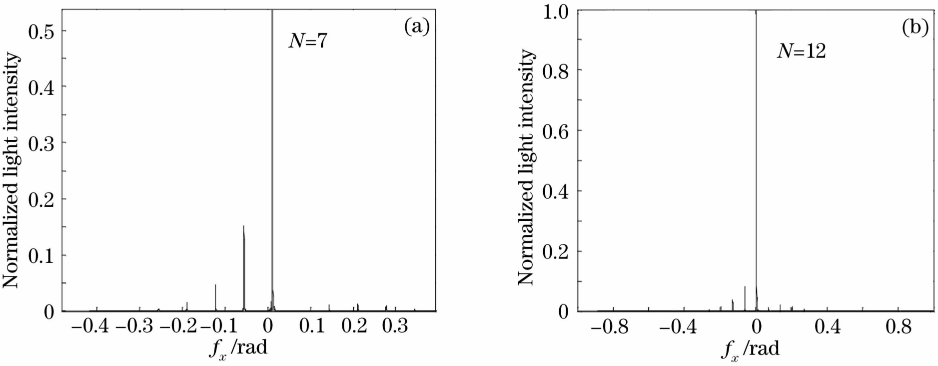

Fig. 3 Normalized far-field patterns got through simulation. (a) Far-field pattern of $N=7$, (b) far-field pattern of $N=12$

Improving the minimum efficiency of the maximum steering angle by raising technics levels is an effective way to solve this problem. A better solution is to use angles just at the left part of the curve. A middle course can be used, that is to use LCOPA associated with other devices, which have large beam deflecting angles. One typical example is the volume holographic step steering[4]. A holographic grating, which can deflect beams into several discrete large angles, is placed after one LCOPA. Thus, the LCOPA just needs to accomplish the task of addressing different discrete large angles in the holographic grating, and then we can make use of just several relatively small angles of the LCOPA, which will enhance the OSE obviously.

3.2 Relations between parameters of the device and the OSE

Parameters of the device include pixel sizes, the uniformity of sizes and fill factors. Figure 4(a) presents the relation between the OSE and pixel sizes. The variation trend indicates that small pixel sizes are beneficial for improving the OSE in the mass, but there are singular points we need to remove during the design. Another parameter affecting the OSE is the uniformity of the pixel sizes. We use a random number series, which are within the interval of $[0.8a, 1.2a]$, to replace the constant value of a. Ten groups of random distribution are used to reflect the relation between the OSE and the uniformity of the pixel sizes. Figure 4(b) shows that the OSE fluctuates around the value of proportional spacing pixels. Thus, we can find an optimal distribution by the methods of optimizing the pixel distribution, in order to gain the maximum OSE. From the influences of the pixel sizes and distribution uniformity, a primary conclusion can be drawn that an optimum design of the pixel distribution is needed, which will contribute a lot to improve the OSE. In fact, researches about this issue have been underway. The method of finite-difference time-domain (FDTD) is an effective way to solve this problem[9].

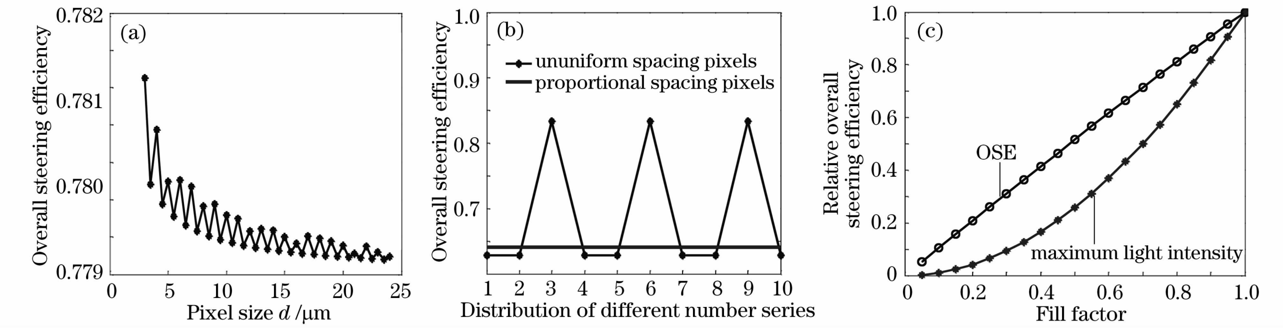

Fig. 4 Effect of device parameters on OSE. (a) Relation curve between OSE and pixel size, (b) relation curve between OSE and uniformity of pixel size, (c) relation curve between OSE and fill factors. $N=6$

Fill factor, which is defined as a/d, is another parameter affecting the OSE significantly. Figure 4(c) represents the improvement of the OSE by raising fill factors. Raising fill factor contributes to two aspects, which are beneficial to elevate the OSE. One is that raising fill factors makes gaps between adjacent steps smaller, leading to a smaller error between practical and ideal phase profiles, at the same time, weakening the effect of phase valley and flyback region[10], which are main causes for intensity loss and will be discussed later. The other is raising fill factors weakens the effect of pixel structure's diffraction, and thus the intensity loss of diffraction decreases.

3.3 Effect of errors between practical and ideal phase profiles

In practical applications, because of the characteristics of the LC material and gaps between two adjacent phase shifters, errors between practical and ideal phase profiles i.e., phase valley[10] and flyback region[2, 10] appear in the phase profile, both of which cause considerable intensity loss.

Figure 5(a) presents three kinds of phase shift. The practical phase distribution can be expressed by[10]

$$\Phi(x)=a_1\exp[a_2(x-a_3)]\sin\left[\frac{2\pi}{T}(x-a_3)\right]+a_4(x-a_3), \tag{11}$$

where a_1 reflects the phase valley depth, a_2 decides the profile of the envelop, a_3 is the initial phase shift, a_4 reflects the slope of the phase profile, and T is the period of the grating. Moreover, the expression is not fit for the flyback region, which is finished by interpolating[10]. Thus, the ideal phase shift expressed by Eq. (6) should be replaced by Eq. (11). We can change the values of a_1 to study the relation between the OSE and the phase valley depth. The result is shown in Fig. 5(b)[10], which illustrates that the OSE decreases rapidly with the increase of phase valley depth. The increase of the phase valley depth causes a larger error between the practical and idea phase profile, which leads to sidelobes with more intensity[10]. Research shows that reducing the space between two adjacent phase shifters is effective for reducing the phase valley depth and raising the OSE[9, 10]. Besides, seeking for a better alternate material and improving the fabricating technique are both possible ways to solve this problem. Compared with the phase valley, the flyback region appears between two adjacent grating periods mostly. It also affects the OSE seriously[2]. Increasing fill factors in an effective way to reduce its effect[10].

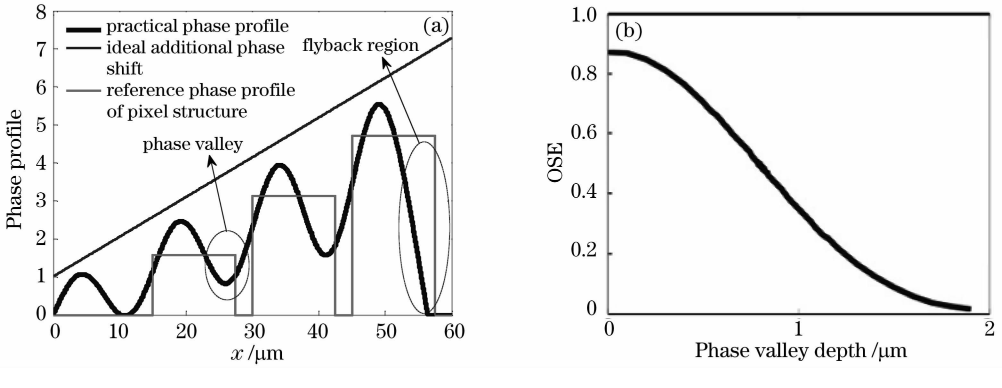

Fig. 5 Effect of errors between practical and ideal phase profiles. (a) Different phase profiles, (b) effect of phase valley depth on OSE

4 Conclusions

A model for analyzing the overall steering efficiency is established, using which we can easily gain the far-field pattern and calculate OSEs under different conditions. Three kinds of factors are found to affect the OSE: steering angles, the parameters of the LCOPA and errors between ideal and practical phase profiles. The OSE decreases greatly with the increase of steering angles, for what a middle course is proposed, which is to use the LCOPA associated with other devices in order to make use of relatively small steering angles to avoid the angles at the range of low efficiencies. Device parameters affecting the OSE represent in three aspects mostly, i.e., the fill factor, the size of electrode pixels and the uniformity of them. Results show that the increase of fill factors improves the OSE obviously by the way of reducing the diffraction loss of the pixel structure and reducing the errors between practical and ideal phase profiles. Reducing the size can improve the OSE in the mass while the proportional spacing between electrodes is not the best choice for improving the OSE. The method of FDTD should be used to find the optimal electrodes distribution. Errors between ideal and practical phase profiles, which represent in the form of phase valley and flyback region, influence the OSE seriously. Increasing fill factors can weaken the effect of phase valley and raising the phase reset weakens the effect of the flyback region obviously.

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