Entanglement between Cavities in System of Two Atoms Trapped in Two Distant Cavities Connected by Optical Fiber

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Abstract The situation that two identical two-level atoms initially entangled with the third atom are separately trapped in two distant single-mode optical cavities, which are coupled by an optical fiber, is considered. The entanglement between cavities is investigated. By means of numerical calculations, through comparing the results if a direct selective measurement is performed or not, the effect of the state-selective measurement on the entanglement and that of the coupling coefficient between atom and cavity on the entanglement are discussed. The results show that the entanglement between the cavities can be strengthened through the state-selective measurement on the atom outside cavities; if the coupling coefficient between atom and cavity increases, the entanglement between cavities can also be strengthened.

Key words quantum optics; optical fiber; selective atomic measurement; quantum entanglement

OCIS codes 270.5580; 020.5580

原子-腔-光纤复合系统中腔场间的纠缠特性

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摘要 考虑将 W 态中的两个二能级原子分别注入用光纤连接的耦合腔 A 和 B 中,并且原子与腔场发生共振相互作用的情况。采用 Negativity 熵来描述两子系统间的纠缠,研究了原子-腔-光纤复合系统中腔场间的纠缠特性。利用数值计算方法,通过对是否进行腔外原子的选择性测量情况下腔场与腔场间的纠缠的比较,讨论了对腔外原子的选择性测量和原子与腔场间的耦合系数变化对纠缠特性的影响。研究结果表明,对腔外原子的选择性测量可增强腔场与腔场间的纠缠,另一方面,随原子与腔场间的耦合系数增大腔场间纠缠增强。

关键词 量子光学；原子-腔-光纤复合系统；选择原子测量；量子纠缠

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1 Introduction

Entanglement is one of the most striking features of quantum mechanics. Entanglement state of two or more particles is not only a key ingredient for the tests of quantum nonlocality, but also a basic resource in achieving tasks of quantum communication and quantum computation. The preparation of entanglement state and the characterization of entanglement for a given quantum state are fundamental problems. So far, many schemes have been devoted to the generation of two-atom or multiple-atom entanglement in cavity quantum electrodynamics (QED)[1~4]. For example, Zheng suggested decoherence-immune generation of highly entangled states for two atoms[1]. In recent years, there has also been a considerable effort to characterize entanglement properties qualitatively and quantitatively. For example, Li *et al.*[5] studied evolution equation of entanglement for bipartite systems. Wu *et al.*[6] studied the entanglement of two moving atoms interacting with a single-mode field via a three-photon process. Chen *et al.*[7] investigated the influences of dipole-dipole interaction and detuning on the sudden death of entanglement between two atoms in the Tavis-Cummings mode. Shan *et al.*[8] discussed the entanglement character of two entangled atoms in Tavis-

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Cummings model. On the other hand, Gerry *et al.*[9] showed that the squeezing can be greatly enhanced via selective atomic measurements. The method of selective atomic measurement has been widely employed in quantum state engineering. For example, Yang *et al.*[10] considered a pair of two-level atoms initially in the Einstein-Podolsky-Rosen (EPR) single state, and put one of the two atoms into a cavity. They drew a conclusion that the emission properties of the atom inside the cavity are much affected by the manipulation of the atom outside the cavity. Wu *et al.*[11] studied nonclassical properties in the resonant interaction of a three-level Λ- type atom with two-mode field in coherent state. Lin *et al.*[12] put forward controllable emission properties of two atoms inside cavities by manipulating the atom outside the cavity. Zhou *et al.*[13] investigated the remote control of quantum statistical properties of light field. The effect of selective atomic measurement on quantum properties of light field has also been studied[14]. So far, as we know, the method of selective atomic measurement is only applied in the system of atom interacting with uncoupled cavity. Recently, the atom-cavity-fiber system has attracted some interest. For example, Yin *et al.*[15] put forward a multiatom and resonant interaction scheme for quantum state transfer and logical gates between two remote cavities via an optical fiber. Zheng *et al.*[16] proposed a generation of two-mode squeezed states for two separated atomic ensembles via coupled cavities. Zhang[17] investigated the entanglement between two atoms in two distant cavities connected by an optical fiber beyond strong fiber-cavity coupling. There are some other results in this area[18~21]. In this paper, three entangled two-level atoms initially in the W state are considered: two of them are separately trapped in two distant single-mode optical cavities connected by an optical fiber and interact resonantly with the cavitty fields. The negative eigenvalues of the partial transposition of density matrix ρ is adopted to quantify the degree of entanglement. Effects of the state-selective measurement on the atom outside cavities and the coupling coefficient between atom and cavity on the entanglement are discussed.

2 State vector evolution of the system

The atom-cavity-fiber system shown in Fig. 1 is introduced. As seen in Fig. 1, two identical two-level atoms, labeled by 1 and 2, are separately trapped in two distant single-mode optical cavities A and B, which are coupled by an optical fiber and initially in vacuum state. It is assumed that the atom-cavity coupling is on resonance. In the rotating-wave approximation, the interaction Hamiltonian of the atom-cavity system can be written as (set $\hbar = 1$)

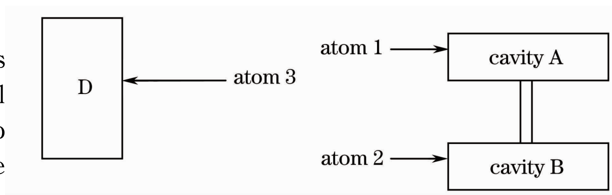

图 1 系统示意图

Fig. 1 Sketch of the system

$$H_{\mathrm{ac}} = \sum_{i=1}^{2} g_i (a_i^{+} s_i^{-} + a_i s_i^{+}), \tag{1}$$

where a_i^{+} and $a_i (i = 1 \text{ or } 2)$ are the creation and annihilation operators of the cavity mode, s_i^{+} and $s_i^{-} (i = 1$ or $2)$ are the atomic rising and lowering operators, and $g_i (i = 1$ or $2)$ is the coupling coefficient between an atom and local cavity field. For simplicity, it is assumed that $g_1 = g_2 = g$.

On the other hand, the coupling between the cavity fields and the fiber modes may be modeled by the interaction Hamiltonian as[15]

$$H_{\mathrm{cf}} = \sum_{j=1}^{\infty} f_j \{ b_j [a_1^{+} + (-1)^j e^{j\theta} a_2^{+}] + H.C \}, \tag{2}$$

where b_j is the annihilation operator for photons in mode j of the fiber, and f_j is the coupling coefficient between the fiber mode j and the cavity mode. The phase θ is induced by the propagation of the field through the fiber of length l: $\theta = 2\pi\omega l / c$, where ω is the frequency of the cavities. Let $\bar{\upsilon}$ be the decay rate of the cavities' field into a continuum of fiber modes. In the short fiber limit $2l\bar{\upsilon}/(2\pi c) \leqslant 1$, only one resonant mode b of the fiber essentially interacts with the cavity modes. The Hamiltonian H_{cf} may be approximated to[16]

$$H_{\mathrm{cf}} = f[b(a_1^{+} + a_2^{+}) + H.C], \tag{3}$$

where f is the cavity-fiber coupling coefficient. Then one can write the total Hamiltonian of the atom-cavity-fiber combined system in the interaction picture as

$$H = H_{\mathrm{ac}} + H_{\mathrm{cf}}. \tag{4}$$

Let us introduce the total excitation operator $\hat{N} = |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| + a_{\mathrm{A}}^{+} a_{\mathrm{A}} + a_{\mathrm{B}}^{+} a_{\mathrm{B}} + b^{+} b$, where $|e_i\rangle$

is the excited state of atom and $|g_i\rangle$ is the ground state of atom. It is easily shown that the excitation operator communicates with the Hamiltonian in Eq. (4). Therefore, the total excitation number is a conserved quantity. When $\hat{N}$ equals one, the evolution of the whole system is confined in the following subspace spanned by the basis state vectors $|\varphi_1\rangle=|g_1\rangle|e_2\rangle|0_A\rangle|0_B|0\rangle_f$, $|\varphi_2\rangle=|e_1\rangle|g_2\rangle|0_A\rangle|0_B|0\rangle_f$, $|\varphi_3\rangle=|g_1\rangle|g_2\rangle|0_A\rangle|1_B|0\rangle_f$, $|\varphi_4\rangle=|g_1\rangle|g_2\rangle|0_A\rangle|0_B|1\rangle_f$, $|\varphi_5\rangle=|g_1\rangle|g_2\rangle|1_A\rangle|0_B|0\rangle_f$, where $|\eta_I\rangle$ denotes n photons in the cavity mode I ($I=$ A, or B), the subscript 1 or 2 denotes atom 1 or 2, the subscript A or B denotes cavity mode A or B, and the subscript f denotes fiber mode.

The state vector evolution of the atom-cavity-fiber system under the action of the Hamiltonian in Eq. (4) can be expressed as

$$\begin{cases} |\varphi_1\rangle\rightarrow A_1|\varphi_1\rangle+B_1|\varphi_2\rangle+C_1|\varphi_3\rangle+D_1|\varphi_4\rangle+E_1|\varphi_5\rangle, \\ |\varphi_2\rangle\rightarrow A_2|\varphi_1\rangle+B_2|\varphi_2\rangle+C_2|\varphi_3\rangle+D_2|\varphi_4\rangle+E_2|\varphi_5\rangle, \end{cases} \tag{5}$$

where

$$A_1=B_2=\frac{1}{2}\left(\cos gt+\frac{g^2}{\alpha^2}\cos\alpha t+\frac{2f^2}{\alpha^2}\right),\quad B_1=A_2=\frac{1}{2}\left(-\cos gt+\frac{g^2}{\alpha^2}\cos\alpha t+\frac{2f^2}{\alpha^2}\right),$$

$$C_1=E_2=-\frac{\mathrm{i}}{2}\left(\sin gt+\frac{g}{\alpha}\sin\alpha t\right),\qquad D_1=D_2=\frac{gf}{\alpha^2}(\cos\alpha t-1),$$

$$E_1=C_2=\frac{\mathrm{i}}{2}\left(\sin gt-\frac{g}{\alpha}\sin\alpha t\right),\qquad \alpha=\sqrt{2f^2+g^2}.$$

Assuming that both the cavities and fiber fields are initially in the vacuum state, and two atoms are initially entangled with the third atom in W state, then the initial state vector of the entire system can be written as

$$|\varphi(t)\rangle=\frac{1}{\sqrt{3}}(|e_1\rangle|g_2\rangle|g_3\rangle+|g_1\rangle|e_2\rangle|g_3\rangle+|g_1\rangle|g_2\rangle|e_3\rangle)|0_A\rangle|0_B\rangle|0\rangle_f. \tag{6}$$

After an interaction time t, the entire system evolves to the state

$$|\varphi(t)\rangle=\frac{1}{\sqrt{3}}[(A|\varphi_1\rangle+B|\varphi_2\rangle+C|\varphi_3\rangle+D|\varphi_4\rangle+E|\varphi_5\rangle)|g_3\rangle+|g_1\rangle|g_2\rangle|e_3\rangle|0_A\rangle|0_B\rangle|0\rangle_f], \tag{7}$$

where $A=A_1+A_2$, $B=B_1+B_2$, $C=C_1+C_2$, $D=D_1+D_2$, $E=E_1+E_2$.

3 Entanglement evolution between cavities

Now let us investigate the entanglement between cavities. There are various equivalent measures of entanglement available for this study. Here the negative eigenvalues of the partial transposition of density matrix $\boldsymbol{\rho}$ are adopted to quantify the degree of entanglement. The state is separable if the eigenvalues of the partial transposition of density operator $\boldsymbol{\rho}$ are positive. However, the state is entangled if one of eigenvalues of $\boldsymbol{\rho}^{\mathrm{T}}$ ($\boldsymbol{\rho}^{\mathrm{T}}$ is the partial transposition of density matrix $\boldsymbol{\rho}$) is negative. For a two-qubit system described by the density operator, the negativity is defined by[22]

$$N=-2\sum_i\mu_i, \tag{8}$$

where μ_i denotes the negative eigenvalues of $\boldsymbol{\rho}^{\mathrm{T}}$. When $N=0$, the atom and cavity are separable, and $N=1$ indicates the maximal entanglement between atom and cavity.

Using Eq. (7), through tracing over the state of fiber mode, that of atom 1, that of atom 2 and that of atom 3, on the basis of $|1_A\rangle|1_B\rangle$, $|1_A\rangle|0_B\rangle$, $|0_A\rangle|1_B\rangle$ and $|0_A\rangle|0_B\rangle$, the density matrix of cavities A and B is expressed as

$$\boldsymbol{\rho}=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & |E|^2 & EC^* & 0 \\ 0 & E^*C & |C|^2 & 0 \\ 0 & 0 & 0 & |A|^2+|B|^2+|D|^2+1 \end{bmatrix}. \tag{9}$$

It can be shown that the degree of entanglement between cavities is given by

$$N=\frac{1}{3}\left[\sqrt{(|A|^2+|B|^2+|D|^2+1)^2+4|CE|^2}-(|A|^2+|B|^2+|D|^2+1)\right]. \tag{10}$$

On the other hand, if atom 3 is selectively measured in state $|g\rangle$, the entire system collapses onto the state

$$|\varphi'(t)\rangle=\frac{1}{\sqrt{2}}(A|\varphi_1\rangle+B|\varphi_2\rangle+C|\varphi_3\rangle+D|\varphi_4\rangle+E|\varphi_5\rangle), \tag{11}$$

Using Eq. (11), through tracing over the state of fiber mode, that of atom 1, that of atom 2 and that of atom 3, on the basis of $|1_A\rangle|1_B\rangle$, $|1_A\rangle|0_B\rangle$, $|0_A\rangle|1_B\rangle$ and $|0_A\rangle|0_B\rangle$, the density matrix of cavities A and B can be obtained as

$$\boldsymbol{\rho}'=\frac{1}{2}\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & |E|^2 & EC^* & 0\\ 0 & E^*C & |C|^2 & 0\\ 0 & 0 & 0 & |A|^2+|B|^2+|D|^2\end{bmatrix}. \tag{12}$$

It can be shown that the degree of entanglement between cavities is given by

$$N'=\frac{1}{2}\left[\sqrt{(|A|^2+|B|^2+|D|^2)^2+4|CE|^2}-(|A|^2+|B|^2+|D|^2)\right]. \tag{13}$$

Considering the coupling coefficient between atom and cavity $g=0.5f$, $1f$, $2f$, $4f$, $8f$, $10f$ separately, the degree of entanglement N between cavities against the scaled time ft is shown in Fig. 2. In Fig. 2, the dashed lines correspond to the case of detecting the atom 3 in the state of $|g\rangle$, and the solid lines correspond to the case of not performing selective measurement on the atom 3. From Fig. 2, we can get three results: firstly, the degree of entanglement between cavities N displays periodical oscillation, and its oscillation period decreases as the coupling coefficient between atom and cavity g increases. Secondly, as g increases, the entanglement between cavities is strengthened. Thirdly, the dashed lines are higher than the solid ones, which shows that the entanglement between cavities is strengthened through the state-selective measurement on the atom outside the cavities.

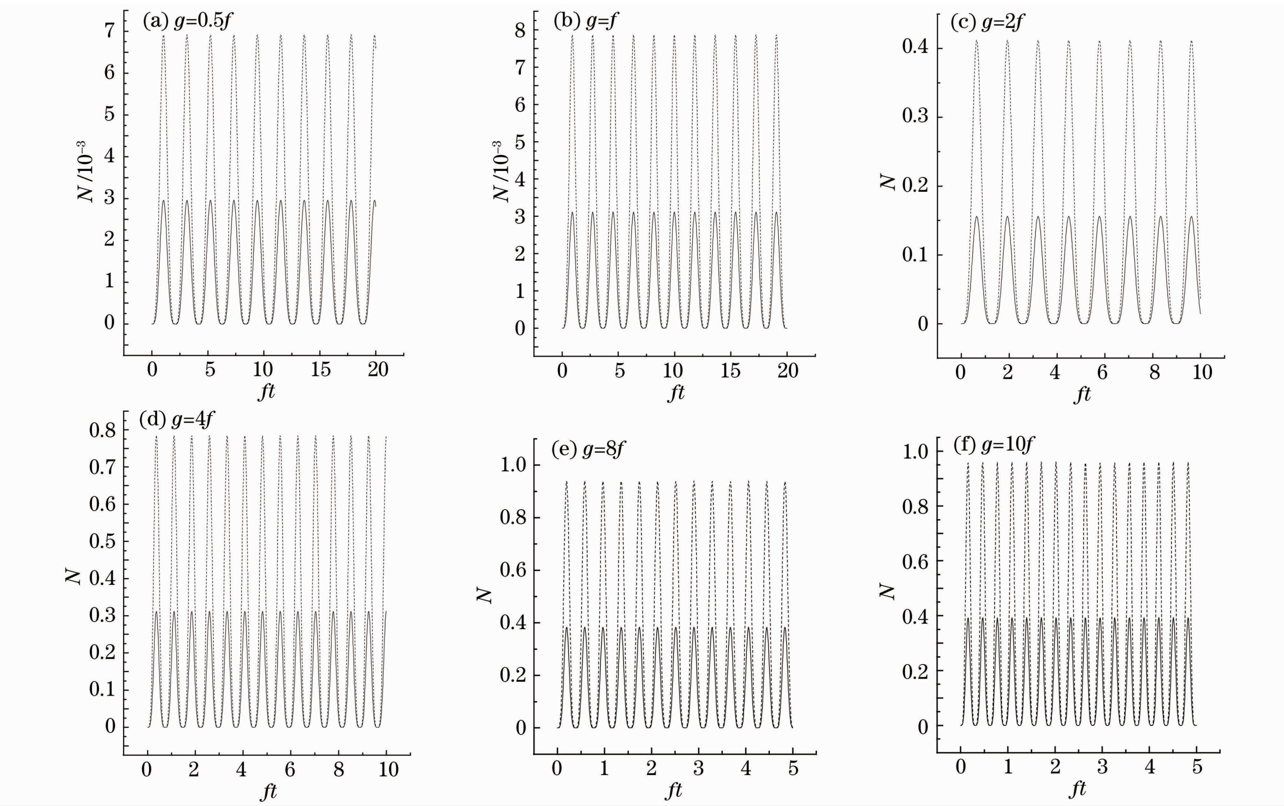

图 2 当 g 取不同值时纠缠度随规范时间 ft 的演化

Fig. 2 Time evolution of the entanglement N between cavities under different g values

4 Conclusion

The situation is considered that in the W state of three two-level atoms, two of which are separated in two initially empty cavities that are connected by an optical fiber, and the other atom is outside cavities. The atoms resonantly interact with the local cavity fields. The evolution of the entanglement between

cavities is investigated by using the negative eigenvalues of the partial transposition of density matrix. The effect of state-selective measurement on the atom outside the cavities on entanglement and that of the coupling coefficient between atom and cavity on entanglement are discussed. The results obtained using the numerical method show that the entanglement between cavities are strengthened by state-selective measurement on the atom outside the cavities, and as the coupling coefficient between atom and cavity increases the entanglement is strengthened. These results will be helpful for the understanding of the quantum entanglement and may play a guiding role in the control of quantum entanglement in quantum computation and quantum information.

References

1 S. B. Zheng. Decoherence-immune generation of highly entangled states for two atoms[J]. *Chin. Phys. B*, 2010, **19**(4): 044204

2 L. H. Lin. Robust generation of high-fidelity entangled states for multiple atoms[J]. *Chin. Phys. B*, 2009, **18**(2): 588～592

3 S. B. Zheng, G. C. Guo. Efficient scheme for two-atom entanglement and quantum information processing in cavity QED [J]. *Phys. Rev. Lett.*, 2000, **85**(11): 2392～2395

4 Chen Meifeng, Ma Songshe. Probabilistic remote preparation of two-atom entangled atate[J]. *Acta Photonica Sinica*, 2008, **37**(1): 188～191

陈美锋，马宋设. 远程制备双原子纠缠态[J]. 光子学报，2008,**37**(1)：188～191

5 Z. G. Li, S. M. Fei, Z. D. Wang *et al.*. Evolution equation of entanglement for bipartite systems[J]. *Phys. Rev. A*, 2009, **79**(2): 024303

6 C. Wu, M. F. Fang. The entanglement of two moving atoms interacting with a single-mode field via a three-photon process [J]. *Chin. Phys. B*, 2010, **19**(2): 020309

7 L. Chen, X. Q. Shao, S. Zhang. The influences of dipole-dipole interaction and detuning on the sudden death of entanglement between two atoms in the Tavis-Cummings model[J]. *Chin. Phys. B*, 2009, **18**(3): 888～893

8 Shan Chuanjia, Xia Yunjie. The entanglement character of two entangled atoms in Tavis-Cummings model[J]. *Acta Physica Sinica*, 2006, **55**(4): 1585～1590

单传家，夏云杰. Tavis-Cummings 模型中两纠缠原子纠缠的演化特性[J]. 物理学报，2006，**55**(4)：1585～1590

9 C. C. Gerry, H. H. Ghosh. Squeezing enhancement in the Jaynes-Cummings model via selective atomic measurements[J]. *Phys. Lett. A*, 1997, **229**(1): 17～22

10 C. P. Yang, G. C. Guo. Controllable emission properties of an atom inside a cavity by manipulating the atom outside the cavity[J]. *Phys. Lett. A*, 1999, **255**(3): 129～132

11 H. Z. Wu, W. J. Su. Nonclassical properties in the resonant interaction of a three level Λ-type atom with two-mode field in coherent state[J]. *Chin. Phys.*, 2007, **16**(1): 106～110

12 Lin Xiu, Li Hongcai. Controllable emission properties of two atoms inside cavities by manipulating the atom outside the cavity [J]. *Acta optica Sinica*, 2001, **21**(10): 1182～1185

林　秀，李洪才. 通过对腔外原子的操作控制腔内原子的发射性质[J]. 光学学报，2001，**21**(10)：1182～1185

13 Zhou Yuan, Zhang Yingjie, Xia Yunjie. Remote control of quantum statistical properties of light field[J]. *Acta optica Sinica*, 2007, **27**(6): 1122～1128

周　原，张英杰，夏云杰. 远程控制光场的量子统计性质[J]. 光学学报，2007，**27**(6)：1122～1128

14 Lu Daoming. Effect of selective atomic measurement on quantum properties of light field[J]. *Acta Optica Sinica*, 2010, **30**(6): 1804～1808

卢道明. 选择原子测量对光场量子特性的影响[J]. 光学学报，2010，**30**(6)：1804～1808

15 Z. Q. Yin, F. L. Li. Multiatom and resonant interaction scheme for quantum state transfer and logical gates between two remote cavities via an optical fiber[J]. *Phys. Rev. A*, 2007, **75**(1): 012324

16 S. B. Zheng, Z. B. Yang, Y. Xia. Generation of two-mode squeezed states for two separated atomic ensembles via coupled cavities[J]. *Phys. Rev. A*, 2010, **81**(1): 015804

17 B. Zhang. Entanglement between two atoms in two distant cavities connected by an optical fiber beyond strong fiber-cavity coupling[J]. *Opt. Commun.*, 2010, **283**(1): 196～199

18 M. J. Hartmann, F. G. S. L. Brandao, M. B. Plenio. Effective spin systems in coupled microcavities[J]. *Phys. Rev. Lett.*, 2007, **99**(16): 160501

19 S. B. Zheng, C. P. Yang, F. Nori. Arbitrary control of coherent dynamics for distant qubits in a quantum network[J]. *Phys. Rev. A*, 2010, **82**(4): 042327

20 S. B. Zheng. Quantum communication and entanglement between two distant atoms via vacuum field[J]. *Chin. Phys. B*, 2010, **19**(6): 064204

21 Z. B. Yang, Y. Xia, S. B. Zheng. Resonant scheme for realizing quantum phase gates for two separate atoms via coupled cavities[J]. *Opt. Commun.*, 2010, **283**(15): 3052～3057

22 S. J. Akhtarshenas, M. Farsi. Negativity as entanglement degree of the Jaynes-Cummings model[J]. *Physica Scripta*, 2007, **75**(5): 608～614