

非均匀光纤中啁啾对艾里脉冲聚焦特性的影响

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摘要 基于变系数非线性薛定谔方程, 对具有初始啁啾的对称艾里脉冲的传输特性进行解析求解和数值研究。解析结果表明, 在周期色散调制下, 对称艾里脉冲的传输轨迹具有一定的周期性, 并且它的传输特性在很大程度上依赖于啁啾参量和主瓣位置。在满足焦点存在的条件下, 通过调节啁啾参量和脉冲主瓣的初始位置, 可以控制对称艾里脉冲相互碰撞过程中产生的焦点数量以及焦点的位置。此外, 本文还考虑了截断参数和克尔非线性效应对啁啾艾里脉冲传输特性的影响。截断参数不影响焦点的数量和位置, 但较大的截断参数在碰撞时会损失更多的能量。非线性效应会导致孤子脱落, 更强的非线性会导致更多孤子产生。

关键词 非线性光学; 非均匀光纤; 对称艾里脉冲; 啁啾参量; 脉冲主瓣; 聚焦

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1 引言

1979年, Berry 和 Balazs^[1] 在量子力学背景下提出了艾里波包的概念。艾里波包作为自由粒子薛定谔方程的解, 其能量是无限的, 因此在实际中无法实现。2007年, Siviloglou 等^[2] 在光学中引入了有限能量艾里光束的概念, 并在实验中实现了它^[3]。随后, 有限能量艾里光束引起了科研工作者的广泛兴趣^[4-6]。有限能量艾里光束具有近似无衍射、自加速以及自修复等特性^[7-11], 已被应用于真空电子加速^[12]、光学等离子体通道^[13]、光微粒操控^[14-16]、光子弹^[17-19]、光路由^[20]等领域。

时间色散和空间衍射在非线性薛定谔方程中具有同样的形式, 因此时间艾里脉冲可以在非线性薛定谔方程的框架中进行处理^[21], 从而艾里脉冲也得到了广泛研究: 在克尔非线性作用下, 孤子从艾里脉冲传输过程中脱落^[22]; 高阶线性和非线性效应对艾里脉冲传输的影响^[23-24]; 减速艾里脉冲在非瞬时立方介质中的传输动力学^[25]; 利用飞秒艾里脉冲在光子晶体光纤中进行超连续谱的产生^[26]; 艾里脉冲在光纤中传输的时间加速自成像效应^[27]; 艾里脉冲和

光孤子在非均匀光纤中的演化特性^[28]; 周期色散调制^[29]以及线性变化色散对艾里脉冲轨迹的控制^[30]。此外, 近期的一些研究表明, 时间变化的光学电势可以控制艾里脉冲的传输轨迹^[31]。

实际上, 激光系统在脉冲产生、传输和放大方面均涉及啁啾。啁啾对激光自聚焦具有显著影响, 因此可用于控制超连续谱的产生以及脉冲压缩等^[32-34]。此外, 初始啁啾对艾里脉冲传输特性的影响也得到了研究, 如: Zhang 等^[35]研究了初始啁啾对光纤中艾里脉冲传输特性的影响, 结果表明, 如果群速度色散和啁啾具有相反的符号, 那么艾里脉冲会在初始压缩阶段后以相反的加速度移动; Shi 等^[36]研究了对称艾里脉冲的多点聚焦特性。但这些结果都是在恒定的色散系数下得到的, 当考虑周期变化的色散作用时, 就会出现更为丰富的动力学特性。因此, 本文以变系数非线性薛定谔方程为模型, 通过解析求解与数值模拟相结合的方法, 研究啁啾艾里脉冲在周期色散调制系统中的相互作用, 并揭示其新的聚焦特性。

2 理论模型与分析

光脉冲在非均匀光纤中的传输可以由(1)式所

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示的变系数非线性薛定谔方程进行描述^[35]。

$$i \frac{\partial U}{\partial Z} + \frac{\delta_2(Z)}{2} \frac{\partial^2 U}{\partial T^2} + \gamma(Z) |U|^2 U = 0, \quad (1)$$

式中: i 为虚数单位; U 是光脉冲的归一化慢变包络; Z 和 T 分别为归一化的传输距离和延迟时间; δ_2 为群速度色散; γ 为非线性系数。群速度色散和非线性系数均为传输距离的函数, 这里取它们的表达式为

$$\delta_2(Z) = \chi \cos(\mu Z), \quad (2)$$

$$\gamma(Z) = N^2 \cos(\mu Z), \quad (3)$$

式中: χ 、 μ 和 N 均为与波动相关的参数, χ 和 N 表示光纤参数波动的幅度, μ 表示光纤参数变化的周期。

首先研究艾里脉冲在线性系统中的传输, 也就是(1)式中 $\gamma(Z) = 0$ 的情况, 即

$$i \frac{\partial U}{\partial Z} + \frac{\delta_2(Z)}{2} \frac{\partial^2 U}{\partial T^2} = 0. \quad (4)$$

初始输入为啁啾对称艾里脉冲, 其表达式为

$$U(0, T) = A_0 \{ A_i(T+d) \exp[a(T+d)] + A_i(-T+d) \exp[a(-T+d)] \} \exp(-iCT^2), \quad (5)$$

式中: A_0 为入射脉冲的振幅; $A_i(T)$ 为艾里函数; a 为截断参数, $0 < a < 1$; d 表示主瓣位置, $d > 0$; C 表示初始啁啾参量。

采用傅里叶变换可以得到(4)式的解的一般形式为

$$U(Z, T) = \frac{A_0}{2\pi} \int_{-\infty}^{+\infty} \tilde{U}(0, \omega) \exp \left[i \frac{\omega^2}{2} \int_0^Z \delta_2(Z') dZ' \right] \exp(-i\omega Z) d\omega, \quad (6)$$

式中: ω 为归一化角频率; $\tilde{U}(0, \omega)$ 是初始入射脉冲 $U(0, T)$ 的傅里叶变换, 其表达式为

$$\tilde{U}(0, \omega) = \frac{A_0}{2\pi} \int_{-\infty}^{+\infty} U(0, T) \exp(i\omega T) dT. \quad (7)$$

将(7)式代入(6)式, 则啁啾对称艾里脉冲的演化可以描述为^[36]

$$|U(Z, T)| = \left| \sqrt{\theta} A_i \left[\frac{T+d-2dC\rho(Z)}{\theta} - \frac{\rho(Z)^2}{4\theta^2} + \frac{i a \rho(Z)}{\theta} \right] \exp \left[\frac{aT}{\theta} \right] \exp \left[\frac{aT+ad-2adC\rho(Z)}{\theta} - \frac{a\rho(Z)^2}{2\theta^2} \right] + \sqrt{\theta} A_i \left[\frac{-T+d-2dC\rho(Z)}{\theta} - \frac{\rho(Z)^2}{4\theta^2} + \frac{i a \rho(Z)}{\theta} \right] \exp \left[\frac{-aT}{\theta} \right] \exp \left[\frac{-aT+ad-2adC\rho(Z)}{\theta} - \frac{a\rho(Z)^2}{2\theta^2} \right] \right|, \quad (8)$$

式中: $\theta = 1 - 2C\rho(Z) \neq 0$; $\rho(Z) = \int_0^Z \delta_2(Z') dZ' = \chi \sin(\mu Z) / \mu$ 。

由以上结果可以看出, 对称艾里脉冲的振幅分布是两个波相互作用的结果。由(8)式可知啁啾对称艾里脉冲的传输轨迹呈周期性, 其轨迹路径为

$$T = -d + 2dC\rho(Z) + \frac{\rho(Z)^2}{4\theta}, \quad (9)$$

$$T = d - 2dC\rho(Z) - \frac{\rho(Z)^2}{4\theta}. \quad (10)$$

联立(9)式和(10)式可以得到对称艾里脉冲在传输过程中与焦点位置相关的方程为

$$\rho(Z_{\pm}) = \frac{2\sqrt{d}}{4C\sqrt{d} \pm 1}. \quad (11)$$

为了讨论方便, 这里令 $\chi = \mu = 1$ 。根据(11)式可以推出对称艾里脉冲在传播过程中碰撞产生焦点的充分条件为: $(4C \pm 1/\sqrt{d}) = 2/\sin(Z_{\pm})$ 。图 1 分别给

出了 $\sin(Z_{\pm}) \geq 0_+$ 和 $\sin(Z_{\pm}) \leq 0_-$ 条件下焦点的分布区域图, 此时, 图 1(a)、(b) 中的参数 C 和 d 分别满足条件 (i) $4C \pm 1/\sqrt{d} \geq 2$ 和条件 (ii) $4C \pm 1/\sqrt{d} \leq -2$ 。由图 1(a) 可以看出: 当满足条件 (i) 时, $C \geq 0$ 。由图 1(a) 还可以看出: 在一个色散调制周期内, 当参数 C 和 d 满足 $4C + 1/\sqrt{d} = 2$ 时 (曲线①), 只存在 1 个焦点; 当参数 C 和 d 满足 $4C - 1/\sqrt{d} = 2$ 时 (曲线②), 存在 3 个焦点; 当参数 C 和 d 满足 $4C + 1/\sqrt{d} > 2$ 且 $4C - 1/\sqrt{d} < 2$ 时 (区域 II), 存在 2 个焦点; 当参数 C 和 d 满足 $4C - 1/\sqrt{d} > 2$ 时 (区域 I), 存在 4 个焦点; 当参数 C 和 d 满足 $4C + 1/\sqrt{d} < 2$ 时 (区域 III), 没有焦点存在。图 1(b) 中 $C < 0$, 与图 1(a) 中结果类似。

当满足以上焦点存在的条件时, 焦点的位置为

$$Z_{\pm j} = (-1)^{j-1} \arcsin \left[\left(2C \pm \frac{1}{2\sqrt{d}} \right)^{-1} \right] + \pi(j-1), \quad (12)$$

式中: $j = 1, 2, 3, \dots$ 。

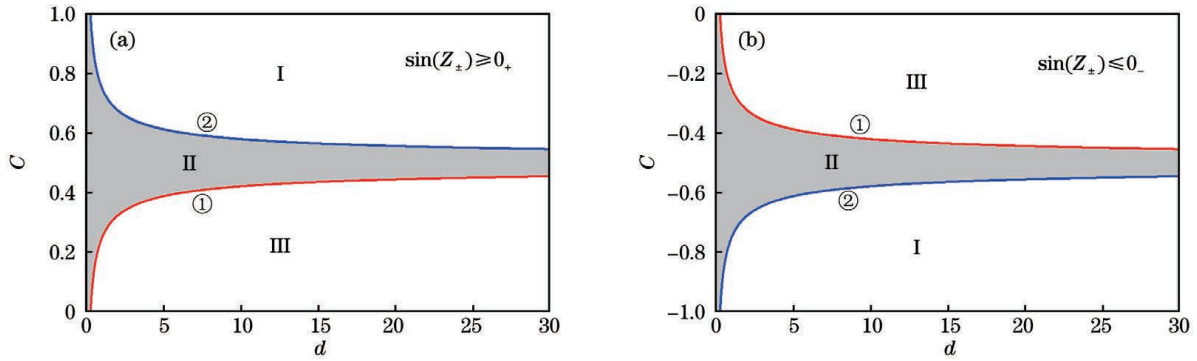


图 1 两个不同条件下焦点的分布区域图。(a) $4C \pm 1/\sqrt{d} \geq 2$; (b) $4C \pm 1/\sqrt{d} \leq -2$

Fig. 1 Distribution area of focus points under two conditions. (a) $4C \pm 1/\sqrt{d} \geq 2$; (b) $4C \pm 1/\sqrt{d} \leq -2$

由(12)式可知:在 $\rho(Z)$ 的一个周期 $[0, 2\pi]$ 内, 当 $C > 0$ 时, $2C \pm \frac{1}{2\sqrt{d}} \geq 0$, 焦点的位置出现在 $\rho(Z)$ 的前半个周期, 对应图 1(a); 当 $C < 0$ 时, $2C \pm \frac{1}{2\sqrt{d}} \leq 0$, 焦点的位置出现在 $\rho(Z)$ 的后半个周期, 对应图 1(b)。因此, 啁啾参量 C 和主瓣位置 d 能够控制产生焦点的位置及个数。

此外, 当(8)式中的 $\theta = 0$ 时, $\frac{1}{C} = 2\sin(Z)$, 有奇点存在, 此时, 脉冲的主瓣轨迹会发生反转^[35]。分析可知: 当 $|C| \geq 0.5$ 时, 在 $\sin(Z)$ 的一个周期内,

(8)式中存在奇点, 艾里主瓣的轨迹会发生反转; 当 $|C| < 0.5$ 时, 不存在奇点, 脉冲在演化中不发生反转。可见, 啁啾参量 C 和主瓣位置 d 对脉冲的传输特性有很大影响。

3 数值结果

为了验证解析结果, 通过数值模拟研究不同正啁啾参量、主瓣位置以及截断参数下对称艾里脉冲的传输特性。

图 2 显示了一个调制周期的传输距离内, 艾里脉冲在不同啁啾参量作用下的传输行为, 其中主瓣位置 $d = 9$ 。由图 2 可见, 啁啾参量可以操控对称

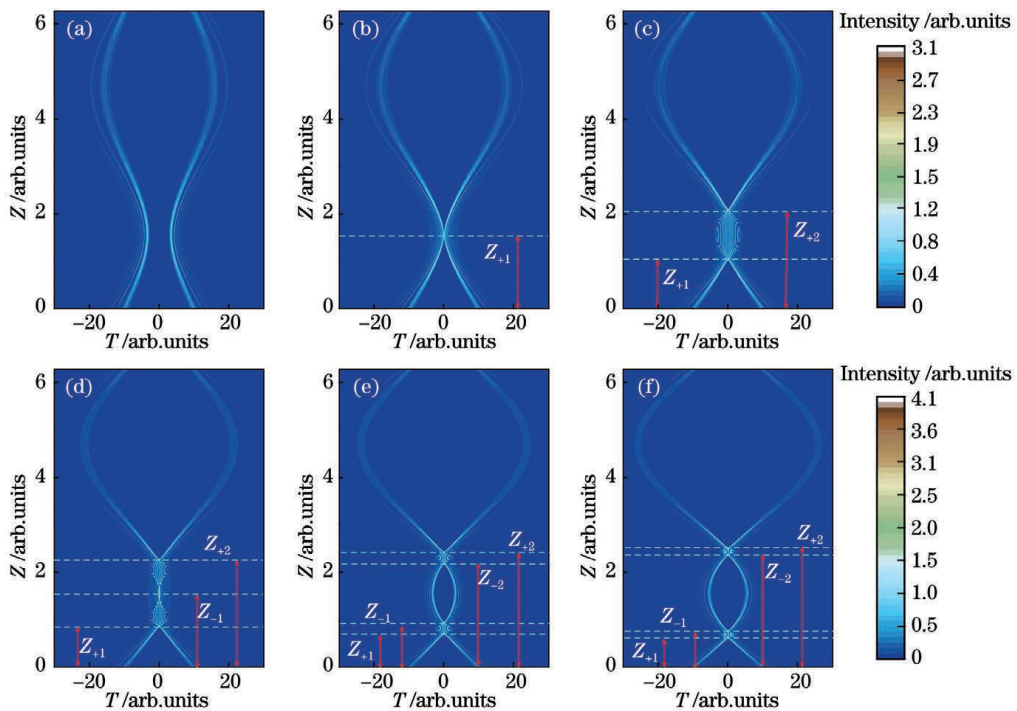
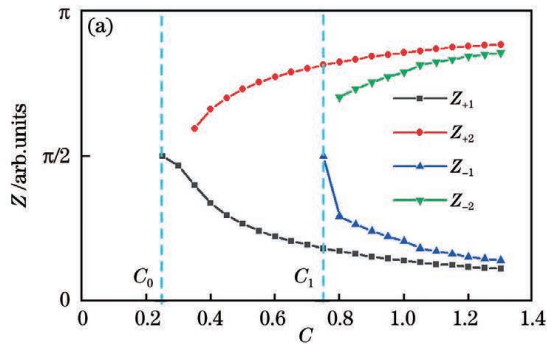


图 2 不同初始啁啾参量作用下对称艾里脉冲的传输演化图 ($d = 9, a = 0.05, A_0 = 1$)。 (a) $C = 0.3$; (b) $C = 0.4167$; (c) $C = 0.5$; (d) $C = 0.5834$; (e) $C = 0.7$; (f) $C = 0.8$

Fig. 2 Transmission evolution of symmetric Airy pulses under different initial chirp parameter values ($d = 9, a = 0.05, A_0 = 1$). (a) $C = 0.3$; (b) $C = 0.4167$; (c) $C = 0.5$; (d) $C = 0.5834$; (e) $C = 0.7$; (f) $C = 0.8$

脉冲之间的相互碰撞,控制脉冲产生焦点的数量:当 $C=0.3$ 时,脉冲在演化中不会出现焦点;随着啾啾参量增加,脉冲的传输轨迹开始发生变化,直到第一次发生碰撞,此时在脉冲的演化中仅可以观察到 1 个焦点,焦点位置为 Z_{+1} ;当 $C=0.5$ 时,脉冲的轨迹发生反转,脉冲在演化中出现 2 个焦点,焦点位置分别为 Z_{+1} 和 Z_{+2} ,两焦点之间形成梳状波区域;随着啾啾参量继续增大,脉冲的轨迹发生 2 次反转,此时脉冲的碰撞次数继续增加,直到产生 4 次碰撞,脉冲焦点个数不再发生变化,焦点位置依次为 Z_{+1} 、 Z_{-1} 、 Z_{-2} 和 Z_{+2} 。

啾啾参量对焦点的位置也会产生影响。图 3 给出了焦点位置随啾啾参量变化的示意图。结果显示:



当 $|C|=C_0$ 时开始出现焦点 ($C_0=0.25$);当 $|C|>C_1$ 时,出现了 4 个焦点 ($C_1=0.75$);焦点位置 Z_{+1} 和 Z_{-1} 随着 $|C|$ 的增大逐渐减小,而 Z_{+2} 和 Z_{-2} 随着 $|C|$ 的增大逐渐增大;当啾啾参量为正时,它们关于 $\frac{\pi}{2}$ 对称;当啾啾参量为负时,它们关于 $\frac{3\pi}{2}$ 对称,并随着传输距离增加周期性重复。可见,啾啾参量可以有效操控艾里脉冲碰撞产生焦点的数量以及焦点产生的位置,并且较大的啾啾参量能够显著提高艾里脉冲的聚焦特性。

图 4 为不同主瓣位置 d 下对称艾里脉冲的演化图,其中啾啾参量 C 分别为 0.4、0.5 及 0.6。为

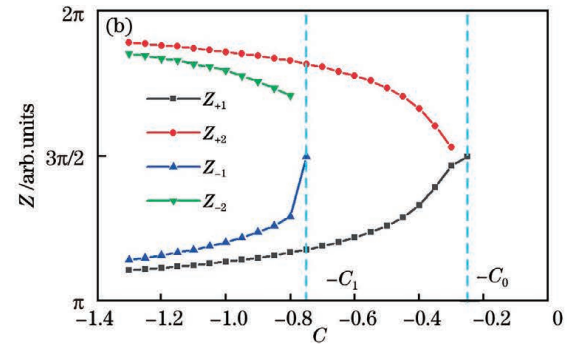


图 3 焦点位置随啾啾参量变化的示意图 ($d=1, a=0.05, A_0=1$)。 (a) $C>0$; (b) $C<0$

Fig. 3 Focus position changes with the chirp parameter ($d=1, a=0.05, A_0=1$). (a) $C>0$; (b) $C<0$

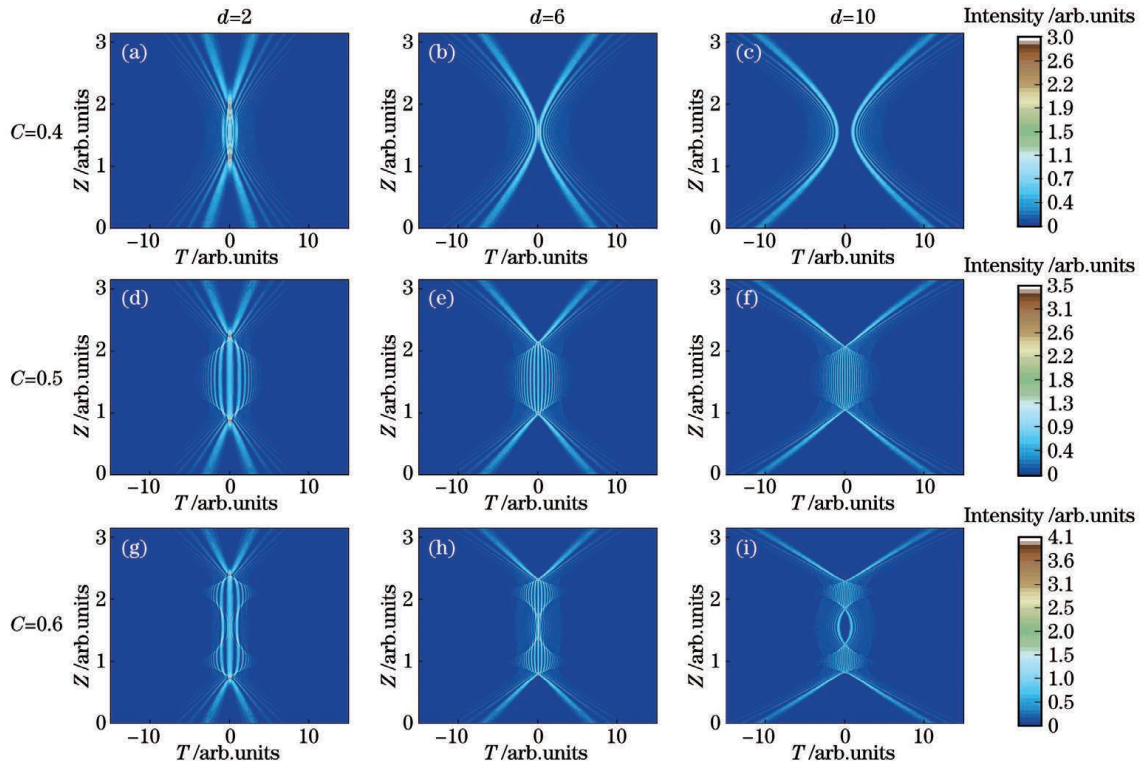


图 4 $C=0.4, 0.5, 0.6$ 时不同主瓣位置 d 下对称艾里脉冲的传输演化图 ($a=0.05, A_0=1$)。 (a)(d)(g) $d=2$;

(b)(e)(h) $d=6$; (c)(f)(i) $d=10$

Fig. 4 Transmission evolution of symmetric Airy pulses under different d values when C is 0.4, 0.5, and 0.6

($a=0.05, A_0=1$). (a)(d)(g) $d=2$; (b)(e)(h) $d=6$; (c)(f)(i) $d=10$

为了方便观察,将传输距离设置为 π 。图 4 表明,主瓣位置对脉冲演化的焦点数量和位置也有一定影响。当 $C=0.4$ 时,脉冲的演化不发生反转,而且当 d 较小时,艾里脉冲发生两次碰撞,随着 d 增大,艾里脉冲之间的距离逐渐增大,直到不再发生碰撞,没有焦点,呈现出对称的抛物轨迹,如图 4(a)~(c) 所示。当 $C=0.5$ 时,脉冲在演化中发生一次反转,主瓣位置对脉冲演化产生的焦点数量没有影响,但是对焦点的位置和梳状波的数量有一定影响,如图 3(d)~(f) 所示;随着参数 d 增加,两焦点之间的距离逐渐靠近,同时梳状波的数量增加。当 $C=0.6$ 时,脉冲反转 2 次,此时至少有 2 个焦点存在;随着 d 增加,反转波不断靠近,出现 3 个甚至 4 个焦点,如图 4(g)~

(i) 所示。因此,当 $|C| > 0.5$ 时,脉冲发生 2 次反转,通过改变主瓣位置 d 能够改变焦点的数量,并且最多出现 4 个焦点。数值模拟结果与理论分析结果一致。

此外,本文还研究了截断参数对艾里脉冲演化的影响。当艾里脉冲具有较大的截断参数时,能量会减少且主要集中在主瓣上,其独特特征迅速消失^[35],因此它的独特特征强烈地依赖于截断参数的值。图 5 给出了三种不同碰撞过程在 π 个传输距离的演化。对于不同的截断参数,脉冲传输过程中焦点的数量和位置没有发生变化;但随着截断参数增大,脉冲的碰撞会损失更多的能量。同时,由于能量的损失,梳状波的数量略有减少。

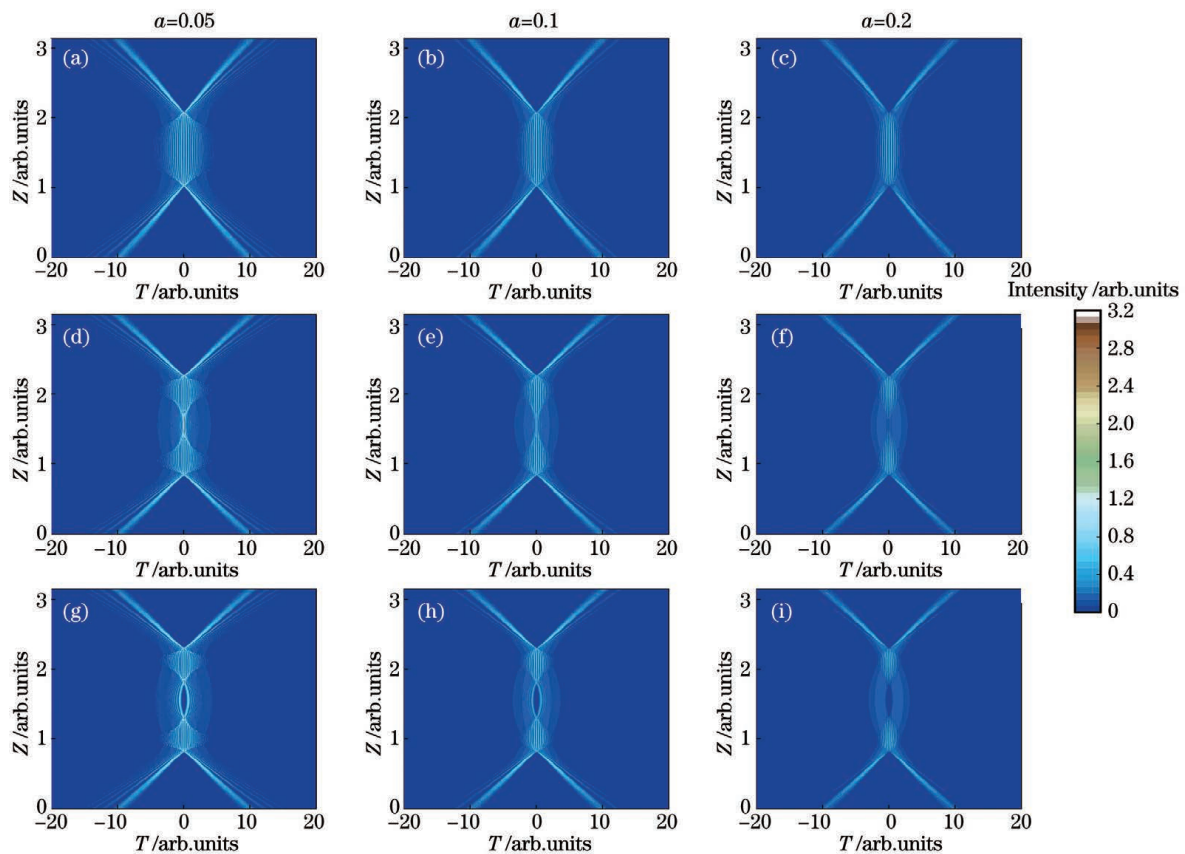


图 5 不同截断参数下对称艾里脉冲的演化过程($A_0=1$)。(a)(b)(c) $C=0.5, d=9$; (d)(e)(f) $C=0.5834, d=9$; (g)(h)(i) $C=0.6, d=9$

Fig. 5 Evolution of symmetric Airy pulses under different truncation parameter values ($A_0=1$). (a)(b)(c) $C=0.5, d=9$; (d)(e)(f) $C=0.5834, d=9$; (g)(h)(i) $C=0.6, d=9$

4 非线性克尔效应的影响

本节讨论非线性效应对艾里脉冲动力学演化特性的影响。图 6 显示了非线性系数 N 取不同值的情况下,脉冲在 π 个传输距离内的动力学特性,其中非线性系数计算公式如(3)式所示。当 $N < 2$ 时,非

线性自聚焦效应较弱,艾里脉冲演化基本没有很大变化。当 $N \geq 2$ 时,艾里脉冲演化发生了巨大变化;在这种情况下,非线性自聚焦效应起主导作用,对称的两个艾里脉冲进入光子脱落状态,在更强的非线性作用下,脱落轨迹受到旁瓣和色散背景连续碰撞的影响,导致多光子产生。 N 越大,脱落的光子数

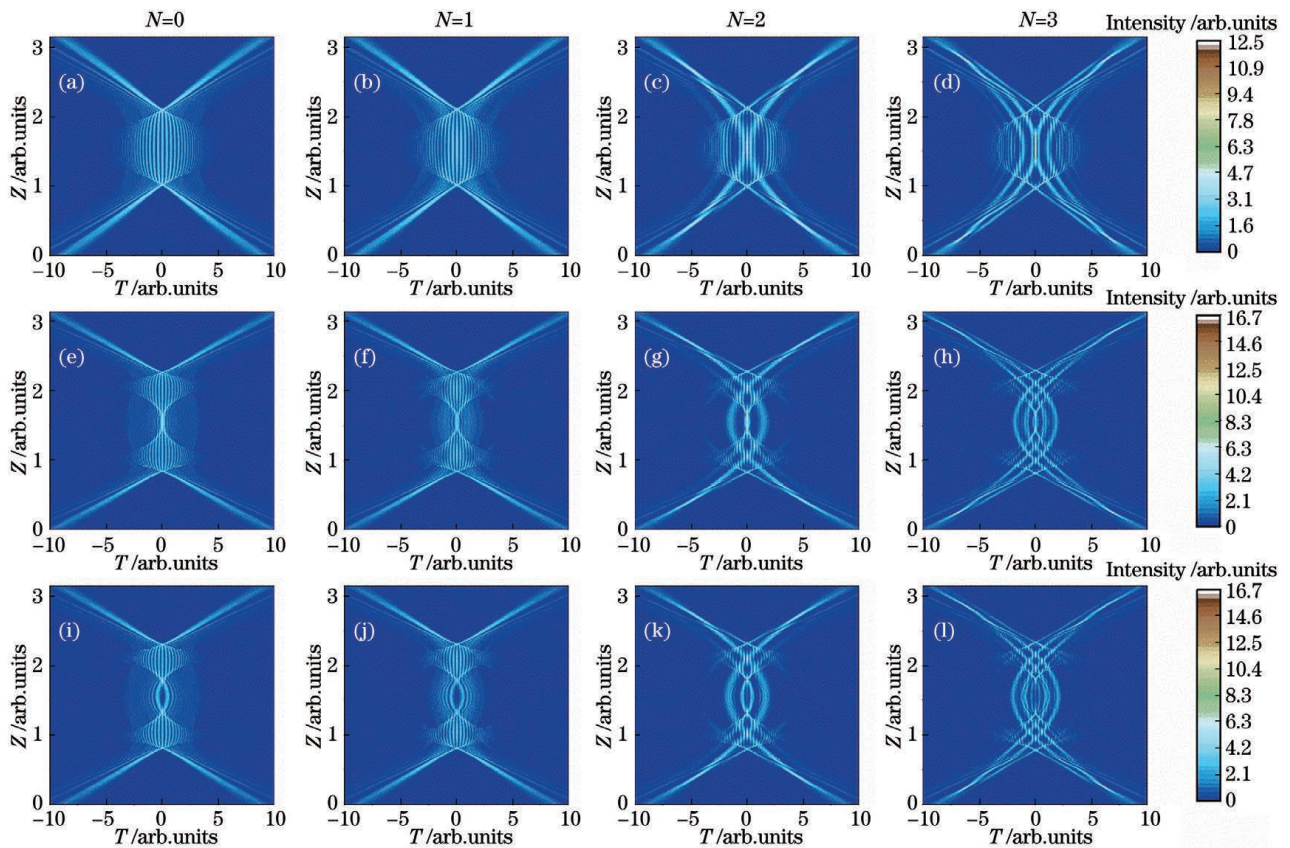


图 6 不同克尔非线性作用下对称艾里脉冲的传输演化图($a=0.05, A_0=2$)。(a)(b)(c)(d) $C=0.5, d=8$;
(e)(f)(g)(h) $C=0.5834, d=9$; (i)(j)(k)(l) $C=0.6, d=8$

Fig. 6 Transmission evolution of symmetric Airy pulses under different Kerr nonlinearities ($a=0.05, A_0=2$).

(a)(b)(c)(d) $C=0.5, d=8$; (e)(f)(g)(h) $C=0.5834, d=9$; (i)(j)(k)(l) $C=0.6, d=8$

越多,如图 6(d)、(h)、(l)所示。

为了进一步阐述以上现象,本文给出了 π 个传输距离内不同非线性系数(非线性系数 N 分别为 0、1、3)下峰值功率的演化图,如图 7 所示。从图 7 中可以看出:当非线性系数 N 较小($N=0$ 和 $N=1$)

时,由于脉冲的演化改变很小,脉冲的峰值功率与线性情况相近;当非线性系数 N 较大($N=3$)时,由于孤子的脱落以及脱落孤子的相互碰撞,峰值功率会出现多个峰值,且非线性系数越大,出现峰值的个数越多。

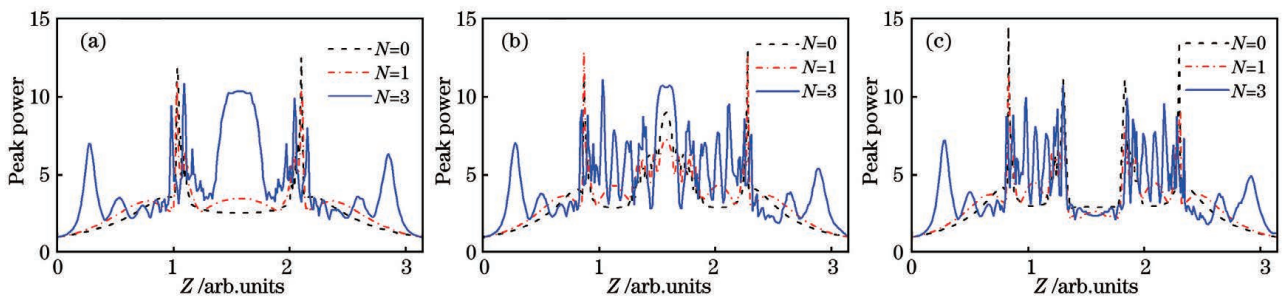


图 7 脉冲的峰值功率演化图($a=0.05, A_0=2$)。(a) $C=0.5, d=8$; (b) $C=0.5834, d=9$; (c) $C=0.6, d=9$

Fig. 7 Peak power evolution of pulse ($a=0.05, A_0=2$). (a) $C=0.5, d=8$; (b) $C=0.5834, d=9$; (c) $C=0.6, d=9$

5 结 论

本文基于变系数非线性薛定谔方程研究了周期色散调制下具有初始啁啾的对称艾里脉冲的动力学

特性。结果表明,啁啾参量 C 与主瓣位置 d 共同决定了对称艾里脉冲传输过程中碰撞产生的焦点的数量、位置。在一个色散周期的演化距离内,当 $|C| < 0.5$ 时,由于脉冲轨迹不发生反转,通过调节主瓣位

置 d 可以产生 1 个或 2 个焦点;当 $|C|=0.5$ 时,脉冲的轨迹会发生一次反转,脉冲在演化中会有 2 个焦点存在,主瓣位置 d 仅能影响焦点的位置,对焦点的数量没有影响;当 $|C|>0.5$ 时,脉冲的轨迹会反转 2 次,此时至少存在 2 个焦点,通过调节主瓣位置 d 能够增加焦点数量到 3 个或 4 个。此外,焦点的数量和位置与截断参数无关,但较大的截断参数在碰撞时会损失更多的能量。当考虑非线性作用时,在弱的非线性条件下,脉冲的传输特性基本没有发生改变。克尔非线性的进一步增加会引起孤子脱落,更强的非线性效应能够导致更多的孤子产生。因此,在周期色散调制下,调节啁啾参量和主瓣位置能够有效控制脉冲传输过程中焦点的个数及位置,与常数色散系统相比能够获得更加丰富的动力学特性。这种周期性的传输过程和聚焦特性为艾里脉冲自重构等性质的研究及应用提供了一定的理论指导。

参 考 文 献

- [1] Berry M V, Balazs N L. Nonspreading wave packets [J]. American Journal of Physics, 1979, 47(3): 264-267.
- [2] Siviloglou G A, Christodoulides D N. Accelerating finite energy Airy beams [J]. Optics Letters, 2007, 32(8): 979-981.
- [3] Siviloglou G A, Broky J, Dogariu A, et al. Observation of accelerating Airy beams [J]. Physical Review Letters, 2007, 99(21): 213901.
- [4] Wang L Y, Huang K K, Zhang X, et al. Manipulation of abruptly autofocusing property of circular Airy beam carrying hard-edge annular aperture [J]. Laser & Optoelectronics Progress, 2020, 57(23): 232601.
王灵一, 黄凯凯, 章显, 等. 硬边环带光阑下圆形艾里光束的自聚焦特性调控 [J]. 激光与光电子学进展, 2020, 57(23): 232601.
- [5] Dai C Y, Han H M, Fan Y P, et al. Generation of tunable Airy beams using deformable lens [J]. Chinese Journal of Lasers, 2020, 47(8): 0805002.
代超宜, 韩洪民, 范引鹏, 等. 基于可变形透镜的可调艾里光束生成 [J]. 中国激光, 2020, 47(8): 0805002.
- [6] Han H M, Fan Y P, Yang Y C, et al. Generation and propagation characteristics of multi-wavelength Airy beams based on deformable mirrors [J]. Acta Optica Sinica, 2021, 41(16): 1626001.
韩洪民, 范引鹏, 杨叶城, 等. 基于变形镜的多波长艾里光束生成与传播特性 [J]. 光学学报, 2021, 41(16): 1626001.
- [7] Siviloglou G A, Broky J, Dogariu A, et al. Ballistic dynamics of Airy beams [J]. Optics Letters, 2008, 33(3): 207-209.
- [8] Broky J, Siviloglou G A, Dogariu A, et al. Self-healing properties of optical Airy beams [J]. Optics Express, 2008, 16(17): 12880-12891.
- [9] Chu X X, Zhou G Q, Chen R P. Analytical study of the self-healing property of Airy beams [J]. Physical Review A, 2012, 85: 013815.
- [10] Cao R, Hua Y, Min C, et al. Self-healing optical pillar array [J]. Optics Letters, 2012, 37(17): 3540-3542.
- [11] Dolev I, Kaminer I, Shapira A, et al. Experimental observation of self-accelerating beams in quadratic nonlinear media [J]. Physical Review Letters, 2012, 108(11): 113903.
- [12] Li J X, Fan X L, Zang W P, et al. Vacuum electron acceleration driven by two crossed Airy beams [J]. Optics Letters, 2011, 36(5): 648-650.
- [13] Polynkin P, Kolesik M, Moloney J V, et al. Curved plasma channel generation using ultraintense Airy beams [J]. Science, 2009, 324(5924): 229-232.
- [14] Zhang P, Prakash J, Zhang Z, et al. Trapping and guiding microparticles with morphing autofocusing Airy beams [J]. Optics Letters, 2011, 36(15): 2883-2885.
- [15] Baumgartl J, Mazilu M, Dholakia K. Optically mediated particle clearing using Airy wavepackets [J]. Nature Photonics, 2008, 2(11): 675-678.
- [16] Zheng Z, Zhang B F, Chen H, et al. Optical trapping with focused Airy beams [J]. Applied Optics, 2010, 50(1): 43-49.
- [17] Abdollahpour D, Suntsov S, Papazoglou D G, et al. Spatiotemporal Airy light bullets in the linear and nonlinear regimes [J]. Physical Review Letters, 2010, 105(25): 253901.
- [18] Panagiotopoulos P, Papazoglou D G, Couairon A, et al. Sharply autofocused ring-Airy beams transforming into non-linear intense light bullets [J]. Nature Communications, 2013, 4: 2622.
- [19] Ren Z J, Wu Q, Zhou W D, et al. Spatially induced Airy-Bessel light bullets [J]. Acta Physica Sinica, 2012, 61(17): 174207.
任志君, 吴琼, 周卫东, 等. 空间诱导产生艾里-贝塞尔光弹研究 [J]. 物理学报, 2012, 61(17): 174207.
- [20] Rose P, Diebel F, Boguslawski M, et al. Airy beam induced optical routing [J]. Applied Physics Letters, 2013, 102(10): 101101.
- [21] Saari P. Laterally accelerating Airy pulses [J]. Optics Express, 2008, 16(14): 10303-10308.

- [22] Fattal Y, Rudnick A, Marom D M. Soliton shedding from Airy pulses in Kerr media[J]. Optics Express, 2011, 19(18): 17298-17307.
- [23] Zhang L F, Zhang J G, Chen Y, et al. Dynamic propagation of finite-energy Airy pulses in the presence of higher-order effects[J]. Journal of the Optical Society of America B, 2014, 31(4): 889.
- [24] Zhang K, Jia W G, Zhu K Z, et al. Study of interaction law between ultra-short Airy pulses in single mode fibers[J]. Acta Optica Sinica, 2016, 36(7): 0706006.
张魁, 贾维国, 朱坤占, 等. 超短艾里脉冲在单模光纤中相互作用规律的研究[J]. 光学学报, 2016, 36(7): 0706006.
- [25] Zhang L F, Huang P W, Conti C, et al. Decelerating Airy pulse propagation in highly non-instantaneous cubic media[J]. Optics Express, 2017, 25(3): 1856-1866.
- [26] Ament C, Polynkin P, Moloney J V. Supercontinuum generation with femtosecond self-healing Airy pulses[J]. Physical Review Letters, 2011, 107(24): 243901.
- [27] Han T W, Wang B, Lu P X. Accelerating self-imaging effect for Airy pulse trains[J]. Physical Review A, 2019, 99(5): 053807.
- [28] Zhang L, Zhang J, Xiao Y. Propagation of chirped Airy pulse and soliton in inhomogeneous optical fiber[J]. Chinese Journal of Lasers, 2017, 44(9): 0906005.
张露, 张健, 肖燕. 啾啾艾里脉冲和孤子在非均匀光纤中的传输[J]. 中国激光, 2017, 44(9): 0906005.
- [29] Wang S F, Fan D F, Bai X K, et al. Propagation dynamics of Airy pulses in optical fibers with periodic dispersion modulation[J]. Physical Review A, 2014, 89(2): 023802.
- [30] Banerjee A, Roy S. Dynamics and selective temporal focusing of a time truncated Airy pulse in varying dispersive media[J]. Journal of the Optical Society of America B, 2020, 37(12): 3819-3828.
- [31] Banerjee A, Roy S. Trajectory manipulation of an Airy pulse near zero-dispersion wavelength under a free-carrier-generated linear potential[J]. Physical Review A, 2019, 100(5): 053816.
- [32] Zhang L F, Fu X Q, Deng J Q, et al. Role of chirp in spatiotemporal modulation instability of broadband pulsed laser[J]. Journal of Optics, 2011, 13(1): 015203.
- [33] Kartazaev V, Alfano R R. Supercontinuum generated in calcite with chirped femtosecond pulses[J]. Optics Letters, 2007, 32(22): 3293-3295.
- [34] Park J, Lee J H, Nam C H. Laser chirp effect on femtosecond laser filamentation generated for pulse compression[J]. Optics Express, 2008, 16(7): 4465-4470.
- [35] Zhang L, Liu K, Zhong H, et al. Effect of initial frequency chirp on Airy pulse propagation in an optical fiber[J]. Optics Express, 2015, 23(3): 2566-2576.
- [36] Shi X H, Huang X W, Deng Y B, et al. Dynamic propagation of symmetric Airy pulses with initial chirps in an optical fiber[J]. Optics Communications, 2017, 399: 16-23.

Influence of Chirp on Focusing Characteristics of Airy Pulses in Nonuniform Fiber

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Abstract

Objective In 1979, the Airy wave packet was first found as a solution to the Schrödinger equation for a free particle. The energy of this solution is infinite; thus, it cannot be realized in practice. In 2007, finite-energy Airy beams (FEABs) were introduced and performed in an experiment for the first time. The tremendous potential application of the FEAB in many areas, such as optical trapping and manipulation, laser filamentation, and nonlinear optics, is due to its unique properties. As the temporal dispersive and spatial diffraction equations are isomorphic, the attributes of spatial Airy beams are directly translated to the corresponding temporal Airy pulses. Therefore, Airy pulses have been extensively studied. In practice, the laser system based on the chirped pulse amplification technology involves chirp in pulse generation, propagation, and amplification. Zhang et al. investigated the effect of initial chirp on Airy pulse propagation in an optical fiber. In addition, multipoint focusing on chirped symmetric Airy

pulses has been studied. These contents are all studied based on the constant dispersion coefficient. In this paper, the nonlinear Schrödinger equation with periodically varying dispersion is used as the theoretical model to investigate the interaction of chirped symmetric Airy pulses and show its new characteristics.

Methods The analytical solution of the linear Schrödinger equation with variable coefficient is obtained using a Fourier transform method and performing complex calculations. The focusing characteristics of the symmetric Airy pulses with an initial chirp are analyzed and discussed based on the solution. We also simulate the linear and nonlinear propagation of Airy pulses in the optical fiber using the well-known split-step Fourier method to confirm the propagation dynamics of the symmetric Airy pulses obtained from the analytical analysis.

Results and Discussions The analytical and numerical results show that the symmetric Airy pulses exhibit a periodic transmission trajectory under periodic dispersion modulation. Additionally, the linear propagation largely depends on the chirp parameters and the position of the main lobe. In one dispersion period, there is no focus in the evolution of the pulse when $4C + 1/\sqrt{d} < 2$; there is one focal point when $4C + 1/\sqrt{d} = 2$; there are two focal points when $4C + 1/\sqrt{d} > 2$ and $4C - 1/\sqrt{d} < 2$; there are three focal points when $4C - 1/\sqrt{d} = 2$; there are four focal points when $4C - 1/\sqrt{d} < 2$ (Fig. 2). The focus position during the collision of the Airy pulses can also be controlled by adjusting the chirp parameter and the initial position of the main lobe of the pulse. When $|C| > C_1$, four focal points appear. The focus positions Z_{+1} and Z_{-1} decrease gradually with the increase in $|C|$; Z_{+2} and Z_{-2} increase gradually with the increase in $|C|$. When $C > 0$, they are symmetrical about $\frac{\pi}{2}$; when $C < 0$, they are symmetrical about $\frac{3\pi}{2}$, and it repeats periodically with the transmission distance (Fig. 3). The effects of truncation parameter and Kerr nonlinearity on the propagation of the chirped Airy pulses are also discussed. It shows that the truncation parameter does not affect the number and the position of the focal points in pulse transmission (Fig. 5). However, as the truncation parameter increases, the collision of the pulse loses more energy. The Kerr nonlinearity effect will cause solitons shedding, and the stronger nonlinearity will generate multiple solitons (Fig. 6).

Conclusions Based on the variable coefficient nonlinear Schrödinger equation, this paper investigates the dynamic characteristics of the symmetric Airy pulses with an initial chirp under periodic dispersion modulation. The results show that the chirp parameter C and the main lobe position d determine the number and positions of focal points generated by the collision during the transmission of the chirped symmetric Airy pulses. Within the evolution distance of a dispersion cycle, when $|C| < 0.5$, one or two focal points can be generated by adjusting the position d of the main lobe as the pulse trajectory does not reverse. When $|C| = 0.5$, the trajectory of the pulse will be reversed once, and there will be two focal points in the evolution of the pulse. The main lobe position d can only affect the position of the focal point; it does not affect the number of focal points. Furthermore, when $|C| > 0.5$, the trajectory of the pulse will be reversed twice, and there are at least two focal points. Moreover, the number of focal points can be increased to three at most four by adjusting the main lobe position d . Additionally, the truncation parameter does not affect the number and the position of the focal points. However, a larger truncation parameter will lose more energy under collision. When the Kerr nonlinear effect is considered, the transmission characteristics of the pulse are nearly unchanged under the small nonlinear effect. As the nonlinearity further increases, it causes soliton shedding. Stronger nonlinear effect can also lead to the generation of multiple solitons and the collision of solitons with side lobes. This periodic transmission process and focusing characteristics provide certain theoretical guidance for the research and the application of properties, such as the self-reconstruction of Airy pulses.

Key words nonlinear optics; nonuniform fiber; symmetric Airy pulses; chirp parameter; pulse main lobe; focusing