

信赖域半径策略优化 L-M 算法在激光追踪仪多站位测量中的应用

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摘要 为了解决 Levenberg-Marquardt(L-M)算法在激光追踪仪多站位测量应用中收敛速度慢的问题,本文提出了信赖域半径策略优化 L-M 算法。通过调整搜索方向的正参数进行信赖域判定,确定每一次迭代最优的方向和步长,使得算法快速收敛到全局最优解。实验结果表明:所提算法有效提高了基于激光追踪仪多站位测量技术的三坐标测量机(CMM)规划测量点体积误差的收敛速度。优化 L-M 算法与 L-M 算法的结果具有一致性,但前者的收敛速度更快、算法精度更高。在进行激光追踪仪站位自标定时,L-M 算法平均迭代 1553 次才能迭代到全局最优解,而信赖域半径策略优化 L-M 算法平均迭代 9 次即可迭代到全局最优解。在求解 CMM 测量点的实际坐标时,L-M 算法的平均精度为 2.30×10^{-7} mm,优化 L-M 算法的平均精度为 8.75×10^{-10} mm。

关键词 测量; 三坐标测量机; 激光追踪仪; 多站位测量; L-M 算法; 信赖域算法

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1 引言

高精度三坐标测量机(CMM)是航空航天、船舶制造、工程机械等高端精密制造领域产品质量控制的关键设备。CMM 是精密测量的代表仪器,随着我国“深入实施制造强国战略的提出”,高端精密制造领域对 CMM 精度的要求越来越高。准确快速地标定 CMM 并对其测量的各项误差进行补偿,是提高 CMM 测量精度的重要途径之一^[1-2]。

目前,CMM 几何误差的检测手段主要有基于激光干涉仪、球杆仪、步距规等的实物基准方法^[3-11],但这些测量方法操作繁琐、耗时长,而且合成误差的模型具有一定的近似性,测试条件与 CMM 工作条件不完全相符。激光追踪仪测量范围在 20 m 以内,测量精度可以达到 $0.2 \mu\text{m} + 0.3 \mu\text{m}/\text{m}$,测量效率可与激光跟踪仪媲美^[12],更适合用于 CMM 和高精度数控设备体积误差的测量和补偿。

Wendtk 等^[13]构建了由 4 台激光追踪仪组成的新型 M3D3 测量系统,实现了 CMM 的精确三维坐标测量,但该系统并未在车间条件下进行精度测试,计量性能有待进一步提升。Camboulives 等^[14]提出了用一台激光追踪仪多站分时测量 CMM 体积误差的技术,即:先利用单台激光追踪仪在连续位置上建立参考测量系统,再利用多边测量原理得到待测点的三维坐标。该技术的不足之处是:通过逐点提取参数完成误差测量,

耗时较长。西安交通大学的查俊团队^[15]提出一种基于激光追踪仪的三轴数控加工刀具的体积误差测量方法,他们先采用三次样条插值法建立刀具路径体积误差模型,然后通过误差分离方法进行体积误差补偿,补偿后的体积误差相比补偿前降低了约 43%。陈洪芳等^[16-17]开展了激光追踪仪多站位测量技术的研究,通过实验验证了该技术在 CMM 体积误差测量和补偿方面具有独特优势,但该技术的测量精度还有待进一步提高。

Levenberg-Marquardt(L-M)算法是解决非线性最小二乘问题的一种优化算法,其因具有梯度法和牛顿法的优点而被广泛应用于飞机装配、机械故障诊断等领域。俞慈君等^[18]通过建立增强参考系统点热变形模型,提出了一种基于 L-M 算法的线性热变形补偿系数矩阵优化方法,该方法能够有效减小激光跟踪仪转站误差和修正转站参数,获得的单位温度下的热变形系数矩阵稳定可靠。该研究表明:L-M 迭代过程虽然受站位选择、测量数据、初值等的影响,但相比传统的梯度下降法具有更快的迭代速度。权凌霄等^[19]提出了遗传算法与 L-M 算法组合的优化方法,该方法提高了 BP 神经网络的学习速度和精确搜索能力,进而提高了电液伺服阀故障的诊断效率和精度。该研究结果表明:L-M 算法的迭代速度相比梯度下降法提高了几十倍甚至上百倍。周爱国等^[20]提出了一种基于加权 L-M 非线性优化的定位算法,该算法充分考虑了拍摄距离对物点计算

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精度的影响。他们利用矩阵论中的广义正交投影法求解多相机约束下物点的世界坐标初值,结合深度信息对目标函数进行加权,提高了近距离相机的置信度。同时,他们根据加权处理 L-M 算法对物点世界坐标进行迭代计算,以获得最优解。研究表明:所提加权 L-M 算法相比传统算法在精度与效率方面有一定提高。

为了进一步提升 CMM 体积误差的测量精度,笔者研究了信赖域半径策略优化 L-M 算法在激光追踪仪多站位测量技术中的应用,进而实现了激光追踪仪多站位测量技术对 CMM 的高效率、高精度标定。

2 信赖域半径策略优化 L-M 算法在激光追踪仪多站位测量中的应用

2.1 激光追踪仪多站位测量系统的构建

建立如图 1 所示的激光追踪仪多站位测量系统。激光追踪仪为海克斯康的 07 2009 TR IF 型 Laser Tracer,其长度测量精度为 $0.2 \mu\text{m} + 0.3 \mu\text{m}/\text{m}$ 。CMM 为海克斯康的 GLOBAL PERFORMANCE 07.10.07,其测量精度为 $(1.9 + 3.0 L/1000) \mu\text{m}$,测量范围为 $700 \text{ mm} \times 1000 \text{ mm} \times 660 \text{ mm}$,其中 L 表示测量距离。基于冗余测量方法,利用一台激光追踪仪采用分时转站的方式构建激光追踪多站位测量模型,并建立激光追踪仪多站位测量系统。

2.2 基于优化 L-M 算法的激光追踪仪站位自标定

激光追踪仪站位自标定是以激光追踪仪站位大致坐标为参考坐标,通过迭代求解出激光追踪仪实际站位坐标的过程。激光追踪仪多站位误差模型可表示为

$$f_i(X_j, Y_j, Z_j) = \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2 + (z_i - Z_j)^2} - d_j - l_{ij}, \quad (1)$$

式中: i 为测量点个数, $i = 1, 2, 3, \dots, n$; j 为激光追踪仪站位个数, $j = 1, 2, 3, \dots, m$; d_j 为激光追踪仪站位到初

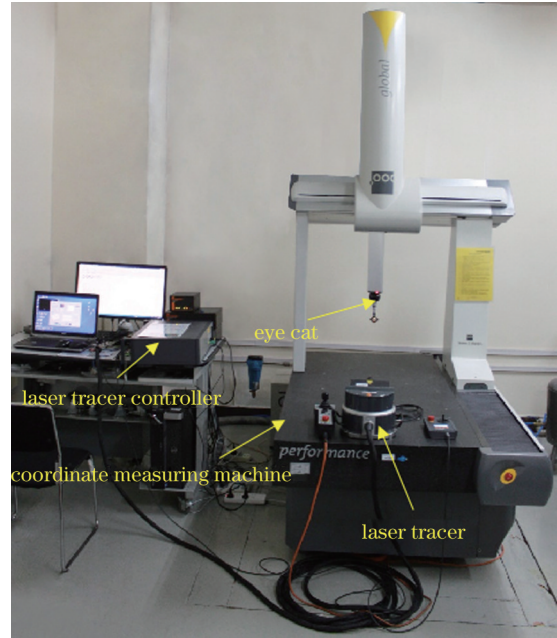


图 1 激光追踪仪多站位测量系统

Fig. 1 Multi-station measurement system of laser tracer

始测量点的距离; l_{ij} 为激光追踪仪在第 j 个站位测量第 i 个测量点时测量的相对干涉长度。激光追踪仪站位坐标为 $P_j(X_j, Y_j, Z_j)$,测量点坐标为 $A_i(x_i, y_i, z_i)$ 。

记 $f(X_j, Y_j, Z_j) = [f_1(X_j, Y_j, Z_j), f_2(X_j, Y_j, Z_j), \dots, f_n(X_j, Y_j, Z_j)]^T$, 则非线性最小二乘问题可以表示为

$$\min_{x \in \mathbb{R}^n} f(X_j, Y_j, Z_j) = \frac{1}{2} \|f(X_j, Y_j, Z_j)\|^2 = \frac{1}{2} \sum_{i=1}^n f_i^2(X_j, Y_j, Z_j). \quad (2)$$

记目标函数的梯度函数为 $g(X_j, Y_j, Z_j)$, 得到

$$g(X_j, Y_j, Z_j) = J(X_j, Y_j, Z_j)^T f(X_j, Y_j, Z_j), \quad (3)$$

其中,

$$J(X_j, Y_j, Z_j) = \begin{bmatrix} \frac{\partial f_1}{\partial X_j} & \frac{\partial f_1}{\partial Y_j} & \frac{\partial f_1}{\partial Z_j} \\ \frac{\partial f_2}{\partial X_j} & \frac{\partial f_2}{\partial Y_j} & \frac{\partial f_2}{\partial Z_j} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial X_j} & \frac{\partial f_n}{\partial Y_j} & \frac{\partial f_n}{\partial Z_j} \end{bmatrix}_{n \times 3} = \begin{bmatrix} \frac{X_j - x_1}{\sqrt{(x_1 - X_j)^2 + (y_1 - Y_j)^2 + (z_1 - Z_j)^2}} & \frac{Y_j - y_1}{\sqrt{(x_1 - X_j)^2 + (y_1 - Y_j)^2 + (z_1 - Z_j)^2}} & \frac{Z_j - z_1}{\sqrt{(x_1 - X_j)^2 + (y_1 - Y_j)^2 + (z_1 - Z_j)^2}} \\ \frac{X_j - x_2}{\sqrt{(x_2 - X_j)^2 + (y_2 - Y_j)^2 + (z_2 - Z_j)^2}} & \frac{Y_j - y_2}{\sqrt{(x_2 - X_j)^2 + (y_2 - Y_j)^2 + (z_2 - Z_j)^2}} & \frac{Z_j - z_2}{\sqrt{(x_2 - X_j)^2 + (y_2 - Y_j)^2 + (z_2 - Z_j)^2}} \\ \vdots & \vdots & \vdots \\ \frac{X_j - x_n}{\sqrt{(x_n - X_j)^2 + (y_n - Y_j)^2 + (z_n - Z_j)^2}} & \frac{Y_j - y_n}{\sqrt{(x_n - X_j)^2 + (y_n - Y_j)^2 + (z_n - Z_j)^2}} & \frac{Z_j - z_n}{\sqrt{(x_n - X_j)^2 + (y_n - Y_j)^2 + (z_n - Z_j)^2}} \end{bmatrix}_{n \times 3}. \quad (4)$$

设 (X_{jk}, Y_{jk}, Z_{jk}) 为第 j 个站位第 k 次迭代结果, 迭代的搜索方向为 h_k , 则有

$$h_k = \arg \min_{h \in \mathbb{R}^3} \|J_k(X_{jk}, Y_{jk}, Z_{jk})h + f_k(X_{jk}, Y_{jk}, Z_{jk})\| + \mu_k I \|h\|^2, \quad (5)$$

式中: $f_k(X_{jk}, Y_{jk}, Z_{jk})$ 为第 j 个站位第 k 次迭代的误差方程矩阵, 简记为 f_k ; $J_k(X_{jk}, Y_{jk}, Z_{jk})$ 为第 j 个站位第 k 次迭代的误差方程的雅可比矩阵, 简记为 J_k ; $h = [h_x, h_y, h_z]^T$, h_x, h_y, h_z 分别为 x, y, z 方向上的迭代步进; μ_k 为调整搜索方向的正参数; I 为 3×3 的单位矩阵。

由于最优性条件满足

$$\nabla \left(\|J_k h_k + f_k\|^2 + \mu_k \|h_k\|^2 \right) = 2 \left[(J_k^T J_k + \mu_k I) h + J_k^T f_k \right] = 0, \quad (6)$$

f_k 在 (X_{jk}, Y_{jk}, Z_{jk}) 处的下降方向为

$$h_k = - (J_k^T J_k + \mu_k I)^{-1} J_k^T f_k, \quad (7)$$

式中: ∇ 为梯度算子。

用 Armijio 搜索准则求解步长, 令 m' 是满足 Armijio 搜索准则应用条件的最小非负整数, 即

$$\frac{1}{2} \sum_{i=1}^n f_i^2 \left[X_{jk} + \beta^{m'} h_k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, Y_{jk} + \beta^{m'} h_k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, Z_{jk} + \beta^{m'} h_k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right] \leq \frac{1}{2} \sum_{i=1}^n f_i^2(X_{jk}, Y_{jk}, Z_{jk}) + \sigma \beta^{m'} h_k g_k^T, \quad (8)$$

式中: β 为常数, 且 $\beta \in (0, 1)$; σ 为常数, 且 $\sigma \in (0, 1)$; g_k 为第 j 个站位第 k 次迭代对应梯度值 $g_k(X_{jk}, Y_{jk}, Z_{jk})$ 的简记。

定义二次函数

$$q_k(h) = f_k(X_{jk}, Y_{jk}, Z_{jk}) + (J_k^T f_k)^T h + \frac{1}{2} h^T (J_k^T J_k) h, \quad (9)$$

基于当前给出的 μ_k , 考虑 $q_k(h)$ 与目标函数的增量

$$\Delta q_k = q_k(h) - q_k(0) = (J_k^T f_k)^T h_k + \frac{1}{2} h_k^T (J_k^T J_k) h_k. \quad (10)$$

用 r_k 表示 $q_k(h)$ 和目标函数的增量之比, 由于 f_k 多为矛盾矩阵, 故而用其范数来计算 r_k , 即

$$r_k = \frac{\| \Delta f_k \|}{\| \Delta q_k \|} = \frac{\| \Delta f_k = f_{k+1}(X_{j(k+1)}, Y_{j(k+1)}, Z_{j(k+1)}) - f_k(X_{jk}, Y_{jk}, Z_{jk}) \|}{\| (J_k^T f_k)^T h_k + \frac{1}{2} h_k^T (J_k^T J_k) h_k \|}. \quad (11)$$

当 r_k 绝对值较大时, 二次函数 $q_k(h)$ 在 (X_{jk}, Y_{jk}, Z_{jk}) 处拟合的目标函数比较好, 参数 μ_k 应取小一些; 当 r_k 绝对值较小时, 二次函数 $q_k(h)$ 在 (X_{jk}, Y_{jk}, Z_{jk}) 处拟合的目标函数比较差, 需增大参数 μ_k 的值来限制 h_k 的模长。

μ_k 选择规则如下:

$$\mu_{k+1} = \begin{cases} \Gamma_1 \mu_k, & r_k \leq \eta_1 \\ \mu_k, & \eta_1 < r_k < \eta_2 \\ \min \{ \Gamma_2 \mu_k, \tilde{\mu} \}, & r_k \geq \eta_2, \|h_k\| = \mu_k \end{cases}, \quad (12)$$

其中 $\Gamma_1 = 0.1, \Gamma_2 = 1, \eta_1 = 0.25, \eta_2 = 0.75, \tilde{\mu}$ 为设定的信赖域半径的上限。

根据信赖域半径的选取规则不断地调整 μ_k , 根据激光追踪仪多站位测量系统的容许误差 ϵ , 迭代求得激光追踪仪站位坐标 $P_j(X_j, Y_j, Z_j)$ 。实现全局域收敛的算法流程如图 2 所示。

图 2 中 A 为 $\frac{1}{2} \sum_{i=1}^n f_i^2 \left[X_{jk} + \beta^{m'} h_k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, Y_{jk} + \beta^{m'} h_k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, Z_{jk} + \beta^{m'} h_k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right]$, B 为 $\frac{1}{2} \sum_{i=1}^n f_i^2(X_{jk}, Y_{jk}, Z_{jk})$, h_i 为 $h_k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ 。

首先选取参数 σ 和 β 、容许误差 ϵ 以及 μ_0 (μ_0 为 μ_k 的初始值), 其中 $\sigma \in (0, 1), \beta \in (0, 1), 0 \leq \epsilon \leq 1, \mu_0 > 0$; 然后计算梯度函数 g_k , 用 $\|g_k\| \leq \epsilon$ 作为判别迭代结束的依据; 接着以方程组 $(J_k^T J_k + \mu_k I) h = -J_k^T f_k$ 求解搜索方向; 以 Armijio 搜索准则求解步长; 最后利用 r_k 选择信赖域半径更新 μ_k , 直至收敛。

2.3 优化 L-M 算法求解测量点的实际坐标

求解 CMM 空间范围内规划测量点的实际坐标, 就是把激光追踪仪站位自标定坐标作为已知条件, 把规划测

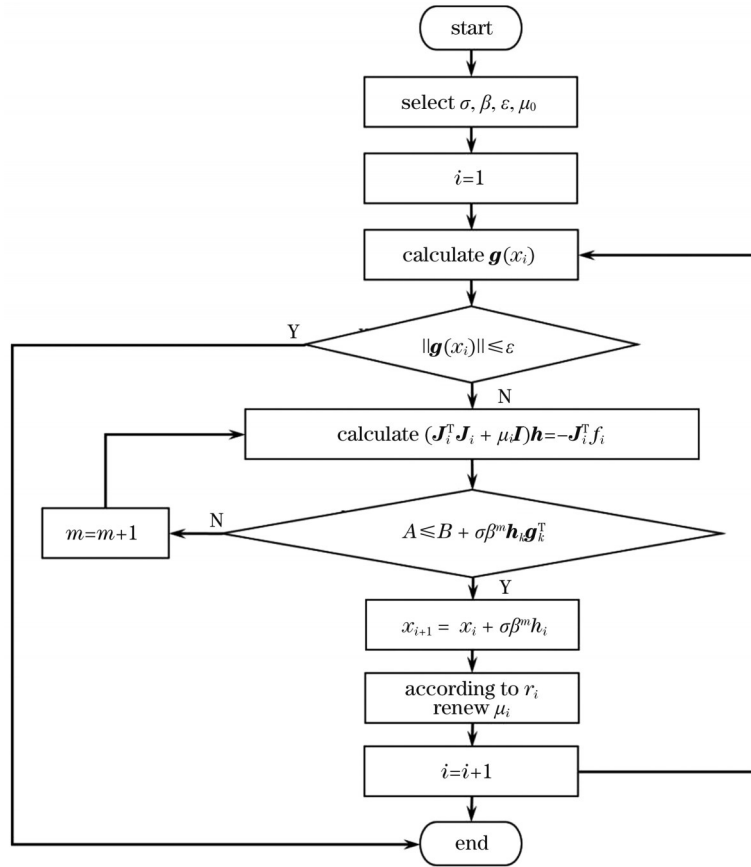


图 2 增加信赖域半径策略的 L-M 算法流程

Fig. 2 L-M algorithm flow for adding trust region radius strategy

量点的名义坐标作为初值进行迭代,进而求解出测量点实际坐标的过程。此时目标函数变为激光追踪仪测距的残差方程,即

$$f_j(x_i, y_i, z_i) = \sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2 + (z_i - Z_j)^2} - d_j - l_{ij}, \quad (13)$$

记 $f(x_i, y_i, z_i) = [f_1(x_i, y_i, z_i), f_2(x_i, y_i, z_i), \dots, f_n(x_i, y_i, z_i)]^T$, 其对应的雅可比矩阵为

$$J(x_i, y_i, z_i) = \begin{bmatrix} \frac{\partial f_1}{\partial x_i} & \frac{\partial f_1}{\partial y_i} & \frac{\partial f_1}{\partial z_i} \\ \frac{\partial f_2}{\partial x_i} & \frac{\partial f_2}{\partial y_i} & \frac{\partial f_2}{\partial z_i} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_m}{\partial x_i} & \frac{\partial f_m}{\partial y_i} & \frac{\partial f_m}{\partial z_i} \end{bmatrix}_{m \times 3} =$$

$$\begin{bmatrix} \frac{x_i - X_1}{\sqrt{(x_i - X_1)^2 + (y_i - Y_1)^2 + (z_i - Z_1)^2}} & \frac{y_i - Y_1}{\sqrt{(x_i - X_1)^2 + (y_i - Y_1)^2 + (z_i - Z_1)^2}} & \frac{z_i - Z_1}{\sqrt{(x_i - X_1)^2 + (y_i - Y_1)^2 + (z_i - Z_1)^2}} \\ \frac{x_i - X_2}{\sqrt{(x_i - X_2)^2 + (y_i - Y_2)^2 + (z_i - Z_2)^2}} & \frac{y_i - Y_2}{\sqrt{(x_i - X_2)^2 + (y_i - Y_2)^2 + (z_i - Z_2)^2}} & \frac{z_i - Z_2}{\sqrt{(x_i - X_2)^2 + (y_i - Y_2)^2 + (z_i - Z_2)^2}} \\ \vdots & \vdots & \vdots \\ \frac{x_i - X_m}{\sqrt{(x_i - X_m)^2 + (y_i - Y_m)^2 + (z_i - Z_m)^2}} & \frac{y_i - Y_m}{\sqrt{(x_i - X_m)^2 + (y_i - Y_m)^2 + (z_i - Z_m)^2}} & \frac{z_i - Z_m}{\sqrt{(x_i - X_m)^2 + (y_i - Y_m)^2 + (z_i - Z_m)^2}} \end{bmatrix}_{m \times 3} \quad (14)$$

由 Armijio 搜索准则可得到满足其应用条件的最小非负整数 m' , 即

$$\frac{1}{2} \sum_{j=1}^m f_j^2 \left[x_{ik} + \beta^{m'} \mathbf{h}_k \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, y_{ik} + \beta^{m'} \mathbf{h}_k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, z_{ik} + \beta^{m'} \mathbf{h}_k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right] \leq \frac{1}{2} \sum_{j=1}^m f_j^2(x_{ik}, y_{ik}, z_{ik}) + \sigma \beta^{m'} \mathbf{h}_k \mathbf{g}_k^T. \quad (15)$$

根据式(7)、(8)、(15)分别求得搜索方向、信赖域半径更新参数和搜索步长,然后不断更新 μ_k 。以CMM空间测量点实际坐标要求的容许误差 ϵ 为判断依据,直至迭代结束求解出测量点的实际坐标 $A_i(x_i, y_i, z_i)$ 。

2.4 初值选取方法

应用信赖域半径策略优化L-M算法求解激光追踪仪多站位测量模型时要确定迭代的初值,在进行激光追踪仪站位自标定时,初值选取为 m 个激光追踪仪站位的大致坐标范围值,即参考坐标 $P_0(X_0, Y_0, Z_0)$;将坐标初值代入式(1),激光追踪仪站位到初始测量点的距离 d_j 、激光追踪仪在第 j 个站位测量第 i 个测量点时测量的相对干涉长度 l_{ij} 、规划测量点坐标 $A_i(x_i, y_i, z_i)$ 为已知量,根据信赖域半径策略优化L-M算法规则进行迭代,可以求出 m 个激光追踪仪站位的实际坐标,从而完成激光追踪仪站位自标定。

在求解CMM空间范围内规划测量点的实际坐标值时,初值选取为规划测量点的理论坐标值,将激光追踪仪站位自标定坐标值作为已知条件代入式(13),根据信赖域半径策略优化L-M算法规则进行迭代,可以求出CMM空间范围内 n 个规划测量点的实际坐标值。

3 实验

搭建的激光追踪仪多站位系统如图1所示,规划的CMM空间测量范围为700 mm×500 mm×450 mm。规划测量点的个数为 $n=183$,激光追踪仪的站位个数为 $m=4$ 。规划测量点的空间分布如图3所示。

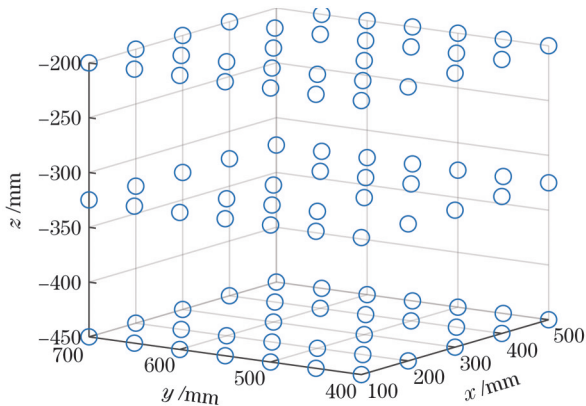


图3 规划测量点的空间分布

Fig. 3 Spatial distribution of the planned measurement points

3.1 激光追踪仪站位自标定实验

利用激光追踪仪多站位测量方法,根据图3所示的规划测量路径,通过激光追踪仪自身测量软件获得

的站位的理论坐标值如表1所示,而利用信赖域半径策略优化L-M算法获得的激光追踪仪站位自标定结果如表2所示。通过比较可以看出,利用信赖域半径策略优化L-M算法求解得到的站位坐标与站位理论坐标一致。

表1 激光追踪仪站位的理论坐标
Table 1 Theoretical coordinates of laser tracer station

Station marker	X/mm	Y/mm	Z/mm
1	-39.5209	-18.2952	-613.1806
2	513.0757	-48.5926	-613.1649
3	263.3054	-40.3469	-465.2685
4	265.0009	1306.7992	-613.4593

表2 信赖域半径策略优化L-M算法求解的站位坐标
Table 2 Station coordinates solved by L-M algorithm with trust region radius strategy

Station marker	X/mm	Y/mm	Z/mm
1	-39.5209	-18.2952	-613.1806
2	513.0757	-48.5926	-613.1648
3	263.3053	-40.3468	-465.2685
4	265.0008	1306.7792	-613.4593

3.2 L-M算法和优化L-M算法的效率对比

由算法产生的迭代序列 $\{x_i\}$ 收敛到最优解 x^* ,若 $J(x^*)^T J(x^*)$ 非奇异,则 $\left(\frac{1}{2} - \sigma\right) J(x^*)^T J(x^*) -$

$\frac{1}{2} \sum_{i=1}^n f_i(x^*) \nabla^2 f_i(x^*)$ 正定,目标函数在 x^* 处连续,可以

根据迭代过程研究L-M算法和信赖域半径策略优化L-M算法的收敛速度。表3是两种算法进行站位自标定时迭代次数对比。

基于L-M算法的激光追踪仪站位自标定过程是由各个站位的参考坐标迭代至站位实际坐标的过程,

表3 L-M算法和信赖域半径策略优化L-M算法进行激光追踪仪站位自标定迭代次数

Table 3 Number of iterations of L-M algorithm and L-M algorithm with trust region radius strategy

Station marker	L-M algorithm	L-M algorithm with trust region radius strategy
1	1727	9
2	1444	9
3	1128	9
4	1914	10

由于站位参考坐标是人为给出的,具有很大的主观性,所以不同站位参考坐标到实际坐标的距离不同,而且迭代次数也不同。L-M算法的平均迭代次数为1553次,信赖域半径策略优化L-M算法的平均迭代次数为9次,极大地提高了收敛速度。

图4(a)是利用L-M算法进行激光追踪仪站位自标定时第一站位坐标 (X_1, Y_1, Z_1) 的迭代过程,图4(b)是利用信赖域半径策略优化L-M算法进行激光追踪

仪站位自标定时第一站位坐标 (X_1, Y_1, Z_1) 的迭代过程。可以看出:激光追踪仪第一站位自标定时,L-M算法需要迭代1727次才能收敛到全局最优解,而信赖域半径策略优化L-M算法仅需9次迭代就能收敛到全局最优解。信赖域半径策略优化L-M算法既保留了L-M算法在迭代过程中不要求雅可比矩阵列满秩的特点,又极大地提高了收敛速度,从而提高了计算效率。

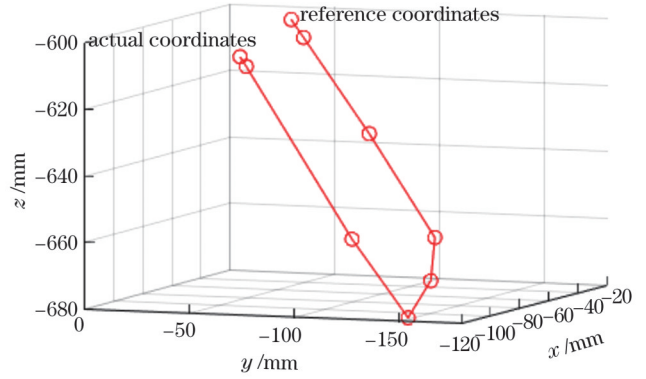
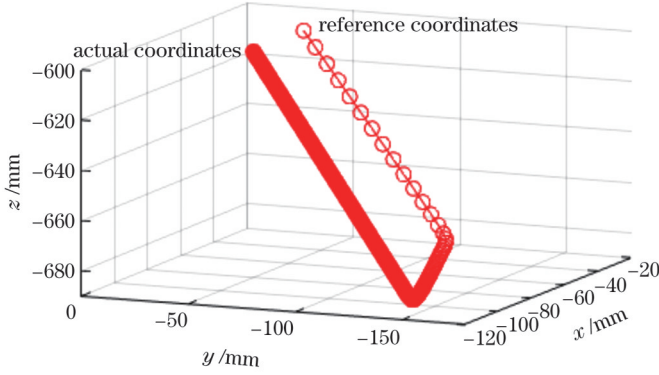


图4 激光追踪仪第一站位自标定的迭代过程。(a)L-M算法;(b)信赖域半径策略优化L-M算法

Fig. 4 Iterative process of laser tracer first station self-calibration. (a) L-M algorithm; (b) L-M algorithm with trust region radius strategy

3.3 L-M算法和优化L-M算法的精度对比

将求得的高精度站位坐标代入式(13),分别利用L-M算法和信赖域半径策略优化L-M算法求解出测量点的实际坐标值,进而得到CMM规划测量点的体积误差为

$$\begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ 0 \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \\ z'_i \\ 1 \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}, \quad (16)$$

式中: $A_i(x_i, y_i, z_i)$ 为测量点的理论坐标值; $A_i(x'_i, y'_i, z'_i)$ 为测量点的实际坐标值; $(\Delta x_i, \Delta y_i, \Delta z_i)$ 为待测点 A_i 的体积误差。

基于L-M算法求出的测量点在X、Y、Z方向上的体积误差的大小如图5所示。X方向的误差范围为 $-0.00306 \sim 0.00473$ mm, Y方向的误差范围为 $-0.00458 \sim 0.00370$ mm, Z方向的误差范围为 $-0.00983 \sim 0.00433$ mm。

利用信赖域半径策略优化L-M算法求出的测量点在X、Y、Z方向上的体积误差的大小如图6所示。X方向的误差范围为 $-0.00318 \sim 0.00494$ mm, Y方向的误差范围为 $-0.00463 \sim 0.00371$ mm, Z方向的误差范围为 $-0.01102 \sim 0.00436$ mm。

L-M算法和信赖域半径策略优化L-M算法都通过Val参数(在公式中将其记为“ V_{al} ”)表示算法的精度,即根据目标函数接近于0的程度来判断迭代结果的好坏。Val参数的定义为

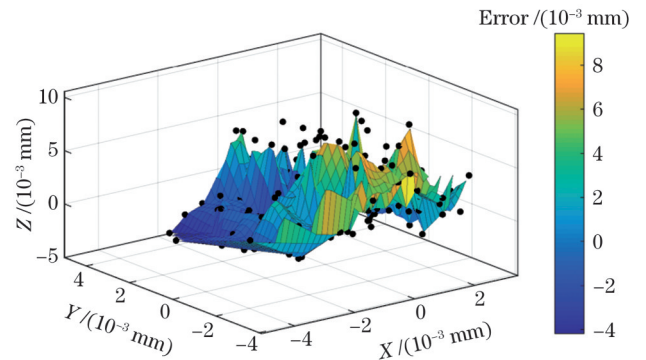


图5 L-M算法求出的测量点的体积误差
Fig. 5 Volume error of measuring points calculated by L-M algorithm

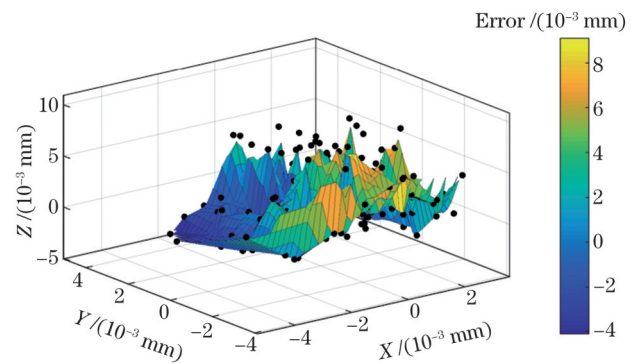


图6 利用信赖域半径策略优化L-M算法求出的测量点的体积误差

Fig. 6 Volume error of measuring points calculated with L-M algorithm with trust region radius strategy

$$V_{al} = \frac{1}{2} \left\| f(x_i, y_i, z_i) \right\|_0 \quad (17)$$

L-M 算法求测量点实际坐标的算法平均精度为 2.30×10^{-7} mm, 标准差为 2.78×10^{-7} mm; 信赖域半径策略优化 L-M 算法求测量点实际坐标的算法平均精度为 8.75×10^{-10} mm, 标准差为 7.47×10^{-10} mm。可以看出, 信赖域半径策略优化 L-M 算法极大地提高了算法精度。

4 结 论

笔者通过建立激光追踪仪多站位测量系统获得测量数据, 根据空间距离公式建立非线性方程组, 利用最小二乘思想建立目标函数, 引入信赖域半径策略对传统的 L-M 算法进行优化, 同时通过调整搜索方向的正参数进行信赖域判定, 确定了每一次迭代最优的方向和步长, 使得算法快速收敛到全局最优解, 实现了基于激光追踪仪多站位测量技术的高效率的 CMM 规划测量点的体积误差测量。

参 考 文 献

- [1] 石照耀, 张斌, 林家春, 等. 坐标测量技术半世纪: 演变与趋势[J]. 北京工业大学学报, 2011, 37(5): 648-656.
Shi Z Y, Zhang B, Lin J C, et al. Half century of coordinate metrology technology: evolution and trends[J]. Journal of Beijing University of Technology, 2011, 37(5): 648-656.
- [2] 张国雄. 三坐标测量机的发展趋势[J]. 中国机械工程, 2000, 11(1): 222-226.
Zhang G X. The development tendency of coordinate measuring machines[J]. China Mechanical Engineering, 2000, 11(1): 222-226.
- [3] 吕学祜, 郭前建, 王昊天, 等. 数控机床误差补偿关键技术综述[J]. 航空制造技术, 2022, 65(11): 104-111, 119.
Lü X H, Guo Q J, Wang H T, et al. Summary of key technologies for error compensation of CNC machine tools[J]. Aeronautical Manufacturing Technology, 2022, 65(11): 104-111, 119.
- [4] 谈宜东, 徐欣, 张书练. 激光干涉精密测量与应用[J]. 中国激光, 2021, 48(15): 1504001.
Tan Y D, Xu X, Zhang S L. Precision measurement and application of laser interference[J]. Chinese Journal of Lasers, 2021, 48(15): 1504001.
- [5] Franco P, Jodar J. Theoretical analysis of straightness errors in coordinate measuring machines (CMM) with three linear axes[J]. International Journal of Precision Engineering and Manufacturing, 2021, 22(1): 63-72.
- [6] Deng M, Li H M, Xiang S T, et al. Geometric errors identification considering rigid-body motion constraint for rotary axis of multi-axis machine tool using a tracking interferometer[J]. International Journal of Machine Tools and Manufacture, 2020, 158: 103625.
- [7] Andre B, Hintze W. Workshop-suited geometric errors identification of three-axis machine tools using on-machine measurement for long term precision assurance[J]. Precision Engineering, 2022, 75: 235-247.
- [8] Wang Z, Wang D L, Yu S D, et al. A reconfigurable mechanism model for error identification in the double ball bar tests of machine tools[J]. International Journal of Machine Tools and Manufacture, 2021, 165: 103737.
- [9] Xu K, Li G L, He K, et al. Identification of position-dependent geometric errors with non-integer exponents for linear axis using double ball bar[J]. International Journal of Mechanical Sciences, 2020, 170: 105326.
- [10] 赵东升, 张健, 贾敏强, 等. 测量数控机床定位误差的两种方法之比较[J]. 计量学报, 2019(S01): 33-35.
Zhao D S, Zhang J, Jia M Q, et al. Comparison of two methods for measuring the error of CNC machine tool[J]. Acta Metrologica Sinica, 2019(S01): 33-35.
- [11] Hemming B, Esala V P, Laukkanen P, et al. Interferometric step gauge for CMM verification[J]. Measurement Science and Technology, 2018, 29(7): 074012.
- [12] 熊涛, 李泷泉, 李琦, 等. 基于组合测量的激光跟踪仪站位规划[J]. 激光与光电子学进展, 2021, 58(17): 1712001.
Xiong T, Li S G, Li Q, et al. Station planning of laser tracker based on combination measurement[J]. Laser & Optoelectronics Progress, 2021, 58(17): 1712001.
- [13] Wendtk K, Franek M, Hartig F. Measuring large 3D structures using four portable tracking laser interferometers[J]. Measurement, 2012, 45(10): 2339-2345.
- [14] Camboulives M, Lartigue C, Bourdet P, et al. Calibration of a 3D working space by multilateration[J]. Precision Engineering, 2016, 44: 163-170.
- [15] Zha J, Wang T, Li L H, et al. Volumetric error compensation of machine tool using laser tracer and machining verification[J]. The International Journal of Advanced Manufacturing Technology, 2020, 108(7/8): 2467-2481.
- [16] Chen H F, Jiang B, Lin H, et al. Calibration method for angular positioning deviation of a high-precision rotary table based on the laser tracer multi-station measurement system[J]. Applied Sciences, 2019, 9(16): 3417.
- [17] Chen H F, Zhang S, Wang G Y, et al. LASSO based compensation method for geometric errors of large coordinate measuring machine[J]. Measurement, 2022, 196: 111157.
- [18] 俞慈君, 宋凯, 李江雄, 等. 基于 Levenberg-Marquardt 算法的线性热变形补偿系数矩阵优化[J]. 浙江大学学报(工学版), 2016, 50(6): 1056-1064.
Yu C J, Song K, Li J X, et al. Optimization of linear thermal deformation compensation coefficient matrix based on Levenberg-Marquardt algorithm[J]. Journal of Zhejiang University (Engineering Science), 2016, 50(6): 1056-1064.
- [19] 权凌霄, 郭海鑫, 盛世伟, 等. 采用“GA+LM”优化 BP 神经网络的电液伺服阀故障诊断[J]. 中国机械工程, 2018, 29(5): 505-510.
Quan L X, Guo H X, Sheng S W, et al. Fault diagnosis of electro hydraulic servo valves based on GA+LM algorithm optimized BP neural networks[J]. China Mechanical Engineering, 2018, 29(5): 505-510.
- [20] 周爱国, 于江洋, 蒲家坤, 等. 基于加权 Levenberg-Marquardt 的多目视觉同名物点定位算法[J]. 激光与光电子学进展, 2021, 58(24): 2415006.
Zhou A G, Yu J Y, Pu J K, et al. Multi-vision homonym location algorithm based on weighted Levenberg-Marquardt[J]. Laser & Optoelectronics Progress, 2021, 58(24): 2415006.

Application of L-M Algorithm with Trust Region Radius Strategy in Laser Tracer Multi-Station Measurement

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Abstract

Objective High-precision CMM (coordinate measuring machine) is a key equipment for product quality control in high-end precision manufacturing fields such as aerospace, shipbuilding, and engineering machinery, and so on. At present, the primary CMM geometric error detection means are some physical reference methods based on laser interferometers, ballbars, and step gauges. However, the above measurement methods are complicated, and the operation is cumbersome and time-consuming. The model of synthetic error has a certain approximation, and the test conditions are not entirely consistent with the working conditions of CMM. The measuring range of laser tracers is less than 20 m, the measuring accuracy can reach $0.2 \mu\text{m} + 0.3 \mu\text{m}/\text{m}$, and the measuring efficiency is close to that of laser tracker. Therefore, the laser tracer is more suitable for the measurement and compensation of CMM and high-grade CNC (computerized numerical control) equipment volume error. In order to solve the problem of slow convergence of Levenberg-Marquardt (L-M) algorithm in the application of laser tracer multi-station measurement, the L-M algorithm based on the strategy of adding trust region radius was proposed to effectively achieve calibration of laser tracing multi-station measurement technology for CMM.

Methods The solution of the laser tracer multi-station measurement system model was divided into two parts: one is the self-calibration of the laser tracer station, the other is to solve the actual coordinates of the planned measurement points within the CMM space. The L-M algorithm which adds trust region radius strategy was used to solve the laser tracer multi-station measurement system model. Based on L-M algorithm, the determination of the trust region was conducted by adding the positive parameters of the search direction, and the optimal iteration direction and iteration step size were determined, making the algorithm can quickly converge to the global domain optimal solution. The process of trust region determination was to first define the quadratic function, and then consider the ratio of the increment of the quadratic function value to the target function value based on the current positive parameters to determine whether the selection of positive parameters is reasonable. When the absolute value of the ratio is large, the positive parameters should be obtained the smaller to increase the modulus length of the search direction. When the absolute value of the ratio is small, the value of the positive parameter should be increased to limit the modulus length of the search direction. The iterative result can be less than the tolerance error of the laser tracer multi-station measurement system by continuously adjusting the positive parameters through the selection rules of trust region radius, realizing global domain convergence to self-calibrate the laser tracer station and obtain the actual coordinates of the planned measurement points within the CMM space.

Results and Discussions The built laser tracer multi-station system was shown in Fig. 1. The measurement paths were planned based on laser tracer multi-station measurement methods. The L-M algorithm and the L-M algorithm with trust region radius strategy were used to solve this laser tracer multi-station measurement system model. The experimental results show that the average number of iterations in the self-calibration process of laser tracer station based on L-M algorithm is 1553, and the average number of iterations of the L-M algorithm with trust region radius strategy is 9. The latter algorithm greatly improves the convergence speed. The Val parameter is used to represent the accuracy of the algorithm when solving the actual coordinates of the planned measurement points within the CMM space. The average accuracy of the L-M algorithm is 2.30×10^{-7} mm with a standard deviation of 2.78×10^{-7} mm, and the average accuracy of the L-M algorithm with trust region radius strategy is 8.75×10^{-10} mm with a standard deviation of 7.47×10^{-10} mm. It could be seen that the latter algorithm improves the measurement accuracy.

Conclusions The measurement data were obtained by establishing a laser tracer multi-station measurement system, the nonlinear equations were established based on the spatial distance formula, and the objective function was established through the least square idea. The traditional L-M algorithm is optimized by introducing the trust region radius strategy. By adjusting the positive parameters of the search direction, the trust region determination is determined, so the optimal iteration direction and iteration step size for each iteration are determined. Thus, the algorithm can rapidly converge to the optimal solution in the global domain. And the high-efficiency and high-precision of the volume error measurement of CMM planned measurement points based on the laser tracer multi-station measurement technology can be realized.

Key words measurement; coordinate measuring machine; laser tracer; multi-station measurement; L-M algorithm; trust region algorithm