

## 三腔复合光力系统的透射谱和四波混频现象研究

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**摘要** 研究了三腔复合光力系统中探测场的透射系数和四波混频现象。结果表明:改变两个光学腔之间的耦合强度,可以使光力诱导透明现象发生显著变化。此外,在共振情况下,通过控制两个光学腔之间的耦合强度并改变机械振子的频率,可以对四波混频谱进行调制。通过双场探测手段,利用四波混频谱中的尖峰位置,实现了机械振子振动频率的精确测量。

**关键词** 量子光学; 复合光力系统; 透射谱; 四波混频; 光力诱导透明

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## 1 引言

腔光力学领域发展迅速<sup>[1-2]</sup>,该领域研究的电磁和机械系统可以实现对光力系统中光学响应的控制<sup>[3-5]</sup>。光力系统由机械谐振器与光学腔耦合组成,提供了操纵机械谐振器和电磁场的平台,为光机械设备的潜在应用铺平了道路,例如声子激光器<sup>[6]</sup>、传感<sup>[7]</sup>、声子压缩<sup>[8]</sup>、压缩光的实现<sup>[9]</sup>等。此外,通过对腔施加强驱动可以增强有效光力耦合强度。当泵浦激光驱动腔至蓝边带时,在光力系统中可观察到量子纠缠<sup>[10]</sup>、微波放大<sup>[11]</sup>等现象。当泵浦激光驱动腔至红边带时,实验上获得了很多重要进展,例如基态冷却<sup>[12]</sup>、量子态转化<sup>[13]</sup>、光力诱导透明<sup>[14-15]</sup>和四波混频<sup>[16]</sup>等。其中,光力诱导透明与电磁诱导透明类似,主要是由于干涉效应或者激发态中的暗态共振引起的。四波混频过程在不同光力系统中也得到了广泛的研究,例如强耦合光力系统中的模式分裂<sup>[17]</sup>、相干机械驱动光力系统<sup>[18]</sup>和双模腔光力系统<sup>[19-20]</sup>等。

近年来,三腔光力系统引起了人们的广泛关注。Wang<sup>[21]</sup>提出了一种在三腔光力系统不可分辨边带区域中将机械谐振器冷却至基态的方案。文献<sup>[22]</sup>研究了一种在三腔光力系统中实现量子限制方向放大器的方案。此外,研究者对复合腔光力系统中的非线性光学现象进行了深入的研究,如复合光子-分子腔光力系统中的光学双稳态和四波混频过程<sup>[23]</sup>。本课题组研究了超导量子比特耦合微波腔和机械谐振器系统的探测场吸收特性<sup>[24]</sup>。陈咏雷等<sup>[25]</sup>研究了复合旋转光力系统中的光学双稳态行为和四波混频现象。文献<sup>[26-27]</sup>分别

研究了复合腔光力系统中的双光子散射以及非旋转波近似效应的光力诱导透明和放大现象。Xing等<sup>[28]</sup>从理论上研究了复合光力系统中的四波混频响应。

基于以上的研究,本文研究了由泵浦光和探测光同时驱动三腔复合光力系统中探测场的透射系数和四波混频现象。主要讨论了辅助腔对探测场的透射系数和四波混频谱的影响。结果发现:利用透明窗口之间的距离,可以实现两个光学腔之间耦合强度的精确测量;增大两个光学腔之间的耦合强度,可以在共振处得到较大透射率。此外,利用四波混频谱中吸收峰的位置,可以准确测量机械振子的振动频率。改变双腔之间的耦合强度和机械振子的频率,四波混频谱会有显著变化。

## 2 模型和理论

本文研究的复合光力系统的物理模型如图 1 所示,一个共振频率为  $\omega_a$  的微波腔  $a$  和一个共振频率为  $\omega_{c_1}$  的光学腔  $c_1$  共同与一个振动频率为  $\omega_m$  的纳米机械振子  $b$  耦合,同时另一个共振频率为  $\omega_{c_2}$  的光学腔  $c_2$  与光学腔  $c_1$  通过交换能量耦合<sup>[29]</sup>。一束振幅为  $E_e$   $[|E_e| = \sqrt{2P_e/(\hbar\Omega_e)}]$ 、频率为  $\Omega_e$ 、功率为  $P_e$  的强泵浦场对微波腔  $a$  进行驱动,其中  $\hbar$  为普朗克常量。一束振幅为  $E_o$   $[|E_o| = \sqrt{2P_o/(\hbar\Omega_o)}]$ 、频率为  $\Omega_o$ 、功率为  $P_o$  的强泵浦场和一束振幅为  $E_p$   $[|E_p| = \sqrt{2P_p/(\hbar\Omega_p)}]$ 、频率为  $\Omega_p$ 、功率为  $P_p$  的弱探测场同时对光学腔  $c_1$  进行驱动。整个系统的哈密顿量( $\hbar = 1$ )<sup>[14,30-32]</sup>为

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$$H = \omega_a a^\dagger a + \omega_m b^\dagger b + \omega_{c_1} c_1^\dagger c_1 + \omega_{c_2} c_2^\dagger c_2 - g_e a^\dagger a (b^\dagger + b) - g_o c_1^\dagger c_1 (b^\dagger + b) + J (c_1^\dagger c_2 + c_1 c_2^\dagger) + i \sqrt{\kappa_{e, \text{ext}}/2} E_e [a^\dagger \exp(-i\Omega_e t) - a \exp(i\Omega_e t)] + i \sqrt{\kappa_{o, \text{ext}}/2} E_o [c_1^\dagger \exp(-i\Omega_o t) - c_1 \exp(i\Omega_o t)] + i \sqrt{\kappa_{o, \text{ext}}/2} E_p [c_1^\dagger \exp(-i\Omega_p t) - c_1 \exp(i\Omega_p t)] \quad (1)$$

式中:  $t$  为时间;  $g_e$  和  $g_o$  分别为纳米机械振子  $b$  与微波腔  $a$  及纳米机械振子  $b$  与光学腔  $c_1$  的耦合强度;  $J$  为腔  $c_1$  与腔  $c_2$  之间的耦合强度; 算符  $a(a^\dagger)$ 、 $b(b^\dagger)$ 、 $c_1(c_1^\dagger)$ 、 $c_2(c_2^\dagger)$  分别为微波腔、纳米机械振子、光学腔  $c_1$  和光学

腔  $c_2$  的湮灭算符(产生算符);  $\omega_a a^\dagger a$ 、 $\omega_m b^\dagger b$ 、 $\omega_{c_1} c_1^\dagger c_1$  和  $\omega_{c_2} c_2^\dagger c_2$  分别为微波腔、纳米机械振子、光学腔  $c_1$  和光学腔  $c_2$  的自由哈密顿量;  $\kappa_{e, \text{ext}}(\kappa_{o, \text{ext}})$  为外部耦合引起的衰减率。

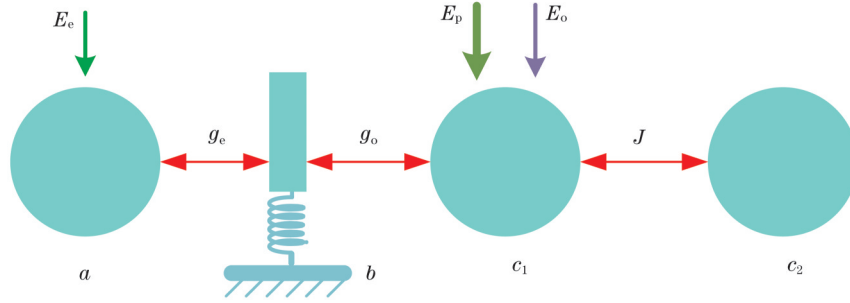


图 1 物理模型示意图

Fig. 1 Schematic of physical model

在频率为  $\Omega_e$  和  $\Omega_o$  的强泵浦场的旋转框架下, 复合光力系统的哈密顿量为

$$H = \Delta_a a^\dagger a + \omega_m b^\dagger b + \Delta_{c_1} c_1^\dagger c_1 + \Delta_{c_2} c_2^\dagger c_2 - g_e a^\dagger a (b^\dagger + b) - g_o c_1^\dagger c_1 (b^\dagger + b) + J (c_1^\dagger c_2 + c_1 c_2^\dagger) + i \sqrt{\kappa_{e, \text{ext}}/2} E_e (a^\dagger - a) + i \sqrt{\kappa_{o, \text{ext}}/2} E_o (c_1^\dagger - c_1) + i \sqrt{\kappa_{o, \text{ext}}/2} E_p [c_1^\dagger \exp(-i\delta t) - c_1 \exp(i\delta t)] \quad (2)$$

式中:  $\Delta_a = \omega_a - \Omega_e$ ,  $\Delta_{c_1} = \omega_{c_1} - \Omega_o$ ,  $\Delta_{c_2} = \omega_{c_2} - \Omega_o$ ;  $\delta$  为探测场和泵浦场之间的失谐量,  $\delta = \Omega_p - \Omega_o$ 。

通过海森堡运动方程  $\frac{dO}{dt} = \frac{1}{i\hbar} [O, H]$  ( $O$  为算符)

和对易关系  $[a, a^\dagger] = 1$ , 得到量子朗之万方程为

$$\frac{da}{dt} = -\left(i\Delta_a + \frac{\kappa_e}{2}\right)a + ig_e a Q + \sqrt{\kappa_{e, \text{ext}}/2} E_e, \quad (3)$$

$$\frac{dc_1}{dt} = -\left(i\Delta_{c_1} + \frac{\kappa_o}{2}\right)c_1 + ig_o c_1 Q - iJc_2 + \sqrt{\kappa_{o, \text{ext}}/2} [E_o + E_p \exp(i\delta t)], \quad (4)$$

$$\frac{dc_2}{dt} = -\left(i\Delta_{c_2} + \frac{\kappa_1}{2}\right)c_2 - iJc_1, \quad (5)$$

$$\frac{d^2 Q}{dt^2} + \gamma_m \frac{dQ}{dt} + \omega_m^2 Q = 2\omega_m g_e a^\dagger a + 2\omega_m g_o c_1^\dagger c_1, \quad (6)$$

式中:  $Q = b^\dagger + b$ ;  $\kappa_e(\kappa_o)$  为微波腔  $a$  (光学腔  $c_1$ ) 的衰减率;  $\gamma_m$  为机械振子  $b$  的衰减率;  $\kappa_1$  为光学腔  $c_2$  的衰减率。

为了求解式(3)~(6), 进行近似代换, 即

$$a(t) = a_0 + a_+ \exp(-i\delta t) + a_- \exp(i\delta t), \quad (7)$$

$$Q(t) = Q_0 + Q_+ \exp(-i\delta t) + Q_- \exp(i\delta t), \quad (8)$$

$$c_1(t) = c_{1_0} + c_{1_+} \exp(-i\delta t) + c_{1_-} \exp(i\delta t), \quad (9)$$

$$c_2(t) = c_{2_0} + c_{2_+} \exp(-i\delta t) + c_{2_-} \exp(i\delta t), \quad (10)$$

式中:  $a_0$  为腔  $a$  在稳态时的振幅;  $a_+$ 、 $a_-$  为系数;  $Q_0$  为稳态时的机械位移;  $Q_+$ 、 $Q_-$  为系数;  $c_{1_0}$  为腔  $c_1$  在稳态时的振幅;  $c_{1_+}$ 、 $c_{1_-}$  为系数;  $c_{2_0}$  为腔  $c_2$  在稳态时的振幅;  $c_{2_+}$ 、 $c_{2_-}$  为系数。

忽略高阶非线性项, 得到稳态值为

$$a_0 = \frac{\sqrt{\kappa_{e, \text{ext}}/2} E_e}{\left(i\Delta_a + \frac{\kappa_e}{2}\right) - ig_e Q_0}, \quad (11)$$

$$Q_0 = \frac{2g_e n_a + 2g_o n_{c_1}}{\omega_m}, \quad (12)$$

$$c_{1_0} = \frac{\sqrt{\kappa_{o, \text{ext}}/2} E_o}{\left(i\Delta_{c_1} + \frac{\kappa_o}{2}\right) - ig_o Q_0 + iJ\alpha}, \quad (13)$$

$$c_{2_0} = \frac{-iJc_1}{i\Delta_{c_2} + \frac{\kappa_1}{2}}, \quad (14)$$

式中:  $n_a$  为腔  $a$  内的光子数;  $n_{c_1}$  为腔  $c_1$  内的光子数;  $\alpha = \frac{-iJ}{i\Delta_{c_2} + \frac{\kappa_1}{2}}$ 。式(11)~(14)决定了腔内光子数( $n_a = |a_0|^2$ ,  $n_{c_1} = |c_{10}|^2$ ), 即

$$n_a = \frac{\kappa_{e, \text{ext}}/2 E_e^2}{\kappa_e^2/4 + \left[ \Delta_a - \frac{2g_e}{\omega_m} (g_e n_a + g_o n_{c_1}) \right]^2}, \quad (15)$$

$$n_{c_1} = \frac{E_o^2 \kappa_o/2}{\frac{\kappa_o^2}{4} + \left[ -\Delta_{c_1} + \frac{J^2 \Delta_{c_2}}{\Delta_{c_2}^2 + \frac{\kappa_1^2}{4}} + \frac{g_o}{\omega_m} (2g_e n_a + 2g_o n_{c_1}) \right]^2 - \left( \frac{J^2 \Delta_{c_2}}{\Delta_{c_2}^2 + \frac{\kappa_1^2}{4}} \right)^2 + \frac{J^4 + J^2 \kappa_1 \kappa_o/2}{\Delta_{c_2}^2 + \frac{\kappa_1^2}{4}}}. \quad (16)$$

此外, 通过求解式(3)~(6), 还可以得到

$$c_{1-} = \frac{\sqrt{\frac{\kappa_{o, \text{ext}}}{2}} i\Phi_3^* \frac{\kappa_{o, \text{ext}}}{2} E_p E_o^2 g_o^2}{\Phi_6^* \Phi_8^* \Phi_7^* \Phi_8^* \left( 1 - \frac{i\Phi_3^* g_e^2 n_a}{\Phi_5^*} + \Phi \frac{i\Phi_3^* g_e^2 n_a}{\Phi_4^*} - \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_7^*} + \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_6^*} \right)}, \quad (17)$$

$$c_{1+} = \frac{\sqrt{\frac{\kappa_{o, \text{ext}}}{2}} E_p}{-i\delta + \left( i\Delta_{c_1} + \frac{\kappa_o}{2} \right) - ig_o Q_0 + iJ\Phi_1} + \frac{ig_o}{-i\delta + \left( i\Delta_{c_1} + \frac{\kappa_o}{2} \right) - ig_o Q_0 + iJ\Phi_1} \times \frac{\sqrt{\frac{\kappa_{o, \text{ext}}}{2}} \Phi_3 g_o \frac{\kappa_{o, \text{ext}}}{2} E_p E_o^2}{\Phi_6^* \Phi_8^* \Phi_8^* \left( 1 - \frac{i\Phi_3^* g_e^2 n_a}{\Phi_4} + \frac{i\Phi_3^* g_e^2 n_a}{\Phi_5} - \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_6} + \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_7} \right)}, \quad (18)$$

式中:  $\Phi_1 = \frac{-iJ}{-i\delta + \left( i\Delta_{c_2} + \frac{\kappa_1}{2} \right)}$ ;  $\Phi_2 = \frac{-iJ}{i\delta + \left( i\Delta_{c_2} + \frac{\kappa_1}{2} \right)}$ ;  $\Phi_3 = \frac{2\omega_m}{-\delta^2 - i\delta\gamma_m + \omega_m^2}$ ;  $\Phi_4 = -i\delta - ig_e Q_0 + \left( i\Delta_a + \frac{\kappa_e}{2} \right)$ ;  
 $\Phi_5 = -i\delta + ig_e Q_0 + \left( -i\Delta_a + \frac{\kappa_e}{2} \right)$ ;  $\Phi_6 = -i\delta + \left( i\Delta_{c_1} + \frac{\kappa_o}{2} \right) - ig_o Q_0 + iJ\Phi_1$ ;  $\Phi_7 = -i\delta + \left( -i\Delta_{c_1} + \frac{\kappa_o}{2} \right) + ig_o Q_0 - iJ\Phi_2$ ;  $\Phi_8 = \left( -i\Delta_{c_1} + \frac{\kappa_o}{2} \right) + ig_o Q_0 - iJ\alpha^*$ ; \*表示取共轭。

引入标准的输入-输出理论<sup>[2]</sup>得到

$$c_{1\text{out}}(t) = \left( E_o - \sqrt{\kappa_{o, \text{ext}}/2} c_{1-} \right) \exp(-i\Omega_o t) + \left( E_p - \sqrt{\kappa_{o, \text{ext}}/2} c_{1+} \right) \exp[-i(\delta + \Omega_o)t] - \sqrt{\kappa_{o, \text{ext}}/2} c_{1-} \times \exp[-i(\delta - \Omega_o)t] = \left( E_o - \sqrt{\kappa_{o, \text{ext}}/2} c_{1-} \right) \exp(-i\Omega_o t) + \left( E_p - \sqrt{\kappa_{o, \text{ext}}/2} c_{1+} \right) \exp(-i\Omega_p t) - \sqrt{\kappa_{o, \text{ext}}/2} c_{1-} \exp[-i(2\Omega_o - \Omega_p)t], \quad (19)$$

式中:  $c_{1\text{out}}(t)$  为输出场算符; 第一项是振幅为  $E_o$ 、频率为  $\Omega_o$  的驱动场; 第二项是振幅为  $E_p$ 、频率为  $\Omega_p$  的探测场; 最后一项是频率为  $2\Omega_o - \Omega_p$  的四波混频场。四波混频场的强度<sup>[16]</sup>定义为

$$F_{\text{FWM}} = \left| \frac{\sqrt{\kappa_{o, \text{ext}}/2} c_{1-}}{E_p} \right|^2 = \left| \frac{i\Phi_3^* \frac{\kappa_{o, \text{ext}}^2}{4} E_o^2 g_o^2}{\Phi_6^* \Phi_8^* \Phi_7^* \Phi_8^* \left( 1 - \frac{i\Phi_3^* g_e^2 n_a}{\Phi_5^*} + \frac{i\Phi_3^* g_e^2 n_a}{\Phi_4^*} - \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_7^*} + \frac{i\Phi_3^* g_o^2 n_{c_1}}{\Phi_6^*} \right)} \right|^2. \quad (20)$$

探测场的透射系数  $t(\Omega_p)$  为输出场与输入场的振幅之比, 即

$$t(\Omega_p) = \frac{E_p - \sqrt{\kappa_{o,ext}/2} c_{1+}}{E_p} = 1 - \frac{\kappa_{o,ext}/2}{-i\delta + \left(i\Delta_{c_1} + \frac{\kappa_o}{2}\right) - ig_o Q_0 + iJ\Phi_1} - \frac{ig_o}{-i\delta + \left(i\Delta_{c_1} + \frac{\kappa_o}{2}\right) - ig_o Q_0 + iJ\Phi_1} \times \frac{\Phi_3 g_o \frac{\kappa_{o,ext}}{4} E_o^2}{\Phi_6 \Phi_8^* \Phi_8 \left(1 - \frac{i\Phi_3 g_e^2 n_a}{\Phi_4} + \frac{i\Phi_3 g_e^2 n_a}{\Phi_5} - \frac{i\Phi_3 g_o^2 n_{c_1}}{\Phi_6} + \frac{i\Phi_3 g_o^2 n_{c_1}}{\Phi_7}\right)} \quad (21)$$

利用以上结果讨论双腔之间的耦合系数和机械振子的频率对三腔复合光力系统中探测场的透射系数和四波混频现象的影响。

### 3 数值结果与讨论

为了更好地研究系统的光学响应,选择实验上可实现的参数<sup>[33-35]</sup>: $\kappa_o = 2\pi \times 1.65$  MHz,  $\kappa_e = 2\pi \times 1.6$  MHz,  $\kappa_{o,ext} = 0.11\kappa_o$ ,  $\kappa_{o,ext} = 0.76\kappa_o$ ,  $J = \kappa_o$ ,  $g_o = 2\pi \times 27$  Hz,  $g_e = 2\pi \times 2.7$  Hz,  $\omega_a = 2\pi \times 7.1$  GHz,  $\omega_m = 2\pi \times 5.6$  MHz,  $\omega_{c_1} = \omega_{c_2} = 2\pi \times 282$  THz,  $\gamma_m = 2\pi \times 4$  Hz。

在两束泵浦激光的作用下,复合系统的光学响应可以通过探测场的透射谱来表征。当 $\Delta_{c_1} = \Delta_{c_2} = -\omega_m$ ,  $\Delta_a = \omega_m$ 时,光学腔 $c_1$ 和 $c_2$ 被驱动至蓝边带,此时微波腔 $a$ 被驱动至红边带。当光学腔 $c_1$ 和 $c_2$ 的耦合强度

$J = 0, 0.5\kappa_o, \kappa_o, 2\kappa_o$ 时探测场的透射谱随探测场-腔场的失谐量 $\Delta_p = \Omega_p - \omega_{c_1}$ 的变化曲线如图 2 所示。当 $J = 0$ 时,如图 2 中实线所示,系统中没有光学腔 $c_2$ 的作用,探测场透射谱呈现洛伦兹线型。但是当两个光学腔之间的耦合强度打开时,即 $J = 0.5\kappa_o, \kappa_o, 2\kappa_o$ 时,位于 $\Delta_p = 0$ 处的洛伦兹峰分裂为两个对称的峰和一个透明窗口<sup>[36-37]</sup>。从图 2 还可以明显看出,随着耦合强度 $J$ 的增大,峰间距逐渐增大。共振处( $\Delta_p = 0$ )探测场的透射率随着 $J$ 的增大而增大。图 3 为峰间距和共振处透射率随耦合强度 $J$ 的变化情况。可以发现,峰间距与耦合强度 $J$ 呈线性关系。此外,当耦合强度 $J$ 增大到某个值时,共振处透射率的增大趋势逐渐缓慢。

接下来讨论该复合系统中的四波混频现象。图 4 绘制了耦合强度 $J = 0, 0.5\kappa_o, \kappa_o, 2\kappa_o$ 时四波混频强度随

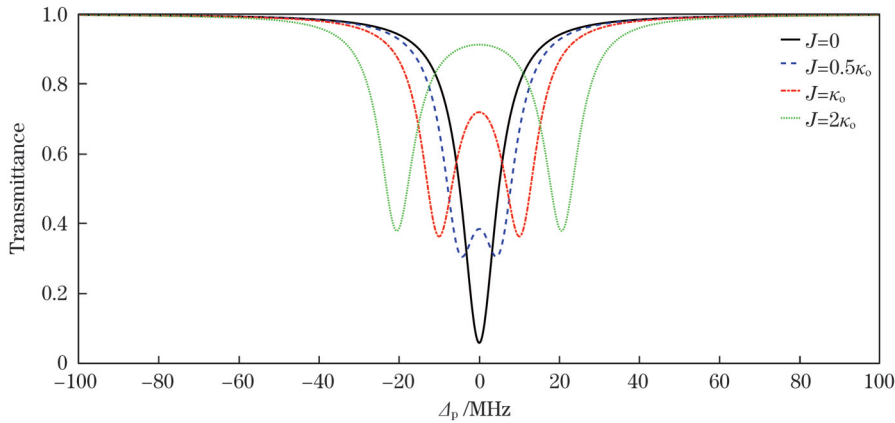


图 2  $J = 0, 0.5\kappa_o, \kappa_o, 2\kappa_o$ 时探测场的透射谱随失谐量 $\Delta_p = \Omega_p - \omega_{c_1}$ 的变化

Fig. 2 Transmission spectrum of probe field versus  $\Delta_p = \Omega_p - \omega_{c_1}$  when  $J = 0, 0.5\kappa_o, \kappa_o, 2\kappa_o$ .

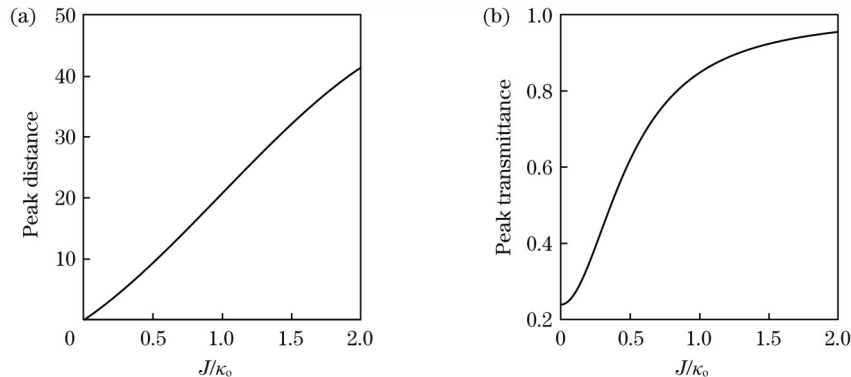


图 3 耦合强度 $J$ 对探测场透射谱的影响。(a)峰间距;(b) $\Delta_p = 0$ 处的峰值透射率

Fig. 3 Influence of coupling strength  $J$  on transmission spectrum of probe field. (a) Peak distance; (b) peak transmittance at  $\Delta_p = 0$



探测场-腔场失谐量  $\Delta_p$  的变化情况。机械振子  $b$  的频率保持在  $\omega_m = 2\pi \times 5.6$  MHz 不变, 在共振的情况下, 即  $\Delta_{c_1} = \Delta_{c_2} = \Delta_a = 0$  时, 从图 4(a) 可以看出, 当  $J = 0$  时, 四波混频谱有三个峰, 一个是  $\Delta_p = 0$  处的洛伦兹峰, 两个是  $\Delta_p = 0$  两侧的分裂峰。两个分裂峰的出现可以用单腔光力系统中的缀饰态理论来解释<sup>[38]</sup>。当系统中加

入辅助腔  $c_2$  后, 从图 4(b)~(d) 可以发现, 四波混频谱发生了显著变化, 出现了双色光力诱导透明现象。图 4(a) 中的洛伦兹峰分裂为两个峰, 且随着耦合强度  $J$  的增大, 这两个峰的峰值逐渐减小, 峰间距逐渐增大。这种现象可以用超模理论<sup>[39]</sup>来解释。原本两侧尖峰的位置不变但峰值逐渐减小。

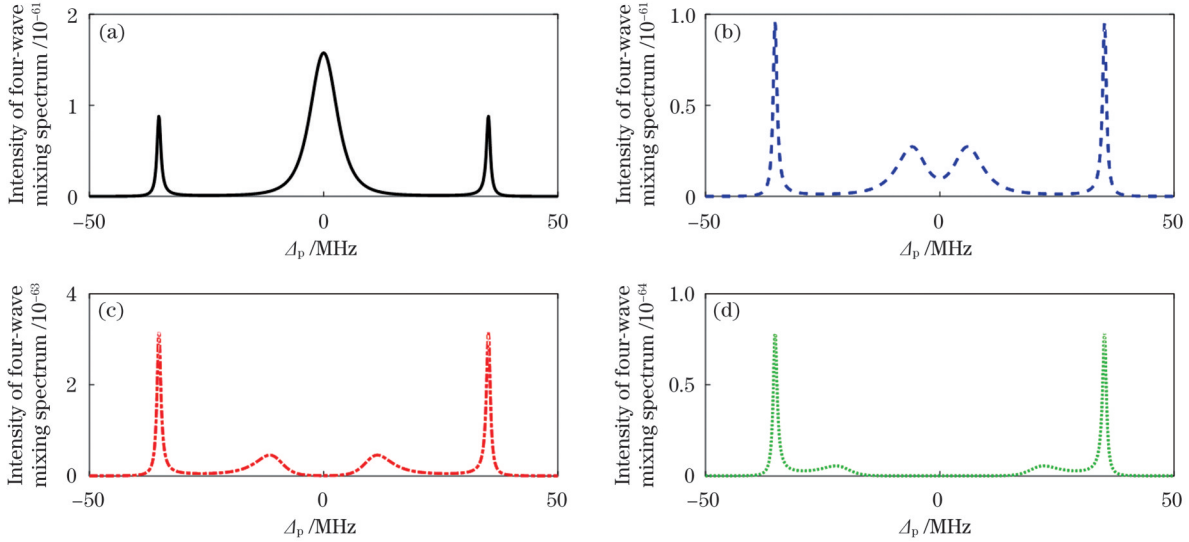


图 4 不同  $J$  下四波混频谱强度随探测场-腔场失谐量  $\Delta_p$  的变化。(a)  $J = 0$ ; (b)  $J = 0.5\kappa_o$ ; (c)  $J = \kappa_o$ ; (d)  $J = 2\kappa_o$ 。

Fig. 4 Intensity of four-wave mixing spectrum versus  $\Delta_p$  under different  $J$ . (a)  $J = 0$ ; (b)  $J = 0.5\kappa_o$ ; (c)  $J = \kappa_o$ ; (d)  $J = 2\kappa_o$ 。

基于图 4 所示的研究情况, 接下来讨论当复合系统中没有辅助腔  $c_2$ , 改变机械振子  $b$  的频率, 如  $\omega_m = 2\pi \times 4.6$  MHz,  $\omega_m = 2\pi \times 5.6$  MHz,  $\omega_m = 2\pi \times 6.6$  MHz 时, 四波混频谱随探测场-腔场失谐量  $\Delta_p$  的变化情况, 结果如图 5 所示。可以看出, 当机械振子  $b$  的频率  $\omega_m$  逐渐增大时, 四波混频谱峰值逐渐减小。同时, 可以观察到四波混频谱位于中间洛伦兹峰两侧的尖峰正处于  $\Delta_p = \pm \omega_m$  位置处, 表明尖峰的位置正好对应机械振子  $b$  的频率。这种现象的物理本质是机械振动模式与两束光场相互作用而产生了量子相干效应<sup>[19,40]</sup>。图 5 表明, 通过双场探测手段可以实现机械振子振动频率的

精确测量。首先使泵浦场频率与光学腔场频率相等 ( $\Delta_p = 0$ ), 然后利用探测场扫描复合系统, 此时四波混频谱中尖峰的位置恰好对应机械振子的频率。

如图 6 所示, 进一步绘制复合系统中加入辅助腔  $c_2$  ( $J = \kappa_o$ ), 并且机械振子  $b$  的频率分别为  $\omega_m = 2\pi \times 4.6$  MHz,  $\omega_m = 2\pi \times 5.6$  MHz,  $\omega_m = 2\pi \times 6.6$  MHz 时, 四波混频谱随探测场-腔场失谐量  $\Delta_p$  的变化。随着机械振子频率的增大, 中间两个对称的分裂峰强度和峰间距均减小, 且两侧尖峰的位置正好对应机械振子的三种不同频率。图 6 结果表明, 改变机械振子的频率会使四波混频谱强度发生变化。

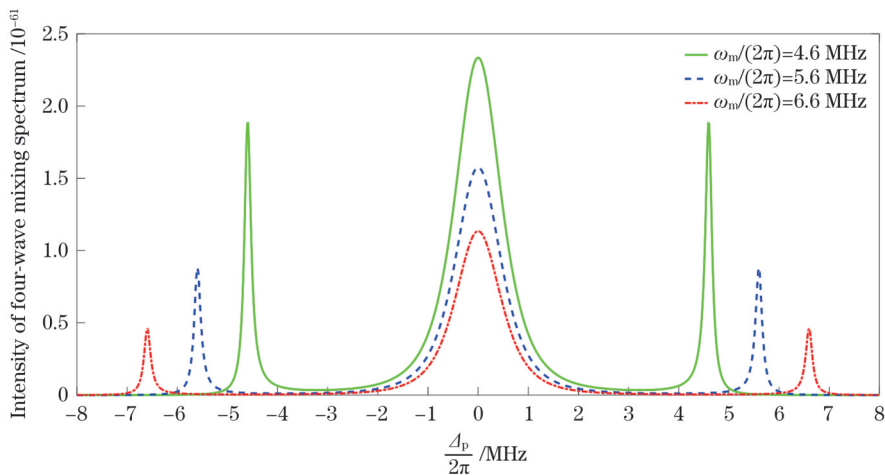


图 5  $J = 0$  时不同机械振子频率下四波混频谱强度随  $\Delta_p / 2\pi$  的变化

Fig. 5 Intensity of four-wave mixing spectrum versus  $\Delta_p / 2\pi$  under different mechanical resonator frequencies when  $J = 0$

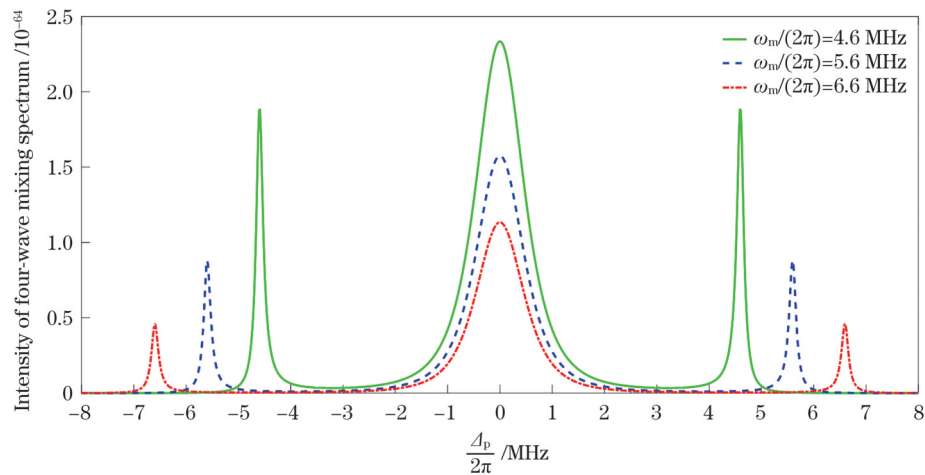


图 6  $J=\kappa_0$  时不同机械振子频率下四波混频谱强度随  $\Delta_p/2\pi$  的变化

Fig. 6 Intensity of four-wave mixing spectrum versus  $\Delta_p/2\pi$  under different mechanical resonator frequencies when  $J=\kappa_0$ .

## 4 结 论

研究了由两个光学腔和一个微波腔以及一个机械振子组成的复合光力系统中的光力诱导透明现象和四波混频现象。当微波腔被驱动到红边带而两个光学腔被驱动到蓝边带时,改变两个光学腔之间的耦合强度,光力诱导透明现象发生变化。此外,讨论了在共振的情况下,通过控制两个光学腔之间的耦合强度和改变机械振子的频率,可以对四波混频谱进行调制。同时,还提出了一种测量机械振子频率的非线性光学方法,即四波混频谱中两侧尖峰的位置正好对应机械振子的频率。

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## Transmission Spectrum and Four-Wave Mixing in Three-Cavity Hybrid Optomechanical System

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### Abstract

**Objective** The rapid development of cavity optomechanical systems has attracted extensive attention, and these systems are widely used in quantum information and precision measurement. In recent years, three-cavity optomechanical systems have attracted considerable attention. Compared with the single-mode optomechanical system, the multiple-mode system provides significantly higher flexible controllability. Furthermore, optomechanically induced transparency and four-wave mixing have been research hotspots in different optomechanical systems. For example, the four-wave mixing in a hybrid optomechanical system is theoretically investigated, which has important implications for the nonlinear optical properties. It is of great significance for theoretically exploring the transmission spectrum and four-wave mixing in a three-cavity optomechanical system.

**Methods** The hybrid optomechanical system consists of a microwave cavity  $a$  with resonance frequency  $\omega_a$  and an optical cavity  $c_1$  with resonance frequency  $\omega_{c_1}$ , which are coupled to a common mechanical resonator  $b$ , while an optical cavity  $c_2$  with resonance frequency  $\omega_{c_2}$  is coupled to the optical cavity  $c_1$ . A strong pump laser beam  $E_c$  with frequency  $\Omega_c$  is applied to the microwave cavity  $a$ . A weak probe laser beam  $E_p$  with frequency  $\Omega_p$  and a strong pump laser beam  $E_o$  with frequency  $\Omega_o$  are applied to the optical cavity  $c_1$  simultaneously. In the rotating frame of the pump fields with frequency  $\Omega_c$  and  $\Omega_o$ , the whole Hamiltonian of the system is obtained. According to the Heisenberg equation and making the ansatz, we finally obtain the transmission spectrum and the four-wave mixing spectrum intensity. Then, we investigate how the evolutions of the transmission spectrum and the four-wave mixing spectrum are affected by the coupling strength and the frequency of the mechanical resonator.

**Results and Discussions** When the optical cavity  $c_2$  is absent in the hybrid optomechanical system ( $J=0$ ), the transmission spectrum of the probe field shows a Lorentzian line shape. However, when  $J \neq 0$ , the Lorentzian peak splits into two symmetrical peaks and a transparent window occurs (Fig. 2). It is clear that with the increase in the coupling strength from  $J=0.5\kappa_o$  to  $J=2\kappa_o$ , the

distance between the peaks increases, and the peak value of the transmission spectrum of the probe field also increases (Fig. 3). We depict the variation of the four-wave mixing spectrum with the detuning of the probe field-cavity field  $\Delta_p$  for  $J=0, 0.5\kappa_o, \kappa_o, 2\kappa_o$  when  $\omega_m=2\pi\times 5.6$  MHz. In the case of  $\Delta_{c_1}=\Delta_{c_2}=\Delta_a=0$ , when  $J=0$ , the four-wave mixing spectrum has three peaks. A Lorentzian peak locates at  $\Delta_p=0$  and two splitting peaks locate on both sides of the Lorentzian peak [Fig. 4(a)]. With the choice of  $J\neq 0$ , a significant change in the four-wave mixing spectrum can be observed. The Lorentzian peak splits into two peaks. As the coupling strength  $J$  increases, the peak value decreases greatly and the distance between the peaks gradually increases [Figs. 4(b)–(d)]. Next, we study how the evolution of the four-wave mixing spectrum is affected by the frequency of the mechanical resonator while the optical cavity  $c_2$  is not in the hybrid optomechanical system ( $J=0$ ). It can be seen that as the frequency of the mechanical resonator increases, the peak value of the four-wave mixing spectrum gradually decreases. Further, the peaks on both sides of the four-wave mixing spectrum located at  $\pm\Delta_p$  just correspond to the frequency  $\omega_m$  of the mechanical resonator (Fig. 5). Moreover, we investigate the four wave mixing spectrum as a function of the detuning  $\Delta_p$  for the frequencies of mechanical resonator of  $\omega_m=2\pi\times 4.6$  MHz,  $\omega_m=2\pi\times 5.6$  MHz, and  $\omega_m=2\pi\times 6.6$  MHz when  $J=\kappa_o$ . As the frequency of the mechanical resonator increases, the peak value and the distance between the two symmetrical splitting peaks in the middle decrease. The positions of the peaks on both sides exactly correspond to the three different frequencies of the mechanical resonator. The results show that the four-wave mixing can be tuned by the frequency of the mechanical resonator (Fig. 6).

**Conclusions** We investigate the optomechanically induced transparency and four-wave mixing in a hybrid optomechanical system composed of two optical cavities, a microwave cavity, and a mechanical resonator. When the microwave cavity is driven to the red sideband and the two optical cavities are driven to the blue sideband, the optomechanically induced transparency can be changed by changing the coupling strength between the two optical cavities. Furthermore, the four-wave mixing spectrum can be modulated by controlling the coupling strength between the two optical cavities and by changing the frequency of the mechanical resonator during resonant detuning  $\Delta_p=0$ . At the same time, a nonlinear optical method for measuring the frequency of a mechanical resonator is provided. The positions of the peaks on both sides of the four-wave mixing spectrum correspond exactly to the frequency of the mechanical resonator. These results have important significance and application prospects in quantum sensing and quantum information processing.

**Key words** quantum optics; hybrid optomechanical system; transmission spectrum; four-wave mixing; optomechanically induced transparency