Effect of Airborne LiDAR Platform's Vibration on Laser Pointing Accuracy

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Abstract In order to reduce the influence of airborne platform's vibration on the accuracy of measurements, the propagation models are established for laser beam pointing error of airborne light detection and ranging (LiDAR). The platform's vibration is decomposed into linear-vibration and angular-vibration. Then error propagation models for the two types of vibration are built respectively. Rules of that positioning accuracy influenced by linear-vibration and angular-vibration are also studied. The results show that linear-vibration of x would bring positioning error of y and z only when there is a slope on the measured ground, while the positioning accuracy of x is isolated from the other two directions. Angular-vibration has a significant influence on the accuracy of horizontal positioning. Positioning accuracies of y and z are only influenced by rolling vibration, while the pitching and yawing vibration are two main source of positioning error of x.

Key words optical devices; airborne LiDAR; pointing accuracy; platform's vibration

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机载平台振动对激光雷达激光束指向精度的影响

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摘要 为了有效地减小机载平台振动对激光雷达(LiDAR)测量数据的精度的影响,针对机载平台振动的特点研究了平台振动引起的激光束指向误差的传递模型。将平台的振动分解为线振动和角振动,分别建立了沿 x、y 和 z 三个方向上的线振动的误差传递模型,以及绕三个轴的俯仰、侧滚和偏航角振动的误差传递模型,分析讨论了各向线振动和角振动对定位精度的影响规律。通过仿真发现载机 x 向线振动仅在坡度存在时才会产生 y 和 z 向的定位误差;而 y 和 z 向线振动不影响 x 向定位精度。载机角振动对水平定位精度影响较大;y 和 z 向定位精度仅受侧滚角振动的影响;俯仰和偏航角振动主要产生 x 向的定位误差。

关键词 光学器件;机载雷达;指向精度;平台振动

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1 Introduction

The pointing accuracy of airborne ligth detection and ranging (LiDAR) laser transmitting system represents the deviation of the laser beam from the intended target[1], which is one of the most important errors of the measurement system and affects 3D coordinate of the laser footprints in geodetic coordinates directly[2-3]. Vibration of airborne platform is a major factor that decreases the pointing accuracy of laser

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beam. Few reports analyzed the influence of airborne platform's vibration on positioning error[4]. The error propagation models for laser beam pointing error, which are based on the characteristics of airborne platform's vibration is built and analyzed.

2 Establishment of error propagation model

2.1 Definition of coordinate systems

The coordinate conversion order in airborne LiDAR system is instantaneous laser coordinate system, laser-scanning coordinate system, airborne coordinate system, inertial platform reference system, local horizontal reference coordinate system, local vertical reference coordinate system and WGS-84 coordinate system[5], shows in Fig 1.

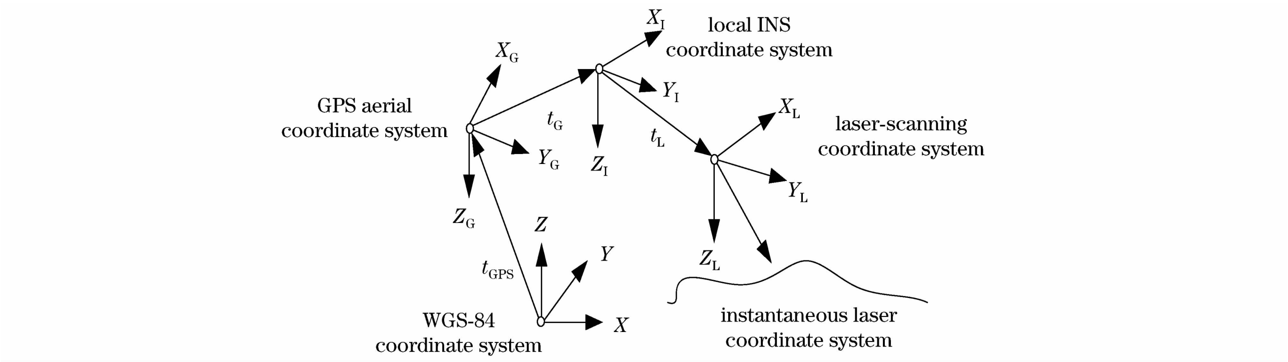

Fig. 1 Diagram of coordinate transformation

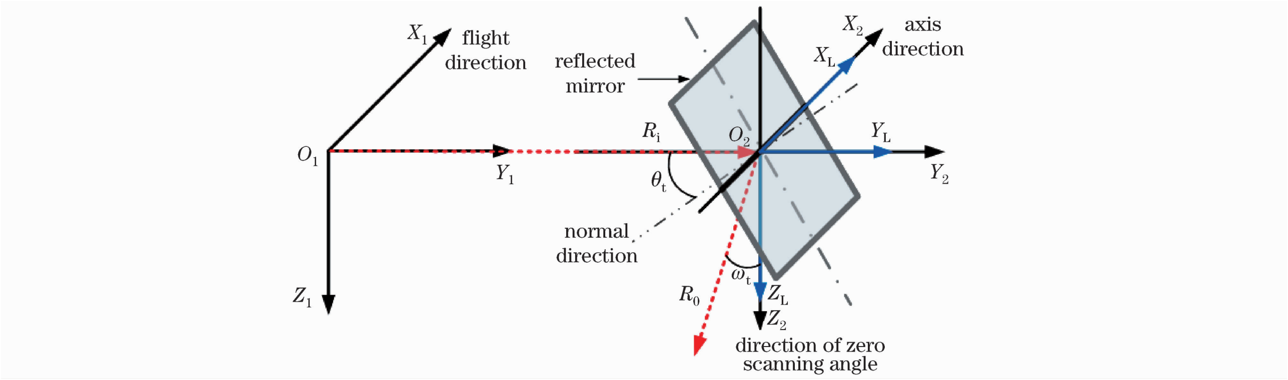

Fig. 2 Relationship of coordinate systems

The coordinate of laser footpoints in WGS-84 coordinate system is

$$\begin{bmatrix} x_{84} \\ y_{84} \\ z_{84} \end{bmatrix} = \boldsymbol{R}_{\mathrm{W}} \boldsymbol{R}_{\mathrm{G}} \boldsymbol{R}_{\mathrm{N}} \left[\boldsymbol{R}_{\mathrm{M}} \boldsymbol{R}_{\mathrm{L}} \begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix} + \begin{bmatrix} \Delta x_{\mathrm{I}}^{L} \\ \Delta y_{\mathrm{I}}^{L} \\ \Delta z_{\mathrm{I}}^{L} \end{bmatrix} - \begin{bmatrix} \Delta x_{\mathrm{I}}^{G} \\ \Delta y_{\mathrm{I}}^{G} \\ \Delta z_{\mathrm{I}}^{G} \end{bmatrix} \right] + \begin{bmatrix} x_{84} \\ y_{84} \\ z_{84} \end{bmatrix}_{\mathrm{GPS}}, \tag{1}$$

where $\boldsymbol{R}_{\mathrm{W}}$ and $\boldsymbol{R}_{\mathrm{G}}$ are coordinate conversion rotation matrixes related to the current location. $\boldsymbol{R}_{\mathrm{N}}$ and $\boldsymbol{R}_{\mathrm{L}}$ are rotation matrixes related to attitude angle and scan angle of measurement or interpolation. $\boldsymbol{R}_{\mathrm{M}}$ is installation error rotation matrix.

2.2 Establish ideal model

To model and calculate more conveniently, laser coordinate system ($O_1-X_1Y_1Z_1$) and scanning mirror coordinate system ($O_2-X_2Y_2Z_2$) are established. Fig. 2 shows the relationships among $O_1-X_1Y_1Z_1$, $O_2-X_2Y_2Z_2$ and $O_{\mathrm{L}}-X_{\mathrm{L}}Y_{\mathrm{L}}Z_{\mathrm{L}}$. They are parallel to each other when there is no vibration.

Assuming that the vector of the incident laser beam is $\boldsymbol{R}_{\mathrm{i}}=[0 \quad 1 \quad 0]^{\mathrm{T}}$ (the same in $O_1-X_1Y_1Z_1$ and $O_2-X_2Y_2Z_2$) and that the normal vector of reflector is $\boldsymbol{N}_{\mathrm{i}}=[0 \quad -\cos\theta_{\mathrm{t}} \quad \sin\theta_{\mathrm{t}}]^{\mathrm{T}}$ in $O_2-X_2Y_2Z_2$, the matrix of the reflector $\boldsymbol{M}_{\mathrm{i}}$[6] is

$$\boldsymbol{M}_{\mathrm{i}}=\begin{bmatrix}1-2N_{\mathrm{i}x}^{2} & -2N_{\mathrm{i}x}N_{\mathrm{i}y} & -2N_{\mathrm{i}x}N_{\mathrm{i}z}\\ -2N_{\mathrm{i}x}N_{\mathrm{i}y} & 1-2N_{\mathrm{i}y}^{2} & -2N_{\mathrm{i}y}N_{\mathrm{i}z}\\ -2N_{\mathrm{i}x}N_{\mathrm{i}z} & -2N_{\mathrm{i}y}N_{\mathrm{i}z} & 1-2N_{\mathrm{i}z}^{2}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & -\cos2\theta_{\mathrm{t}} & \sin2\theta_{\mathrm{t}}\\ 0 & \sin2\theta_{\mathrm{t}} & \cos2\theta_{\mathrm{t}}\end{bmatrix}, \tag{2}$$

where θ_{t} is the instantaneous incident angle. So the vector of the exit laser beam $\boldsymbol{R}_{\mathrm{o}}$ can be obtained.

$$\boldsymbol{R}_{\mathrm{o}}=\boldsymbol{M}_{\mathrm{i}}\boldsymbol{R}_{\mathrm{i}}=\begin{bmatrix}1 & 0 & 0\\ 0 & -\cos2\theta_{\mathrm{t}} & \sin2\theta_{\mathrm{t}}\\ 0 & \sin2\theta_{\mathrm{t}} & \cos2\theta_{\mathrm{t}}\end{bmatrix}\begin{bmatrix}0\\ -\cos\theta_{\mathrm{t}}\\ \sin\theta_{\mathrm{t}}\end{bmatrix}=\begin{bmatrix}0\\ -\cos2\theta_{\mathrm{t}}\\ \sin2\theta_{\mathrm{t}}\end{bmatrix}. \tag{3}$$

The relationship between θ_{t} and the instantaneous scanning angle ω_{t} is

$$\theta_{\mathrm{t}}=\frac{\omega_{\mathrm{t}}}{2}+\frac{\pi}{4}\quad\text{or}\quad\omega_{\mathrm{t}}=2\theta_{\mathrm{t}}-\frac{\pi}{2}. \tag{4}$$

So the vector of the exit laser beam $\boldsymbol{R}_{\mathrm{o}}$ in $O_2-X_2Y_2Z_2$ is

$$\boldsymbol{R}_{\mathrm{o}}=\begin{bmatrix}0\\ -\cos(\omega_{\mathrm{t}}+\frac{\pi}{2})\\ \sin(\omega_{\mathrm{t}}+\frac{\pi}{2})\end{bmatrix}=\begin{bmatrix}0\\ -\cos\omega_{\mathrm{t}}\\ -\sin\omega_{\mathrm{t}}\end{bmatrix}. \tag{5}$$

Transfer $\boldsymbol{R}_{\mathrm{o}}$ from $O_2-X_2Y_2Z_2$ into WGS-84

$$r_{\mathrm{oWGS\text{-}84}}=R_{\mathrm{W}}R_{\mathrm{G}}R_{\mathrm{N}}(R_{\mathrm{M}}R_{\mathrm{O}}+t_{\mathrm{L}}-t_{\mathrm{G}})+P_{\mathrm{WGS\text{-}84}}. \tag{6}$$

2.3 Establish error propagation models

2.3.1 Positioning error influenced by linear-vibration

Linear-vibration is error of platform's position, which is along an axis. The linear-vibration error will make the laser footpoints coordinate deviate from the ideal scanning line[7]. Positioning error caused by linear-vibration can be represented by the following equation when the topography is flat:

$$\begin{cases}e_x=\mathrm{d}x=\int\Delta V_x(t)\mathrm{d}t\\ e_y=\mathrm{d}y=\int\Delta V_y(t)\mathrm{d}t,\\ e_z=\mathrm{d}z=\int\Delta V_z(t)\mathrm{d}t\end{cases} \tag{7}$$

where e_x, e_y, e_z is the positioning error of the direction of x, y, z. $\mathrm{d}x$, $\mathrm{d}y$, $\mathrm{d}z$ are the linear deviators of the three directions.

When the topography has a certain gradient during flight, $\mathrm{d}x$, $\mathrm{d}y$, $\mathrm{d}z$ are not equal to e_x, e_y, e_z. Fig. 3(a) shows the positioning error affected by the airborne linear-vibration when there is only $\mathrm{d}x$. The flight height is H; the instantaneous scanning angle is ω_{t}; the ground slope angle is α; The angle between airborne flight direction and the normal section of the slope is β. The gradient angle of the slope cut by $X-Z$ plane is

$$\tan\gamma=\cos\beta\tan\alpha. \tag{8}$$

According to the geometrical relationship, the positioning error produced by airborne linear-vibration in x direction is

$$\begin{cases}e_x=\mathrm{d}x\\ e_y=-\mathrm{d}x\tan\alpha\tan\omega_{\mathrm{t}}\cos\beta\\ e_z=-\mathrm{d}x\tan\alpha\cos\beta\end{cases}. \tag{9}$$

Fig. 3(b) shows the influence of linear-vibration on positioning accuracy, when there is only $\mathrm{d}y$. According to geometrical relationships, the positioning errors of linear-vibration of airborne in y is

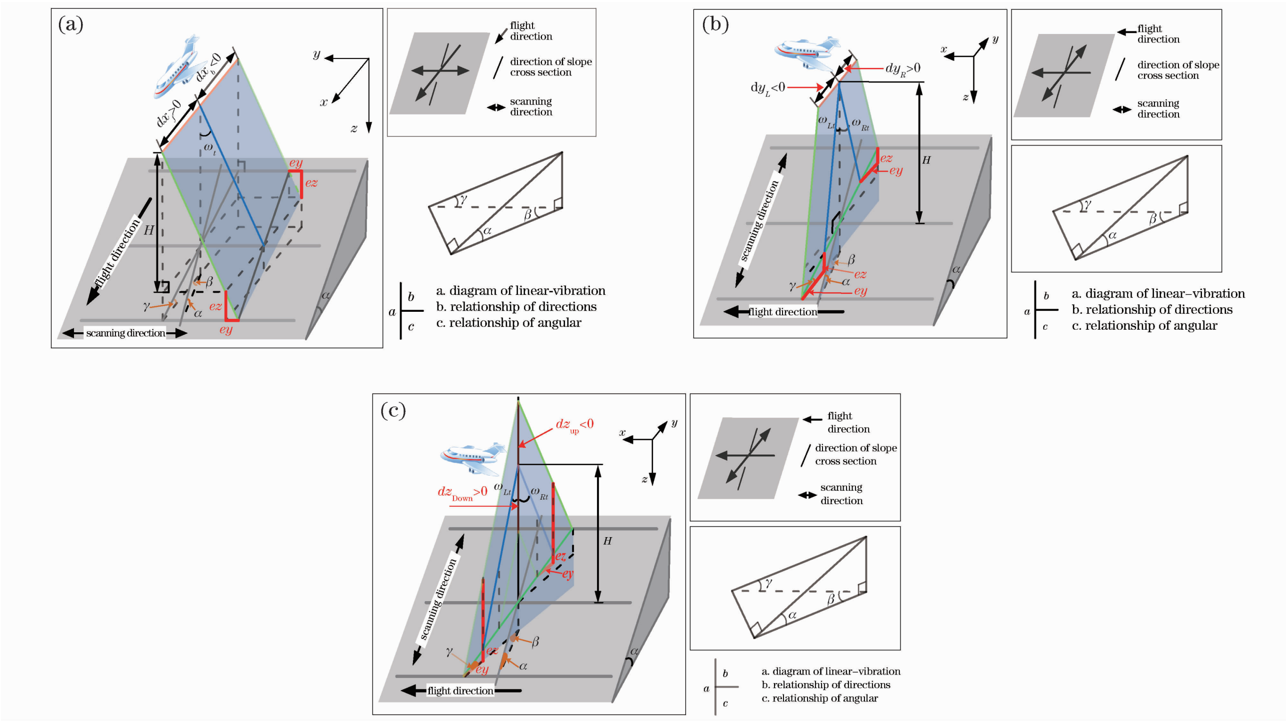

Fig. 3 Influence of linear-vibration on positioning accuracy in (a) x, (b) y and (c) z direction

$$\begin{cases} e_x = 0 \\ e_y = \mathrm{d}y \dfrac{\cos\omega_t}{\cos\omega_t - \sin\omega_t \cos\beta\tan\alpha} \\ e_z = \mathrm{d}y \dfrac{\cos\omega_t \cos\beta\tan\alpha}{\cos\omega_t - \sin\omega_t \cos\beta\tan\alpha} \end{cases}. \tag{10}$$

Fig. 3 (c) shows the influence of linear-vibration on positioning accuracy, when there is only dz. According to geometrical relationships, the positioning errors of linear-vibration of airborne in z is

$$\begin{cases} e_x = 0 \\ e_y = \mathrm{d}z \dfrac{\sin\omega_t \cos[\arctan(\cos\beta\tan\alpha)]}{\cos[\omega_t + \arctan(\cos\beta\tan\alpha)]} \\ e_z = \mathrm{d}z \left\{1 + \dfrac{\sin\omega_t \sin[\arctan(\cos\beta\tan\alpha)]}{\cos[\omega_t + \arctan(\cos\beta\tan\alpha)]}\right\} \end{cases}. \tag{11}$$

2.3.2 Positioning error influenced by angular-vibration

Angular-vibration is error of platform's attitude angle, which is around an axis. Diagram of angular-vibration are shown in Fig. 4. The flight altitude of airborne is H, and the instantaneous scanning angle is ω_t. r, p, h are values of rolling angle, pitching angle and yawing angle respectively. Δr, Δp, Δh are errors of rolling angle, pitching angle and yawing angle respectively. Laser ranging value is S.

According to geometrical relationships in Fig. 4, the influence of angular-vibration of airborne on positioning errors is

$$\begin{cases} e_x = S\cos\omega_t[\sin(p+\Delta p) - \sin p] + S\sin\omega_t[\sin(h+\Delta h) - \sin h] \\ e_y = S[\sin(\omega_t + r + \Delta r) - \sin(\omega_t + r)] + S\sin\omega_t[\cos(h+\Delta h) - \cos h] \\ e_z = S[\cos(\omega_t + r + \Delta r) - \cos(\omega_t + r)] + S\cos\omega_t[\cos(p+\Delta p) - \cos p] \end{cases}. \tag{12}$$

3 Analysis of error propagation model

Models established above are analyzed in different conditions and the results are shown in Fig. 5 to Fig. 8.

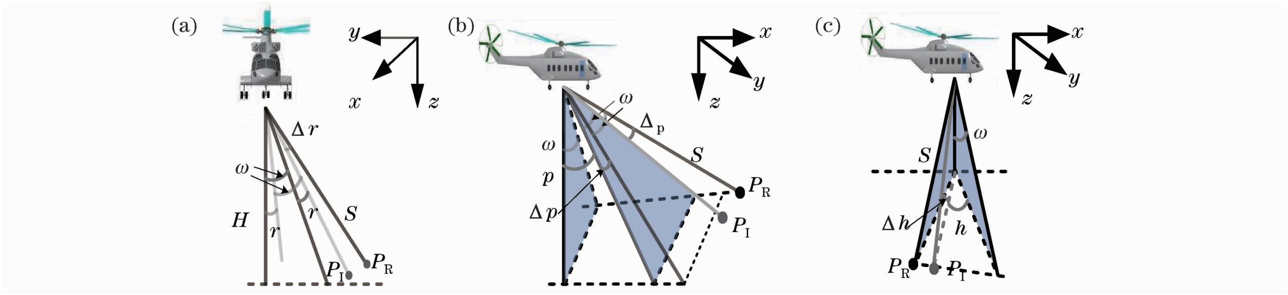

Fig. 4 Diagram of angular-vibration. (a) Rolling angular error; (b) pitching angular error; (c) yawing angular error

In Fig. 5(a), it can be find that the change of scanning angle only affects the positioning error in y direction, and the positioning error in y direction changes linearly from positive to negative when scanning angle increases; the error is zero when the instantaneous scanning angle is 0°. In Fig. 5(b) the positioning errors in y and z direction change linearly from negative to positive with the increase of slope angle; the positioning error in y direction is less than that in z direction when the slope is not zero and in contrast the errors in y and z direction are both zero. It can be seen from Fig. 5(c) that the positioning error in y direction is less than that in z direction; the errors in y and z direction both increase firstly and then decrease with the increase of the angle β between flight direction and the slope normal section, and there is the maximum error when β is 0.

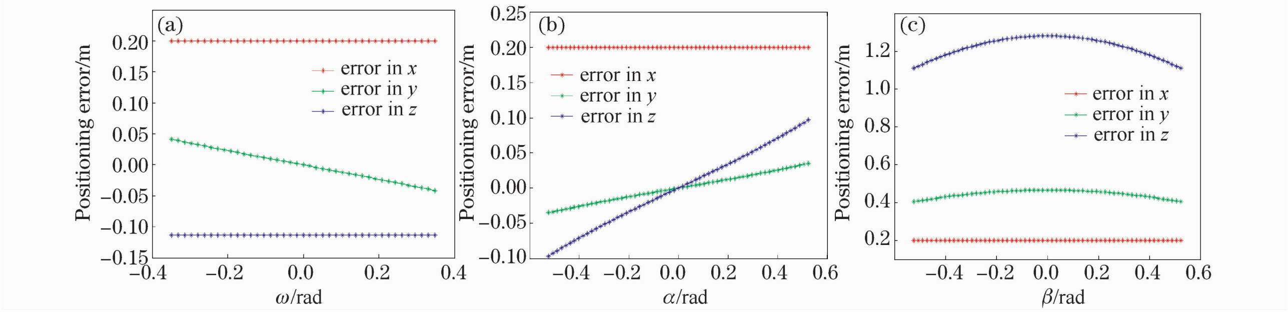

Fig. 5 Influence of linear-vibration on positioning accuracy in x. (a) dx=0.2 m, α=30°, β=10°; (b) dx=0.2 m, β=10°, ω=20°; (c) dx=0.2 m, α=30°, ω=20°

In Fig. 6 (a) with the increases of scanning angle, the positioning errors of y and z are negative and decrease. In Fig. 6(b) with the increases of slope angle, the positioning error is always negative and decreases in y. However, the value of positioning error of z changes gradually from negative to positive, which is always greater than the value of positioning error of z. In Fig. 6(c) the positioning error of y is negative, while the positioning error of z is positive; the positioning error of y is always less than the value of positioning error of z. With the increase of β, the positioning error of y decreases firstly and then increases. However, the positioning error of z increases firstly and then decreases.

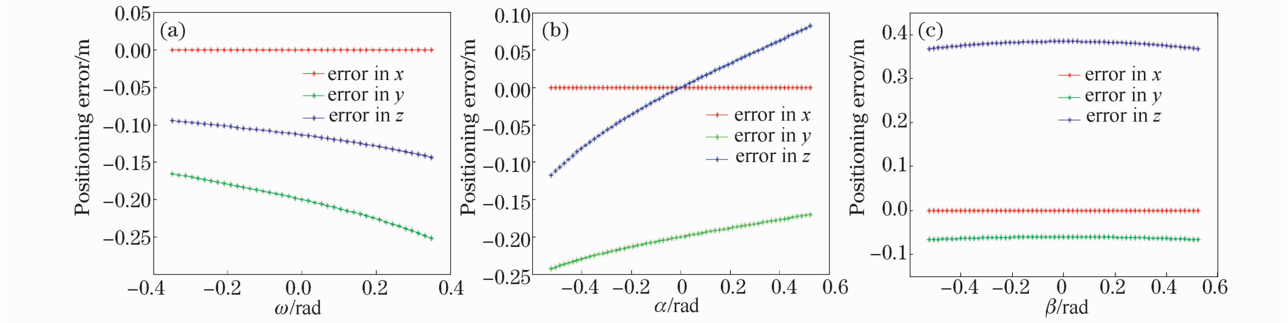

Fig. 6 Influence of linear-vibration on positioning accuracy in y. (a) dy=0.2 m, α=30°, β=10°; (b) dy=0.2 m, α=30°, ω=20°; (c) dy=0.2 m, β=10°, ω=20°

In Fig. 7(a) with the increase of scanning angle, the positioning error of y changes linearly from negative to positive, and when the value of scanning angle is 0°, the positioning error of y is 0. The positioning error of z increases linearly, which is larger than that of y. In Fig. 7(b) the positioning errors of y and z are always positive. With the increases of slope angle, the positioning error of y increases firstly and then decreases. The positioning error of z decreases linearly, which is larger than that of y. In Fig. 7(c), the positioning errors of y and z are always positive and the positioning error of z is always larger than the value of positioning error of y. With the increase of β, the positioning error of y decreases firstly and then increases. However, the value of positioning error of z almost does not change.

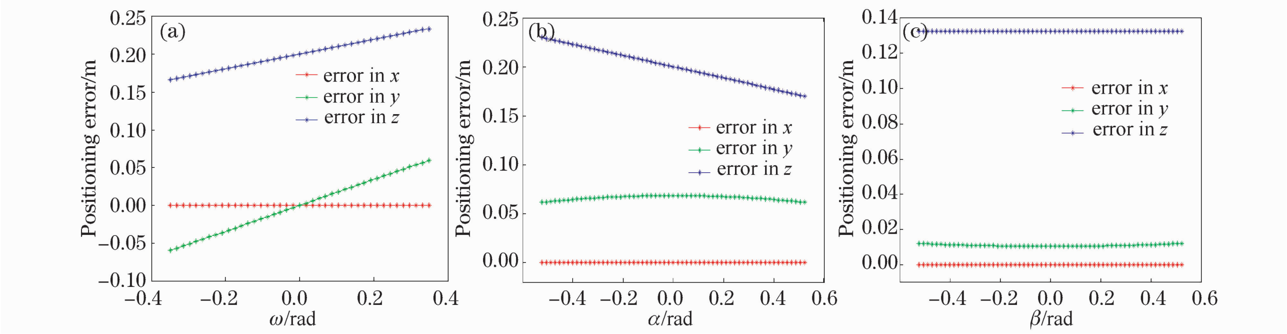

Fig. 7 Influence of linear-vibration on positioning accuracy in z. (a) $dz=0.2$ m, $\alpha=30°$, $\beta=10°$; (b) $dz=0.2$ m, $\alpha=30°$, $\omega=20°$; (c) $dz=0.2$ m, $\beta=10°$, $\omega=20°$

The influences of Δr, Δp, Δh on positioning errors of x, y, z are obvious in Fig. 8(a). When Δr, Δp, Δh are positive, positioning error of x is positive, which increases with scanning angle increasing. Positioning error of y is also positive, which increases firstly and then decreases with scanning angle increasing. The positioning error of z changes linearly from positive to negative when the scanning angle increases. In Fig. 8(b), when Δp, Δh are constants and Δr increases, the positioning error of x is constant, the positioning errors of y and z increase, and their changing laws are invariable with scanning angle changing. In Fig. 8(c), when Δr, Δh are constants and Δp increases, the positioning error of y is

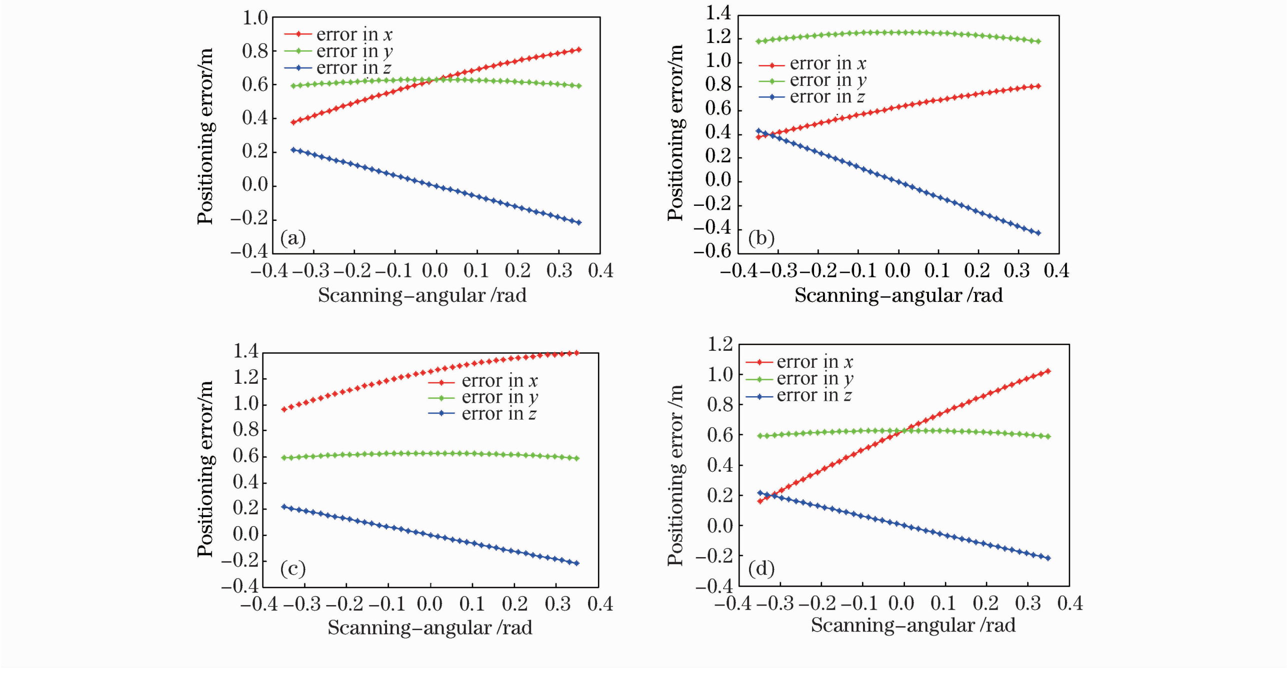

Fig. 8 Influence of angular-vibration on positioning accuracy. (a) $\Delta r=\Delta p=\Delta h=0.02°$; (b) $\Delta r=0.04°$, $\Delta p=\Delta h=0.02°$; (c) $\Delta r=\Delta h=0.02°$, $\Delta p=0.04°$; (d) $\Delta r=\Delta p=0.02°$, $\Delta h=0.04°$

constant, the positioning error of z doesn't change; the positioning error of x increases and its changing laws are also invariable with scanning angle changing. In Fig. 8(d), when Δr, Δp are constants and Δh increases, the positioning error of z is constant, and the positioning error of y doesn't change. When $\omega_t < 0$, the positioning error of x decreases with ω_t increasing. While when $\omega_t > 0$, the positioning error of x increases with ω_t increasing, and its changing law is invariable.

The studies show that linear-vibration along x will bring positioning errors of y and z only when there is a slope, while the positioning accuracy of x is independent of linear-vibration of the other two axes. Angular-vibration affects the horizontal positioning accuracy fairly. Positioning accuracy of y and z are influenced only by rolling vibration, while the pitching and yawing vibration cause mainly positioning errors of x.

4 Conclusions

The relationships of several reference coordinate systems are given. Laser beam positioning model under ideal conditions is built as foundation for building error propagation model that caused by the vibration of airborne platform. The vibration of platform is resolved into linear-vibration and angular-vibration. Based on the qualitative and quantitative analysis, error propagation rules are found out. These results are helpful for the calibration of the raw LiDAR point-cloud data.

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