

夏克-哈特曼波前传感器数字仿真平台的设计和验证

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摘要 夏克-哈特曼波前传感器是自适应光学系统的重要组成部分,其检测精度直接影响着整个系统的性能。数值模拟技术具有成本低、易于实现等优点,因此,采用数值模拟技术建立了夏克-哈特曼波前传感器的仿真平台,并对其进行了测试。测试结果表明,该仿真平台能很好地表征湍流相位的统计特征,并根据自适应光学系统的工作环境优选光斑定位算法及参数,同时处理特殊情况(如子孔径缺光),以保证波前复原的准确性。此外,该仿真平台可以分析不同畸变波前和噪声水平下的光斑定位误差和波前复原误差,为进一步研究和优化自适应光学系统参数以及提高整个系统的检测精度和性能提供了一个非常有用的工具。

关键词 传感器; 傅里叶光学; 夏克-哈特曼波前传感器; 数值模拟; 畸变波前; 噪声模拟

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1 引言

激光在传输中经过大气湍流时引起的波前畸变限制了光学系统的分辨率,自适应光学系统通过对畸变波前的探测与校正保证良好的成像效果^[1]。夏克-哈特曼波前传感器在自适应光学系统的波前探测中具有广泛应用^[2-4],而畸变波前的测量一直是光学领域的重要课题。

数值模拟技术具有成本低、易于实现等优点,是学术研究与工程应用中常用的技术手段。数值模拟技术可用于自适应光学系统的性能分析、参数设计和优化,也可以仿真工程中不易或不能实现的实验,且具有重复性,在大气湍流的研究中得到了广泛应用^[5-7]。因此,本文建立了夏克-哈特曼波前传感器数值仿真平台并对其进行了测试验证。该仿真平台主要包括波前产生、波前探测、波前复原三方面,囊括了多种畸变波前、多种子孔径结构、多种噪声的模

拟以及多种光斑定位算法和波前复原方法,可根据实际需求调整待探测的任意波前像差、大气湍流波前的湍流参数、哈特曼波前探测器的结构参数(CCD靶面的像素数、子孔径排布方式、子孔径数、子孔径像素数、衍射光斑所占像素数等)、背景与噪声的种类和强度、算法种类及参数。

2 理论分析与数值模拟

夏克-哈特曼波前传感器的数字仿真平台流程如图 1 所示,其中,FFT 表示快速傅里叶变换。首先,产生分析所需的畸变波前。然后,模拟夏克-哈特曼波前传感器的工作过程以获得光斑图像;对光斑图像进行预处理并根据信噪比(SNR)选取最优算法,以得到各子孔径的斜率。最后,选择合适的波前复原算法得到待探测的畸变波前。该过程中如果出现 SNR 极低或缺光等无法探测的情况则需采取特殊的处理方法,以保证整个自适应光学系统的稳定运行。

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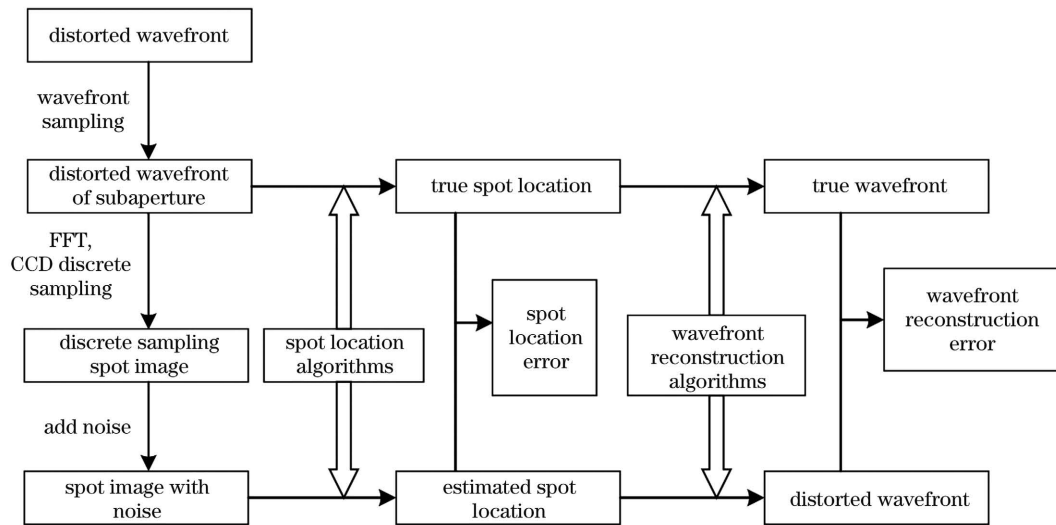


图 1 夏克-哈特曼波前传感器的数字仿真平台流程图

Fig. 1 Flow chart of the digital simulation platform for Shack-Hartmann wavefront sensor

2.1 波前产生

夏克-哈特曼波前传感器的数字仿真平台中已经完善的波前包括经过大气湍流的畸变波前、动态随机波前、函数调制波前、静态波前、平面波前(振幅为 1)。

Kolmogorov 湍流随机相位屏的产生方法有功率谱反演法、Zernike 多项式法、随机中点位移法等。实验采用 Zernike 多项式法产生相位屏^[8],该方法产生的随机相位屏在时间维度具有非相关性,可用于多帧图像数据的统计分析;且其低空间频率成分与理论值相符,高空间频率成分随 Zernike 阶数的

增加逐渐改善。仿真中采用 Karhunen-Loeve(K-L)函数快速构造波前,受大气湍流影响的波前 $\varphi(r)$ 可表示为服从一定已知模式和统计规律的二维函数,即 Zernike 多项式的二维形式

$$\varphi(r) = \sum_{j=1}^{\infty} a_j Z_j(r), \quad (1)$$

式中, $Z_j(r)$ 为 Zernike 多项式的项, a_j 为各项对应的系数, j 为阶数。Noll^[9] 从能量的角度出发推导出了任意两项 Zernike 多项式系数的协方差 $E(a_j, a_{j'})$, 可表示为

$$E(a_j, a_{j'}) = \frac{K_{zz'} \delta_z \Gamma[(n+n'-5/3)/2](D/r_0)^{5/3}}{\Gamma[(n-n'+17/3)/2]\Gamma[(n'-n+17/3)/2]\Gamma[(n+n'+23/3)/2]}, \quad (2)$$

式中, n, n' 分别为 Z_j 与 $Z_{j'}$ 的径向频率数, a_j 与 $a_{j'}$ 为 Zernike 多项式的任意两项系数, D 与 r_0 分别为

望远镜的口径和大气相干长度, Γ 为伽马函数, δ_z 与 $K_{zz'}$ 可表示为

$$\delta_z = (m = m') \wedge [X_{\text{parity}}(j, j')] \vee (m = 0), \quad (3)$$

$$K_{zz'} = \frac{\Gamma(14/3)[(24/5)\Gamma(6/5)]^{5/6}[\Gamma(11/6)]^2}{2\pi^2} \times (-1)^{(n+n'-2m)/2} \sqrt{(n+1)(n'+1)}, \quad (4)$$

式中, m 和 m' 分别为 Z_j 和 $Z_{j'}$ 的角向频率数, $X_{\text{parity}}(j, j')$ 表示阶数 j, j' 的多项式奇偶性相同, \wedge 为逻辑与符号, \vee 为逻辑或符号。由(2)式~(4)式可得到任意阶 Zernike 多项式系数的协方差矩阵,即各项 Zernike 多项式系数之间具有相关性。为了获得湍流波前,需对 Zernike 多项式进行转换。仿真中利用 K-L 函数展开波前,该多项式系数统计

独立,通过转换可获得 Zernike 多项式系数矩阵。

不符合湍流统计规律的随机波前可由(1)式即 Zernike 多项式直接产生,其 Zernike 系数是在 $(-1, 1)$ 之间均匀分布的伪随机数,帧与帧之间的 Zernike 系数是随机独立的, Zernike 多项式阶数可任意设定。

函数调制波前是通过函数调制某部分像差的大

小,如用随时间变化的 \sin 函数调制倾斜像差,是在 X, Y 方向添加随时间变化的 \sin 函数,即对 Zernike 多项式的第 2、3 阶系数进行随时间变化的 \sin 函数调制。静态波前也能通过 Zernike 多项式产生,可用于分析光斑图像的 SNR,且 Zernike 多项式阶数可任意设定。添加噪声时,保持波前不变,噪声随机变化。

2.2 夏克-哈特曼波前传感器的仿真

当波前通过夏克-哈特曼波前传感器时,经过微透镜阵列分割采样,将每个子透镜作为一个子孔径并将光束聚焦到 CCD 上形成光斑阵列图像。当平面波 $U_1(x, y)$ 垂直入射到微透镜时,可将微

透镜作为相位变换器,经微透镜后的振幅分布可表示为

$$U_2(x, y) = U_1(x, y)p(x, y)\exp\left[-j\frac{k}{2f}(x^2 + y^2)\right], \quad (5)$$

$$p(x, y) = \begin{cases} 1, & \text{within the aperture of the micro-lens} \\ 0, & \text{others} \end{cases}, \quad (6)$$

式中, k 为波数, f 为焦距, (x, y) 为对应位置的坐标。利用菲涅耳衍射公式推导出焦平面 CCD 上 (u, v) 处的强度分布为

$$I_f(u, v) = \frac{1}{\lambda^2 f^2} \left| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U_1(x, y)p(x, y)\exp[-j2\pi(f_x x + f_y v)] dx dy \right|^2 = \frac{1}{\lambda^2 f^2} |\mathcal{F}\{U_1(x, y)p(x, y)\}|^2, \quad (7)$$

式中, $f_x = u/(\lambda f)$, $f_y = v/(\lambda f)$, λ 为波长, \mathcal{F} 为 FFT 符号。可以发现,对入射波进行 FFT 就能得到焦平面上的强度分布。根据实际光斑阵列图像中的信号光子噪声、背景光子噪声、CCD 相机噪声的分布类型,可在仿真中添加高斯随机噪声、泊松噪声。此外,考虑到系统中遇到的特殊情况,可选择添加某部分强光背景、椒盐噪声。通过调整信号光强度或噪声水平,可获得不同 SNR 的图像。

哈特曼图像具有信号分布集中的特点,文献 [10] 中分析了三种适合哈特曼图像的 SNR 定义,包括总能量 SNR、均值 SNR、峰值 SNR。哈特曼单孔径图像的 SNR 可表示为

$$\begin{cases} R_{SN}^m = (\sum_{u=1}^N \sum_{v=1}^N I_{uv} - N^2 \mu_n) / (N^2 \sigma_n) \\ R_{SN}^p = (I_{\max} - \mu_n) / \sigma_n \\ R_{SN}^e = 20 \log \left[(\sum_{u=1}^N \sum_{v=1}^N I_{uv} - N^2 \mu_n) / \sigma_n \right] \end{cases}, \quad (8)$$

式中, R_{SN}^m 、 R_{SN}^p 、 R_{SN}^e 分别为子孔径的均值 SNR、峰值 SNR、能量 SNR, I_{uv} 为像素点 (u, v) 处的灰度值, I_{\max} 为子孔径内的灰度值最大值, μ_n 为子孔径内的噪声均值, σ_n 为子孔径内噪声的均方根值, N 为近似艾里斑的宽度。

光斑阵列的定位准确度与精度是夏克-哈特曼波前传感器探测技术的关键,因此,在仿真中需要根

据不同的 SNR 水平自动选取最优算法进行光斑定位。仿真中涉及的光斑定位算法包括阈值质心法 (T-COG)、阈值加权质心 (T-WCOG)、阈值幂指数质心 (T-IWC)、加窗预处理算法、互相关因子法 (CORR)^[10], 同时可通过仿真平台设置光斑定位算法涉及到的各个参数^[11]。

2.3 波前复原方法

自适应光学中常用的波前复原方法有区域法、Zernike 模式法、直接斜率法,由于只需考虑前端哈特曼探测部分,因此采用区域法与 Zernike 模式法进行波前复原。

Zernike 模式波前复原方法可表示为 $\mathbf{A} = \mathbf{D}^+ \mathbf{G}$, 即

$$\begin{bmatrix} a_1 \\ \vdots \\ a_q \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1q} \\ \vdots & & \vdots \\ Z_{2p,1} & \cdots & Z_{2p,q} \end{bmatrix}^+ \begin{bmatrix} g_1 \\ \vdots \\ g_{2p} \end{bmatrix}, \quad (9)$$

式中, \mathbf{A} 为 Zernike 系数列向量, \mathbf{D} 为复原矩阵, $Z_{2p,q}$ 为 Zernike 函数偏导数,可根据夏克-哈特曼波前传感器的结构参量获得, \mathbf{D}^+ 为 \mathbf{D} 的广义逆矩阵, \mathbf{G} 为各子孔径 X, Y 方向的斜率列向量, p 与 q 分别为子孔径数和 Zernike 多项式的阶数。

Southwell 模型如图 2 所示,相邻栅格点的相位差与相邻栅格点间中点的斜率对应。区域法波前复原方法可表示为

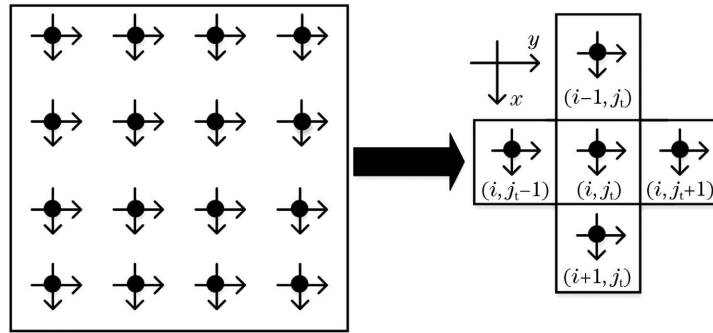


图 2 Southwell 模型
Fig. 2 Southwell model

$$\begin{cases} \frac{1}{2}(S_{i+1,j_t}^x + S_{i,j_t}^x) = \frac{1}{h}(\rho_{i+1,j_t} - \rho_{i,j_t}), & i = 1, \dots, M-1, j_t = 1, \dots, M \\ \frac{1}{2}(S_{i+1,j_t}^y + S_{i,j_t}^y) = \frac{1}{h}(\rho_{i,j_t+1} - \rho_{i,j_t}), & i = 1, \dots, M, j_t = 1, \dots, M-1 \end{cases}, \quad (10)$$

式中, S 为栅格点间的斜率, ρ 为栅格点的相位, $M \times M$ 为待估计相位点, $h = D/M$ 。(10)式可以通过最小二乘法解出待估计相位点。此外,当某部分子孔径的 SNR 极低或缺光导致无法对光斑进行准

确定位时,可根据系统的性能和需求对这部分子孔径采取子孔径斜率置零复原法和子孔径去除复原法^[12]。

波前复原误差(E_{WR})可表示为

$$\begin{cases} E_{WR} = \sqrt{\sum_{i=1}^{N_w} \sum_{j=1}^{N_w} [(W_{rij} - W_{oij}) \cdot A_{ij} - u_w]^2 / N_A} \\ u_w = \sum_{i=1}^{N_w} \sum_{j=1}^{N_w} (W_{rij} - W_{oij}) \cdot A_{ij} / N_A \\ A_{ij} = \begin{cases} 1, & \text{effective point} \\ 0, & \text{invalid point} \end{cases} \end{cases}, \quad (11)$$

式中, W_{oij} 、 W_{rij} 分别为原始波前像差与重构波前像差, N_w 为波前像差图像宽度所占的像素数, N_A 为有效孔径内的计算像素数。

2.4 仿真平台结构与界面

夏克-哈特曼波前传感器仿真平台的结构如图 3 所示,包括光斑阵列图、光斑定位算法、波前复原方法。其中,WT 为窗口阈值,光斑阵列图的产生包含波前生成、噪声添加、传感器结构参数的设置以及光强调制。夏克-哈特曼传感器仿真平台的控制界面如图 4 所示,其中,图 4(a)为模拟波前的控制界面,可生成平面波前、静态波前、随机波前、调制波前以及湍流畸变波前;图 4(b)为光斑定位的控制界面,可控制子孔径结构、光斑阵列图以及光斑定位方法;图 4(c)为波前复原的控制界面,包含波前复原矩阵与波前复原误差的计算。

3 仿真平台验证

夏克-哈特曼波前传感器的结构参数可根据实验系统或需求设置,仿真平台可根据这些参数自动划分检测区域,再根据相机焦距调整子孔径衍射光斑所占的像素数,图 5 为 3 类子孔径布局的分布。第 1 类 CCD 靶面的尺寸为 240 pixel \times 240 pixel,子孔径数为 10 \times 10,子孔径的尺寸为 22 pixel \times 22 pixel,如图 5(a)所示;第 2 类 CCD 靶面的尺寸为 200 pixel \times 200 pixel,子孔径数为 12 \times 12,子孔径的尺寸为 16 pixel \times 16 pixel,如图 5(b)所示;第 3 类 CCD 靶面的尺寸为 240 pixel \times 240 pixel,子孔径数为 14 \times 14,子孔径的尺寸为 16 pixel \times 16 pixel,如图 5(c)所示。

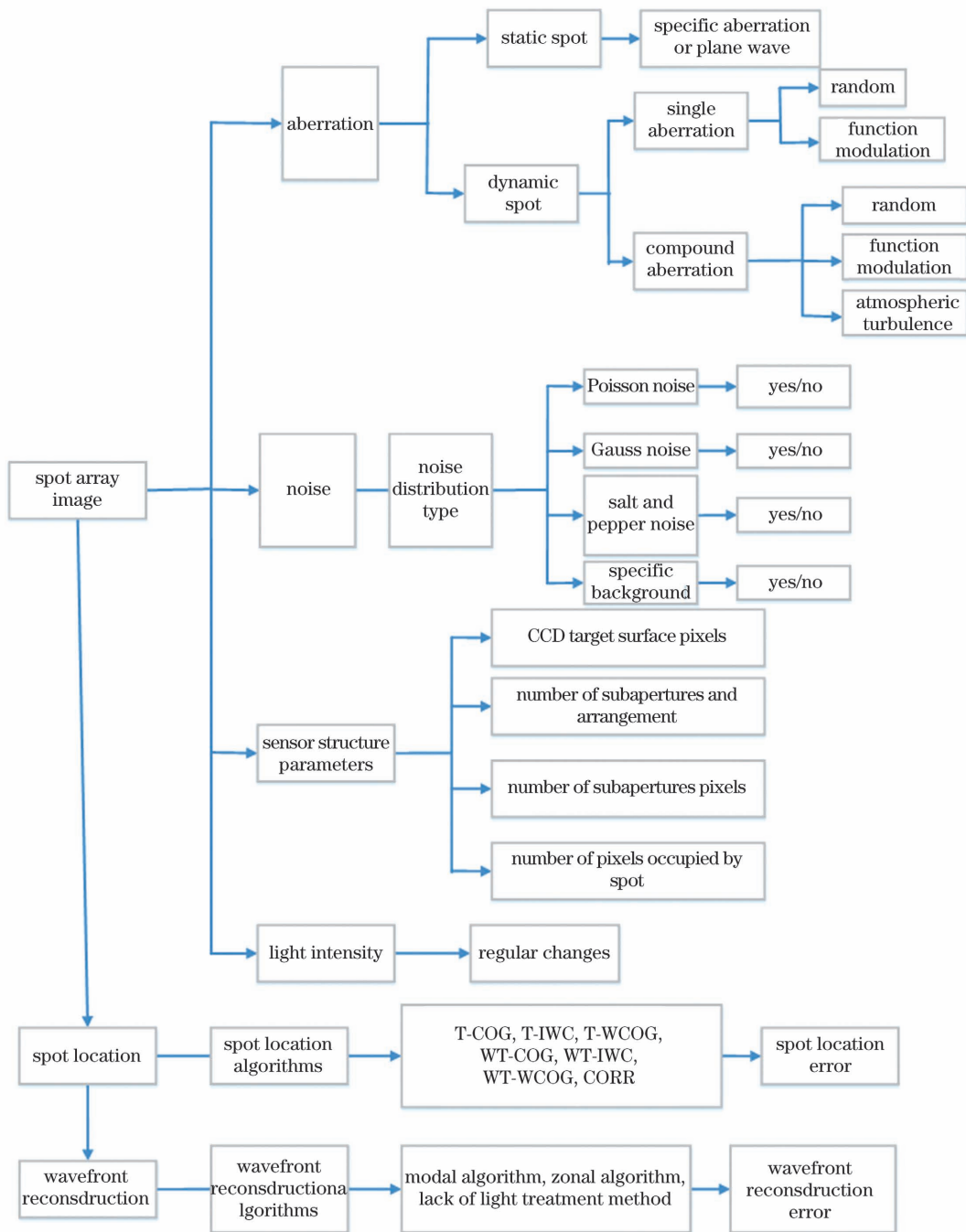


图 3 夏克-哈特曼波前传感器仿真平台结构

Fig. 3 Structure of the simulation platform for Shack-Hartmann wavefront sensor

在仿真图像中添加各类噪声的示意图如图 6 所示,可以发现,各类噪声涉及的参数可根据需要任意调节。图 6 仅添加了单一噪声,还可以根据实际需求任意选择多种噪声叠加,其中,ADU 为灰度值。

为了验证哈特曼传感器仿真平台的测试结果,设置 Kolmogorov 湍流波前的仿真参数为 65 阶 Zernike 多项式,望远镜的口径 $D = 120$ cm,大气相干长度 $r_0 = 12$ cm。单帧图像的尺寸为 $200 \text{ pixel} \times$

200 pixel ,光瞳直径为 210 pixel ,子孔径的排布方式为 12×12 ,子孔径尺寸为 $16 \text{ pixel} \times 16 \text{ pixel}$ 。波前复原方法采用模式法复原且考虑到噪声的影响,复原阶数为 60 阶。哈特曼传感器仿真过程中的典型图像如图 7 所示,Kolmogorov 湍流波前的理论方差与模拟方差如图 8 所示。可以发现,湍流波前的模拟方差与理论方差一致,符合 Kolmogorov 湍流的统计规律。

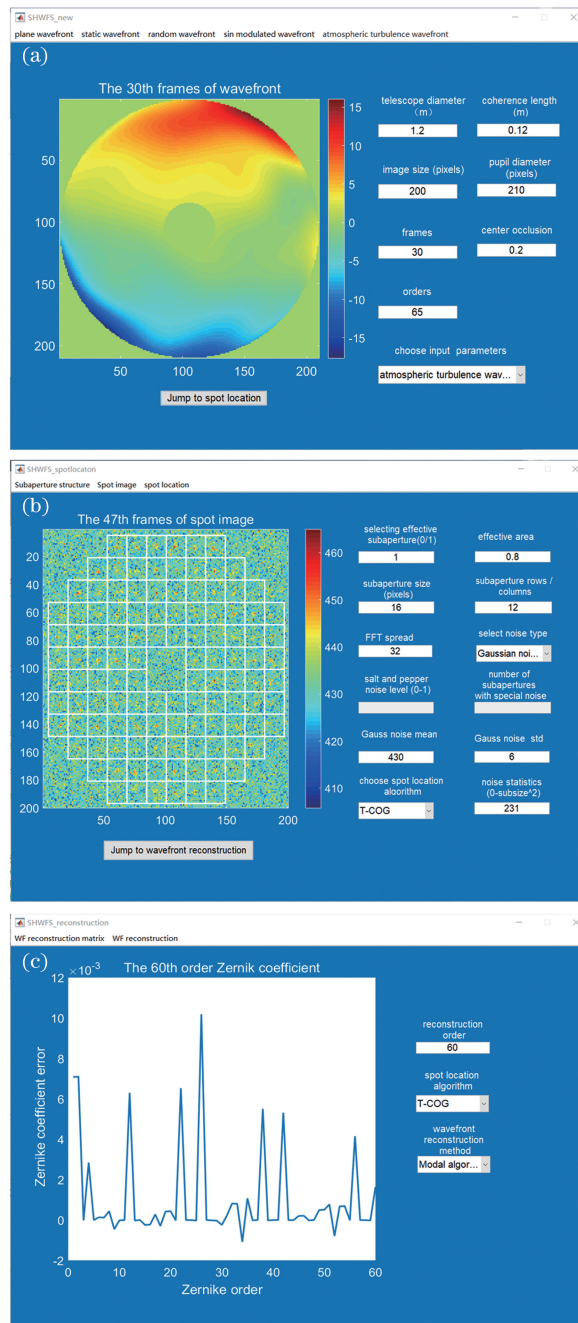


图 4 夏克-哈特曼波前传感器仿真平台的控制界面。(a)模拟波前控制界面;(b)光斑定位控制界面;(c)波前复原控制界面
 Fig. 4 Control interface of the Shack-Hartmann wavefront sensor simulation platform. (a) Control interface of the simulation wavefront; (b) control interface of the spot location; (c) control interface of the wavefront reconstruction

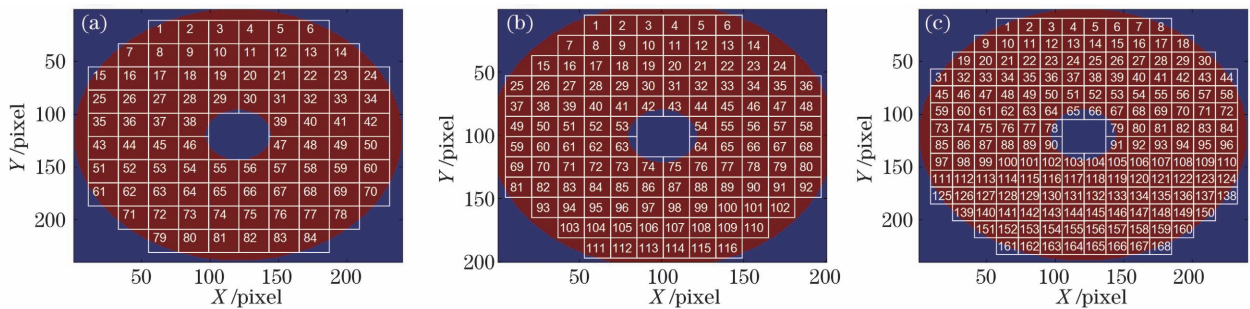


图 5 子孔径分布。(a)类别 1;(b)类别 2;(c)类别 3

Fig. 5 Distribution of sub-apertures. (a) Category 1; (b) category 2; (c) category 3

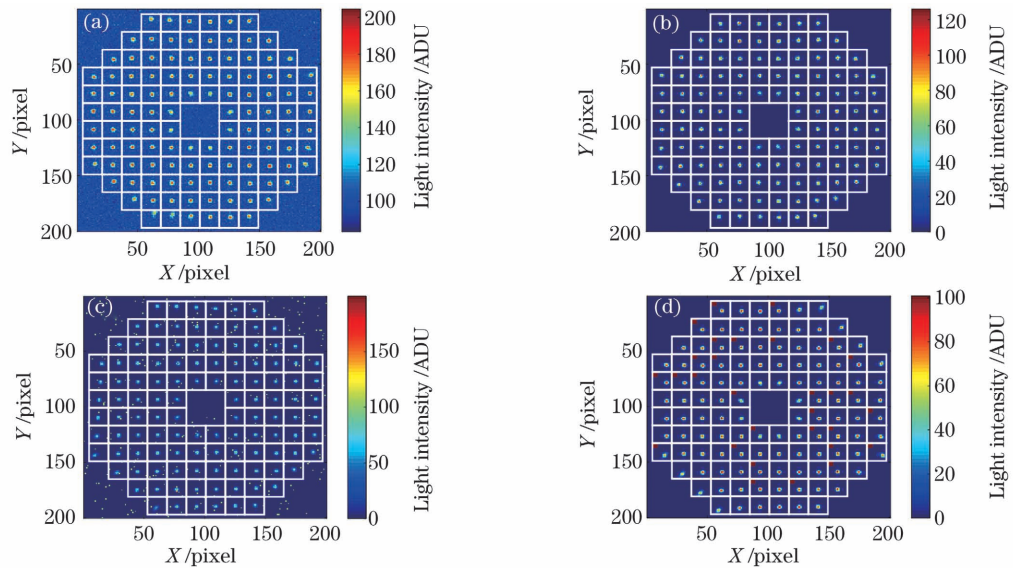


图 6 不同噪声的添加。(a)高斯噪声;(b)泊松噪声;(c)椒盐噪声;(d)特殊噪声

Fig. 6 Addition of different noises. (a) Gaussian noise; (b) Poisson noise; (c) salt and pepper noise; (d) special noise

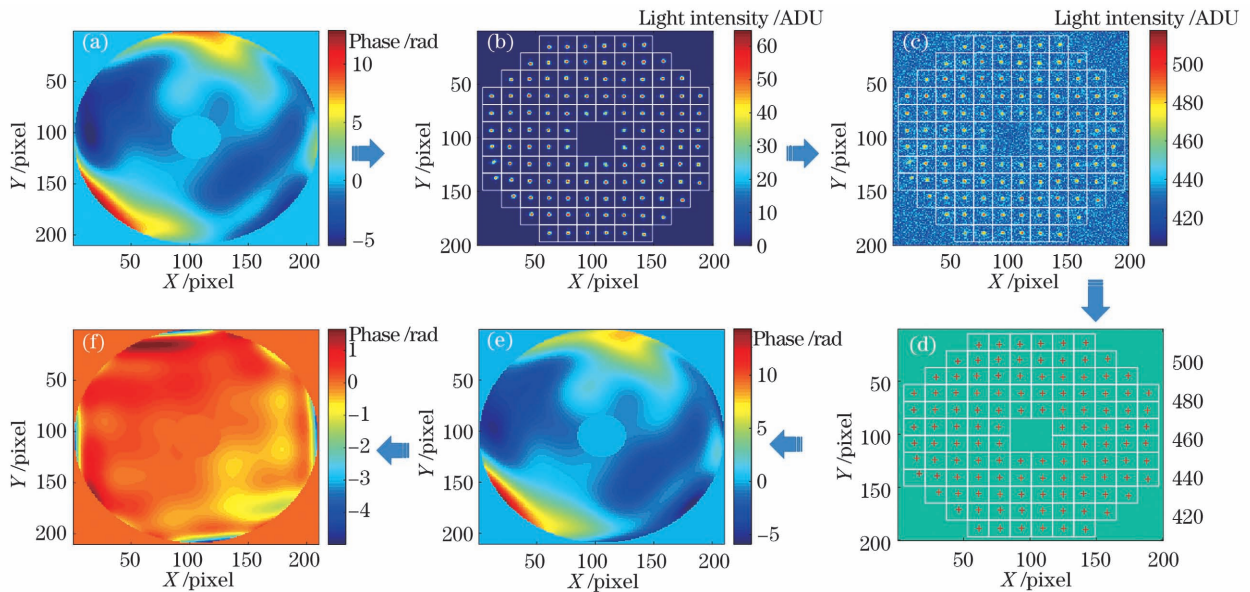


图 7 夏克-哈特曼波前传感器的仿真过程。(a)畸变波前;(b)无噪声的光斑图像;(c)有噪声的光斑图像;(d)光斑定位;(e)波前复原;(f)波前残差

Fig. 7 Simulation process of the Shack-Hartmann wavefront sensor. (a) Distorted wavefront; (b) noise-free spot image; (c) noisy spot image; (d) spot location; (e) wavefront reconstruction; (f) wavefront residual

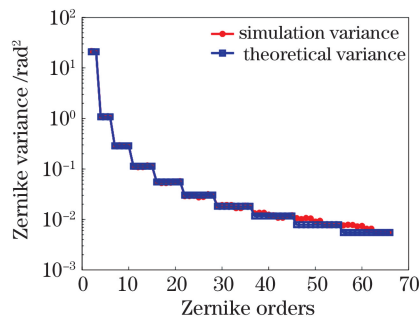


图 8 Kolmogorov 湍流波前的理论方差与模拟方差
Fig. 8 Theoretical and simulated variances of the Kolmogorov turbulent wavefront

采用 T-COG 算法进行光斑定位并得到光斑定位误差,光斑定位误差曲线如图 9(a)所示。其中,横坐标 SNR_m 表示各子孔径 SNR 的均值。可以发现,光斑定位误差随 SNR 的降低而增大,在低 SNR 时误差很大,不能进行准确定位。因此,仿真中集成了多种光斑定位算法,以根据需求选取最合适的算法。图 9(b)为以 100 帧图像为一个周期的 sin 函数调制波前的质心计算结果,可以发现,质心运动轨迹很好地服从 sin 函数分布,验证了光斑定位的准确性。

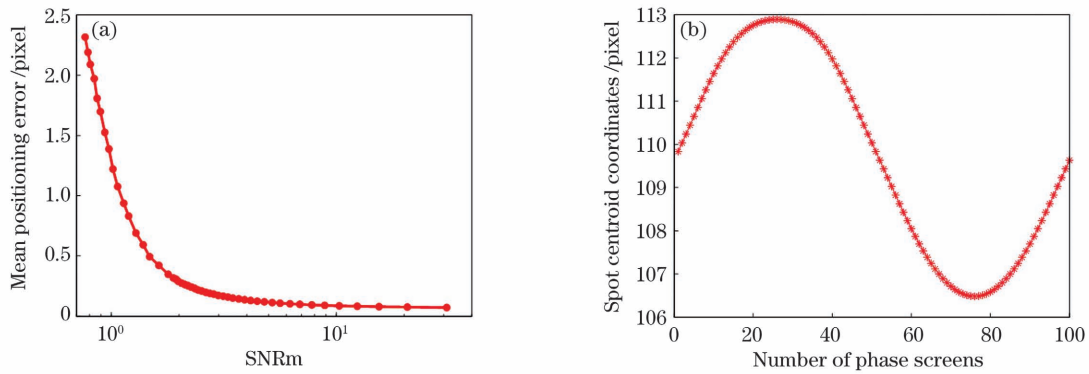


图 9 光斑定位的验证结果。(a)不同 SNR 下的平均定位误差;(b)调制后的光斑质心运动轨迹

Fig. 9 Verification result of spot positioning. (a) Average positioning error under different SNR; (b) modulated light spot centroid movement trajectory

1000 帧光斑阵列图像无噪声时的模式法波前复原误差如图 10(a)所示,不同 SNR 水平下波前复原的误差如图 10(b)所示。可以发现,无噪声的波

前复原误差精度达到 10^{-2} ,且波前复原误差随 SNR 的降低而增大,与图 9(a)中的光斑定位误差变化趋势一致,验证了波前复原的准确性。

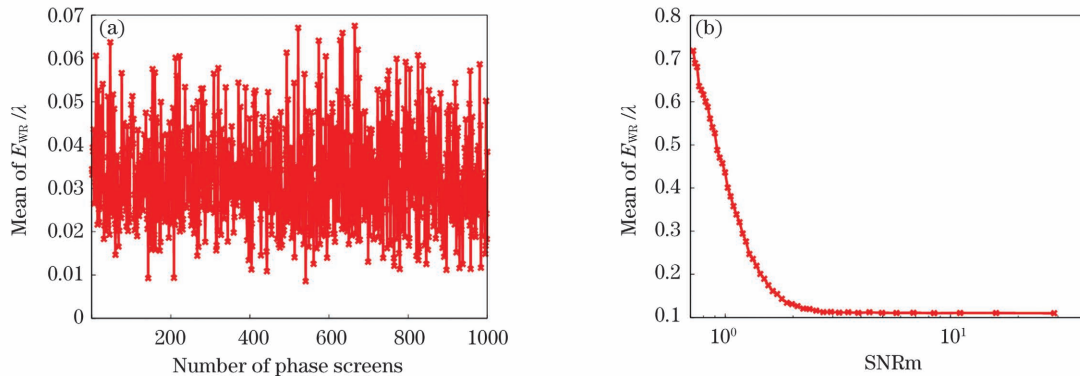


图 10 波前复原的验证。(a)无噪声时波前误差;(b)不同 SNR 下的波前复原误差

Fig. 10 Verification of wavefront reconstruction. (a) Wavefront reconstruction error with no noise; (b) wavefront reconstruction error under different SNR

4 结 论

建立了夏克-哈特曼波前传感器仿真平台,包括生成光斑阵列图像、光斑定位、波前复原。其中,光斑阵列图像的生成主要包含波前生成、噪声添加以及传感器结构参数、光强调制。验证结果表明,该仿真平台模拟生成的 Kolmogorov 湍流随机相位屏符合理论统计规律,可根据夏克-哈特曼波前传感器结构参数自动划分检测区域,且能准确定位光斑和复原波前,波前复原误差精度达到 10^{-2} 。该仿真平台可根据不同 SNR 水平选择最优的光斑定位算法与波前复原方法。此外,整个仿真平台涉及的参数均可调节优化,为进一步研究畸变波前、夏克-哈特曼波前传感器结构参数的设计以及光斑定位算法与波前复原方法的参数优化提供了一个有效的工具。

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Design and Verification of Digital Simulation Platform for Shack-Hartmann Wavefront Sensors

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Abstract

Objective Shack-Hartmann wavefront sensors (SHWFSs) are widely used in the wavefront measurement of adaptive optics systems. Their detection accuracy directly affects the correction performance of the entire system. The optical path of light waves propagating in the atmosphere changes owing to atmospheric turbulence, resulting in wavefront aberrations. The influence of atmospheric turbulence distorts the laser wavefront, leading to the flicker and drift of the optical signal at the receiving end, which severely interferes with target detection. Atmospheric turbulence shows strong randomness and poor reproducibility. Therefore, it is necessary to conduct a numerical simulation of atmospheric turbulence distortion wavefront to study laser atmospheric transmission and the correction of adaptive optics systems. Second, to study and analyze the design, performance estimation, algorithm optimization, and other aspects of SHWFS, it is necessary to simulate various parts of SHWFS, including the entire process from the useful signal input to output. Our goal is to establish a digital simulation platform for SHWFS that is flexible, easy to adjust, easy for statistical analysis, and convenient to provide data for the optical system. Additionally, the simulation platform can analyze spot positioning and wavefront reconstruction errors under different distorted wavefronts and noises. Furthermore, it can examine and optimize optical system parameters and improve the detection accuracy and performance of the entire system.

Methods Because numerical simulation technology has the advantages of low cost, easy implementation, and repeated experiments, the simulation platform for SHWFS is established using this technology. The simulation platform for SHWFS involves three modules: wavefront generation, SHWFS detection, and wavefront

reconstruction. Using the theoretical analysis of each module, the corresponding mathematical model is established to realize the simulation of each module. The Zernike polynomial method is used to generate a distorted wavefront conforming to the Kolmogorov turbulence statistical characteristics. Additionally, the random dynamic, static, and function-modulated wavefronts are generated using the Zernike polynomial method. Different spot location algorithms are used to achieve the position of the spot quickly and accurately under different signal-to-noise ratio conditions. Based on the requirements of the optical system, different wavefront reconstruction algorithms are selected to realize fast and accurate wavefront reconstruction. Finally, the correctness of the simulation platform is verified by comparing the simulation results with the theoretical results.

Results and Discussions The structure of the simulation platform for SHWFS includes the generation of the spot array image, spot positioning, and wavefront reconstruction. The generation of the spot array image includes wavefront generation, noise addition, sensor structure parameters, and light emphasis (Fig. 3). The control interface of the simulation platform for SHWFS is divided into three interfaces: control interface for wavefront simulations, control interface for spot positioning, and control interface for wavefront reconstruction (Fig. 4). Parameters can be set on the control interface as required. The added noise includes Gaussian, Poisson, salt and pepper, and special background noise. A single type of noise can be added or several types of noise can be superimposed based on actual requirements (Fig. 6). Figure 8 shows the verification of the statistical characteristics of the Kolmogorov turbulence. This figure shows that the simulated variance of the turbulent wavefront is consistent with the theoretical variance, which conforms to the statistical law of the Kolmogorov turbulence. The results of the spot positioning and wavefront reconstruction errors obtained using the thresholding center of gravity (T-COG) algorithm for spot positioning and the modal algorithm for wavefront reconstruction confirm the correctness of spot positioning and wavefront reconstruction (Figs. 9 and 10).

Conclusions The proposed simulation platform has the advantage of flexibility and changeability. It can also set and optimize the parameters of each part involved in the SHWFS detection process based on the requirements. The simulation platform contains five parts. The first part involves four types of distortion wavefront simulations. The second part involves the arbitrary selection of the superposition of multiple noises, mainly involving four types of noise: Gaussian, Poisson, salt and pepper, and special background noise. The third part consists of various subaperture structure simulations, which can automatically divide the detection area. The fourth part involves seven types of spot location algorithms commonly used in engineering, i. e., T-COG, thresholding weighted center of gravity, thresholding intensity-weighted centroiding, windowed thresholding center of gravity, windowed thresholding intensity-weighted centroiding, windowed thresholding weighted center of gravity, and cross-correlation. The final part involves the two types of wavefront reconstruction algorithms. When a special situation causes partial subaperture lack of light, the subaperture slope zero reconstruction and subaperture removal reconstruction methods can be adopted to handle this situation. The control interface of the SHWFS simulation platform is mainly divided into three interfaces: wavefront generation, spot positioning, and wavefront reconstruction. The control interface is simple and easy to operate. In this study, the SHWFS digital simulation platform is tested and test results show that the simulation platform can well characterize the statistical characteristics of the turbulent phase. Additionally, it can optimize the spot positioning algorithms and parameters based on the working environment of the adaptive optical system. It can also ensure the correctness of the wavefront reconstruction (the accuracy of the wavefront restoration error is 10^{-2}) and handle special cases of subaperture lack of light.

Key words sensors; Fourier optics; Shack-Hartmann wavefront sensor; numerical simulation; distorted wavefront; noise simulation

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