Movement of the Core and the Centroid of Off-Center Gaussian Vortex Beams during Propagation

Ding Panfeng　Pu Jixiong

(*School of Information Science and Engineering*，*Huaqiao University*，*Xiamen*，*Fujian* 361021，*China*)

Abstract　While using the spiral phase plate (SPP) to produce Gaussian vortex beams，the center of the beam and the center of the SPP should be superposed. However，the center of the beam is always away from the center of the SPP to some extent in practice. The real output light from the phase plate are off-center Gaussian vortex beams. For conventional vortex beams，the core and the centroide are the same point and always located at the center during propagation. The off-center Gaussian vortex beam is different from the conventional vortex beams and its propagation is investigated in this paper. Research result shows that the core and the centroid of the off-center vortex beam are separate and moving during propagation. The directions of movement of the core and the centroid are decided by the sign of the topological charge. The movement distance of the core has no connection with the magnitude of the topological charge. However，the movement distance of the centroid is related to the magnitude of the topological charge.

Key words　physical optics；centroid；vortex beam；off-center；movement；topological charge

OCIS codes　260.6042；260.1960；050.4865

离心高斯涡旋光束在传输中暗核和质心的运动

丁攀峰　蒲继雄

（华侨大学信息科学与工程学院，福建 厦门 361021）

摘要　使用螺旋相位板产生高斯涡旋光束时，要求光束中心与螺旋相位板中心对准重合。然而实际上光束中心总会在一定程度上偏离螺旋相位板中心，此时从相位板输出的光束即为离心高斯涡旋光束。对于传统的涡旋光束而言，暗核和质心为同一点，在传输中一直位于光束的中心位置。离心高斯涡旋光束则不同于传统的涡旋光束，对其传输进行了研究。研究结果表明，离心高斯涡旋光束的暗核和质心的位置分离，在传输过程中会发生移动。移动方向由拓扑电荷的符号确定，暗核的移动距离与拓扑电荷的大小无关，质心的移动距离则与拓扑电荷的大小相关。

关键词　物理光学；质心；涡旋光束；离心；移动；拓扑电荷

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1　Introduction

The concept of vortex beams was firstly introduced by Nye *et al*.[1] in 1974. Because of its special properties and potential applications[2-4]，optical vortex beams attracted a great deal of attention from researchers[5-13]. At present，a convenient and reliable method to produce this special beam is to use spiral phase plate (SPP). However，while the original Gaussian beam from laser is projected onto SPP，the center of the original beam and the center of the SPP are not superposed entirely，i.e. the generated vortex beam is off-center. It is found that the intensity distribution of the off-center vortex beam changes asymmetrically，the core of the beam moves during propagation and the sign of the topological charge decides the moving direction[14]. Besides the intensity distribution，the phase distribution and the centroid are equally important for an optical beam. Recently，the changes of the centroid position of optical beams are intensively investigated while propagating in turbulent atmosphere，and it is pointed out that the centroid position is independent of turbulence，but mainly determined by the centroid positions at the source plane and the elements of optical transfer matrix[15]. In this letter，we will focus our discussion on the exact phase distribution and the position of the centroid of off-center vortex beams during propagation in free space. The details of the movement characteristics of the core and the centroid are investigated.

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作者简介：丁攀峰(1980—)，男，博士，硕士生导师，主要从事光束传输及偏振光束等方面的研究。

E-mail：dingpanfeng@163.com

2 Electric Field of the Off-center Gaussian Vortex Beam During Propagation

Generally, it is assumed that the electric field in the source plane is a Gaussian field with optical vortex. If the center of phase (vortex core) is off the geometric center, the field can be expressed as ($m>0$)

$$E(x',y',z=0)=E_0\exp\left(-\frac{x'^2+y'^2}{\sigma^2}\right)\left(\frac{\sqrt{(x'-a)^2+y'^2}}{\sigma}\right)^m\left(\frac{x'-a+\mathrm{i}y'}{\sqrt{(x'-a)^2+y'^2}}\right)^m, \tag{1}$$

where E_0 and σ are the characteristic amplitude and beam width in the source plane, m is the topological charge, and the vortex core is located at the point of $(a, 0)$ in the Cartesian coordinate system. The superscript denotes the coordinates of the points in the source plane of $z=0$. According to the diffraction theory[16], the electric field in the observation plane after propagation can be obtained by integration as

$$E(x,y,z)=-\frac{\mathrm{i}}{\lambda z}\exp(\mathrm{i}kz)\iint E(x',y',z=0)\exp\left\{\frac{\mathrm{i}k}{2z}[(x-x')^2+(y-y')^2]\right\}\mathrm{d}x'\mathrm{d}y', \tag{2}$$

where λ is the wavelength and $k=2\pi/\lambda$. Thus by substituting Eq.(1) into Eq.(2), we obtain

$$E(x,y,z)=C_0\iint\exp\left[-\alpha^2(x'^2+y'^2)-\frac{\mathrm{i}k}{z}(xx'+yy')\right](x'-a+\mathrm{i}y')^m\mathrm{d}x'\mathrm{d}y', \tag{3}$$

where

$$\begin{cases} C_0=\dfrac{-\mathrm{i}E_0}{\lambda z\sigma^m}\exp\left[\mathrm{i}kz+\dfrac{\mathrm{i}k(x^2+y^2)}{2z}\right] \\ \alpha^2=\dfrac{1}{\sigma^2}-\dfrac{\mathrm{i}k}{2z} \end{cases}. \tag{4}$$

By setting $u=x'+\frac{\mathrm{i}kx}{2z\alpha^2}$, $v=y'+\frac{\mathrm{i}ky}{2z\alpha^2}$, Eq.(3) can be written as

$$E(x,y,z)=C_0\exp\left[-\frac{k^2(x^2+y^2)}{4\alpha^2z^2}\right]\iint\exp[-\alpha^2(u^2+v^2)](u+\mathrm{i}v+Z)^m\mathrm{d}u\mathrm{d}v, \tag{5}$$

where

$$Z=\frac{-\mathrm{i}k}{2z\alpha^2}\left[x-a+\mathrm{i}\left(y-\frac{2z}{k\sigma^2}a\right)\right]. \tag{6}$$

By integral transform technique, we obtain

$$E(x,y,z)=\frac{E_0}{\sigma^m}\left(\frac{-\mathrm{i}k}{2z\alpha^2}\right)^{m+1}\exp\left[\mathrm{i}kz+\left(\frac{\mathrm{i}k}{2z}-\frac{k^2}{4z^2\alpha^2}\right)(x^2+y^2)\right]\left[x-a+\mathrm{i}\left(y-\frac{2z}{k\sigma^2}a\right)\right]^m. \tag{7}$$

Eq.(7) is the analytical expression of the electric field of the off-center Gaussian vortex beam during propagation. If the topological charge is negative ($m<0$), the expression can be obtained with the same method to be

$$E(x,y,z)=\frac{E_0}{\sigma^{-m}}\left(\frac{-\mathrm{i}k}{2z\alpha^2}\right)^{-m+1}\left(\frac{-\mathrm{i}k}{2z\alpha^2}\right)^{-m}\exp\left[\mathrm{i}kz+\left(\frac{\mathrm{i}k}{2z}-\frac{k^2}{4z^2\alpha^2}\right)(x^2+y^2)\right]\left[x-a-\mathrm{i}\left(y+\frac{2z}{k\sigma^2}a\right)\right]^{-m}. \tag{8}$$

3 Movement of the Vortex Core and the Phase Distribution of the Beam During Propagation

3.1 Movement of the Vortex Core

Eqations (7) and (8) contain three parts. The first part shows the common constant for all the points in the observation plane; the second part is actually the function of x^2+y^2 and displays the propagation character of Gaussian beam; the third part decides the position of vortex core, which can be expressed in the Cartesian coordinate system as

$$\begin{cases} x_o=a,\ y_o=\dfrac{2z}{k\sigma^2}a & (m>0) \\ x_o=a,\ y_o=-\dfrac{2z}{k\sigma^2}a & (m<0) \end{cases}. \tag{9}$$

It is obvious that the core is moving while propagating. The movement distance is proportional to propagation length and has no connection with the magnitude of topological charge. It can be found that the move speed is higher if the wavelength is larger and the beam width of the original optical source is smaller. If we project the position of the core on the x-y plane, it is obvious that the moving trace of the vortex core is tangential to the line connecting the original vortex core and the geometric center (origin of coordinates) of the beam in the source plane, clockwise for positive topological charge and anticlockwise for negative topological charge. Fig.1 presents the intuitionistic map of the movement dynamics of the vortex core. The arrows show the moving directions of the core.

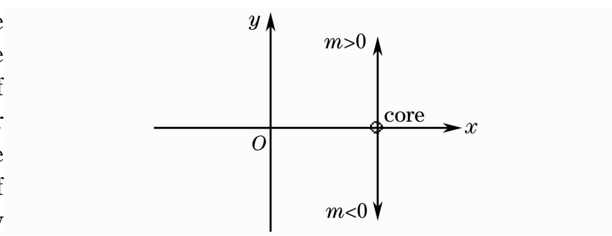

Fig.1 Movement of the vortex core while propagating

3.2 Phase Structure

From Eqs. (7) and (8), the phase distribution of the beam can be obtained. The equiphase line of the electric field in the observation plane can be expressed as

$$\begin{cases} \dfrac{2zk(x^2+y^2)}{k^2\sigma^4+4z^2}+m\arctan\left(\dfrac{y-2az/k\sigma^2}{x-a}\right)=C & (m>0) \\ \dfrac{2zk(x^2+y^2)}{k^2\sigma^4+4z^2}+m\arctan\left(\dfrac{y+2az/k\sigma^2}{x-a}\right)=C & (m<0) \end{cases}, \tag{10}$$

where C denotes the magnitude of different equiphase lines. To provide straightforward information of the phase distributions of the electric field of the vortex beam during propagation, simulation is done on the condition that $\lambda=632.8$ nm, $\sigma=0.5$ mm, $a=0.05$ mm.

The character of the equiphase line is determined by the propagation distance and the topological charge. In Fig. 2(a)~(c), the topological charges are positive and the equiphase lines bend clockwise; while in Fig. 2(d), the topological charge is negative and the equiphase line bends anticlockwise.

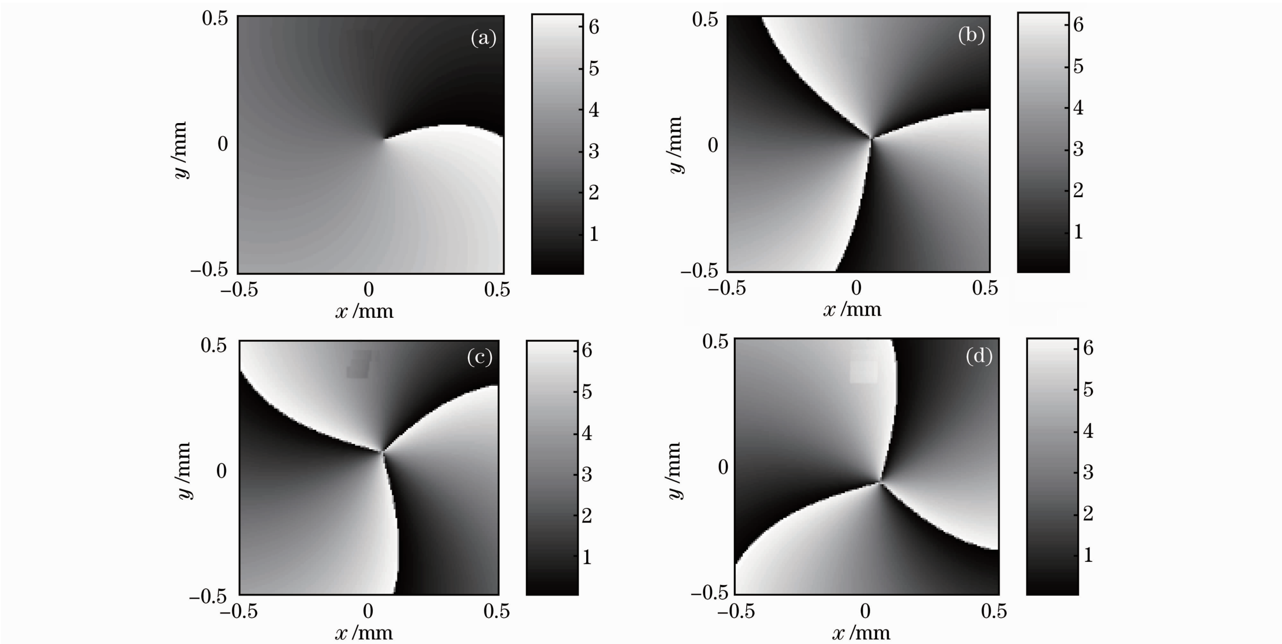

Fig. 2 Phase distributions of the off-center vortex beam while propagating.
(a) $m=1$, $z=500$ mm; (b) $m=3$, $z=500$ mm; (c) $m=3$, $z=1500$ mm; (d) $m=-3$, $z=1500$ mm

4 Movement of the Centroid

The centroid of the off-center vortex beam is given by the first order moment as[17]

$$x_c(z)=\frac{\iint x|E(x,y,z)|^2\mathrm{d}x\mathrm{d}y}{\iint |E(x,y,z)|^2\mathrm{d}x\mathrm{d}y},\quad y_c(z)=\frac{\iint y|E(x,y,z)|^2\mathrm{d}x\mathrm{d}y}{\iint |E(x,y,z)|^2\mathrm{d}x\mathrm{d}y}. \tag{11}$$

By substituting Eq. (7) into Eq. (11), we can perform the integration and obtain the centroid of the off-center vortex beam with different topological charges as

$$\begin{cases} x_c(z)=-\dfrac{a}{1+2t}, & y_c(z)=-\dfrac{pa}{1+2t} & (m=1) \\ x_c(z)=-\dfrac{2a(1+t)}{1+4t+2t^2}, & y_c(z)=-\dfrac{2pa(1+t)}{1+4t+2t^2} & (m=2), \\ x_c(z)=-\dfrac{3a(3+6t+2t^2)}{3+18t+18t^2+4t^3}, & y_c(z)=-\dfrac{3pa(3+6t+2t^2)}{3+18t+18t^2+4t^3} & (m=3) \end{cases} \tag{12}$$

where $t=a^2/\sigma^2$, $p=2z/(k\sigma^2)$. For high order off-center vortex beam, the analytical expression of the centroid is a bit verbose and omitted. It can be found in Eq. (12) that the centroid is also moving while propagating and the movement distance is proportional to the propagation length too. If we project the position of the centroid on the x-y plane, it is obvious that the moving trace of the centroid is tangential to the line connecting the original centroid and the geometric center (origin of coordinates) of the beam in the source plane. Fig. 3 presents the intuitionistic map of the movement dynamics of the centroid.

Although the movement characteristic of the centroid is similar to that of the vortex core (the moving direction is tangential to "the original line" and is decided by the sign of the topological charge; the movement distance is proportional to the propagation length), the difference between them is clear that the movement of the core is independent on the magnitude of the topological, however the position and movement distance of the centroid are closely associated with the magnitude of the topological charge, which is shown in Eq.(12).

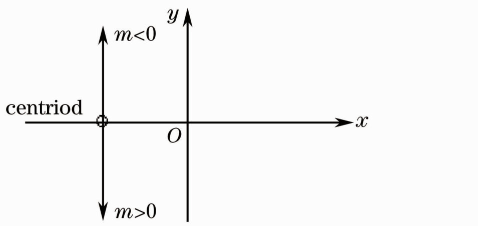

Fig.3 Movement of the centroid while propagating

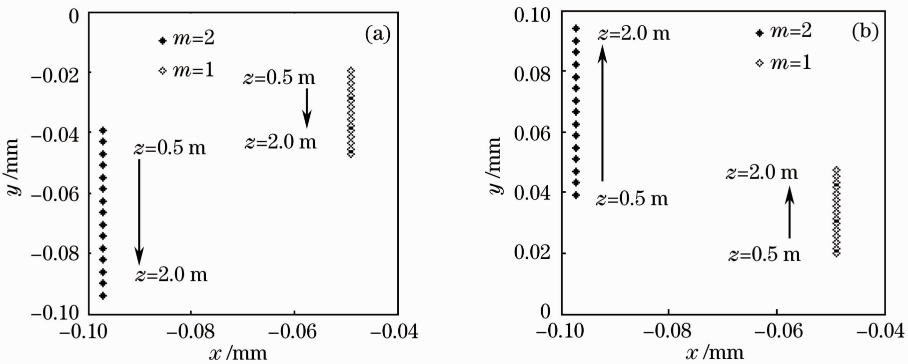

Fig.4 Simulation of the movement of the centroid while propagating. (a) $m>0$; (b) $m<0$

To provide straightforward information of the movement of the centroid of the vortex beam during propagation, simulation is performed on the condition that $\lambda = 632.8$ nm, $\sigma = 0.5$ mm, $a = 0.05$ mm.

The points in Fig.4 denote the positions of the centroid and the arrows show the moving directions of the centroid. While the beam propagates from $z = 0.5$ m to $z = 2.0$ m, the centroid of the first-order off-center vortex beam moves only 0.03 mm and the centroid of the second-order off-center vortex beam moves 0.05 mm. If the topological charge is 3, the movement distance of the centroid will be nearly 0.5 mm (this is not shown in the Fig.4 because the movement distance is too big). From discussions above, it can be concluded that the centroid of the off-center vortex beam with big topological charge moves faster than that with small topological charge during propagation if other parameters are the same.

5 Conclusion

Analytic expressions of the electric field of the off-center vortex beam while propagating are derived. The movement of the core, the phase structure and the movement of the centroid are discussed. It is found that the bending direction of the equiphase line of the electric field depends on the sign of the topological charge in the cross section, and the core and centroid are both moving during propagation. If we project the position of the core and the centroid on the cross section, it can be found that 1) the moving traces of them are both straight lines; 2) the moving directions of them are both decided by the sign of the topological charge; 3) the magnitude of the topological charge has no effect on the movement characteristic of the core, but it determines the movement characteristic of the centroid.

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