Study on Resolution Limit of Total-Reflection X-Ray Optics with Heisenberg Uncertainty Principle

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Abstract　Total-reflection X-ray optics play an important role in X-ray microscopy technology, and the study on their resolution limit is helpful for both designers and users. Theoretical study on the resolution limit of total-reflection X-ray optics is presented based on the Heisenberg uncertainty principle. The theoretical results show that the resolution limit of total-reflection X-ray optics depends on material. The focal spot size limits of total-reflection X-ray optics made of nickel, lead glass and borosilicate glass are 3.2, 4.2 and 6.6 nm, respectively.

Key words　X-ray optics; resolution limit; Heisenberg uncertainty principle; total reflection X-ray optics
OCIS codes　340.7470; 340.7480; 340.7460

利用海森堡不确定性原理研究全反射 X 射线光学器件的焦斑极限

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摘要　全反射 X 射线光学器件在 X 射线显微技术中具有重要的应用，有关其焦斑极限的研究对该器件的设计者和使用者具有指导意义。利用海森堡不确定性原理研究了全反射 X 射线光学器件的焦斑极限。理论结果表明：全反射 X 射线光学器件的焦斑极限与器件的材料有关；利用镍金属、铅玻璃和硼硅酸盐玻璃制成的全反射 X 射线光学器件的焦斑极限分别为 3.2、4.2 和 6.6 nm。

关键词　X 射线光学；焦斑极限；海森堡不确定性原理；全反射 X 射线光学器件
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1　Introduction

X-rays can be focused by materials. There are several X-ray focusing optics which can provide a microbeam with a diameter smaller than 100 nm. Such optics may be divided into four categories such as diffractive optics[1], refractive optics, waveguides, and reflective optics[2] which include the Bragg reflection X-ray optics and the total-reflection X-ray optics. Fresnel zone plates based on the diffraction theory can focus X-rays into a focal spot with a 30 nm diameter at 8.3 keV[3]. A Fresnel zone plate lens with a 12 nm resolution is designed for soft X-ray microscopy[4]. The multilayer Laue lens using diffraction from a multilayer structure illuminated in transmission geometry can provide a focal spot with a diameter of 16 nm at 19.4 keV[5]. Refractive lenses can give a focal spot down to 47 nm in diameter at 20.7 keV[6]. X-ray waveguides can be used to obtain a focal spot with size of 40 nm×25 nm[7]. A sub-15 nm beam can be confined by two crossed X-ray waveguides[8]. A precisely figured total-reflection optics can provide a microbeam with size of 25 nm at 15 keV[9]. An upstream deformable mirror and a multilayer coated Kirkpatrick-Baez (KB) mirror pair have been used to achieve 7 nm one-dimensional

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(1D) focusing at 20 keV[10]. The focal spot size of the monocapillary total-reflection X-ray optics is as small as 90 nm at 10 keV[11]. The resolution limit of X-ray focusing optics attracts the concerns from their designers and users. Suzuki[12] has discussed the resolution limit of refractive lens and Fresnel lens in X-ray region. Mimura *et al.*[13] have shown that the laterally graded multilayer mirror has the potential to exceed the Schwinger limit. Bergemann *et al.*[14] have discussed the resolution limit of waveguide X-ray optics, and they also have regarded the circular taper monocapillary X-ray optics as the waveguide X-ray optics. They found that the effective numerical aperture of waveguides was limited by $\theta_c = \sqrt{2\delta}$, the critical angle of total external reflection θ_c for a material with refractive index $n = 1 - \delta + i\beta$, and believed that the same limitations to focusing held for all X-ray optics. However, Schroer *et al.*[15] have calculated the smallest spot size of adiabatically focusing refractive X-ray lenses and indicated that these adiabatically focusing lenses were shown to have a relatively large numerical aperture, and could focus hard X-rays down to a lateral size of 2 nm. such a size is well below the theoretical resolution limit for focusing with waveguides by Bergemann *et al.*[14]. Moreover, Evans-Lutterodt *et al.*[16-17] have used the compound kinoform hard-X-ray lenses to exceed the critical angle limit. There are also discussions on some concrete types of total-reflection X-ray optics. For example, Kirkpatrick *et al.*[18] have discussed the resolution limit of the KB mirror pairs. A similar discussion of combined total-reflection mirror, such as tandem-toroidal mirror[19], has been carried out. As mentioned above, Bergemann *et al.*[14] have discussed the resolution limit of taper monocapillary total-reflection X-ray optics from the point of view of the waveguideX-ray optics.

In this paper, we discuss the resolution limit of total-reflection X-ray optics using the Heisenberg uncertainty principle. The study does not focus on a particular type of total-reflection X-ray optics, as it applies to any total-reflection X-ray optics.

2 Resolution limit of total-reflection X-ray optics

X-rays are part of the electromagnetic radiation spectrum, and are therefore characterized by wave-particle duality. We use the relationship of the position and momentum of X-ray photon based on quantum mechanics to evaluate the consequences of the critical angle for total external reflection θ_c on the resolution limit of total-reflection X-ray optics. Let us suppose that x and p are the position and momentum of photon, respectively. The Heisenberg uncertainty principle can be written as

$$\Delta x \Delta p_x \geqslant \hbar/2, \tag{1}$$

where $\hbar = h/(2\pi)$ is the reduced Planck constant.

As show in Fig. 1, for the X-ray beam with a divergence 2θ, we have

$$\Delta x \Delta p_x = \Delta x \cdot p \sin\theta \geqslant \frac{\hbar}{2} = \frac{h}{4\pi} = \frac{p\lambda}{4\pi}. \tag{2}$$

so that

$$\Delta x \geqslant \frac{\lambda}{4\pi \cdot \sin\theta}, \tag{3}$$

here λ is the wavelength of X-rays.

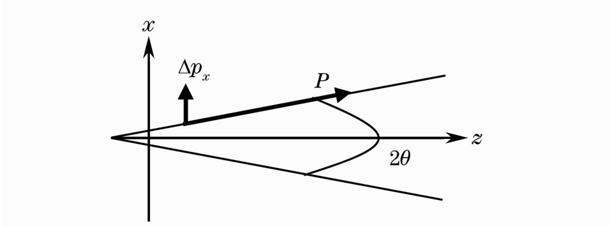

Fig.1 Scheme of X-ray beam with divergence 2θ

In total-reflection X-ray optics, X-rays are reflected by many reflecting particles on the surface of the optics, and then overlap at the focal plane. The divergence of X-rays which undergoes total reflection by a certain reflecting particle on the surface of the optics is θ_c. Therefore, for the wave packet in the X-ray beam reflected by a certain reflecting particle on the surface of the optics, we have

$$\Delta x \geqslant \frac{\lambda}{4\pi \cdot \sin\frac{\theta_c}{2}} = S. \tag{4}$$

This indicates that even if different X-rays, which are reflected respectively by various reflecting points on the surface of the optics, overlap precisely at the same focal plane (focal spot), the minimum size of this focal spot is not smaller than S. And therefore, S is the resolution limit of total-reflection X-ray optics for the X-rays with wavelength λ.

The critical angle θ_c can be written as

$$\theta_c = \sqrt{2\delta} = \frac{\lambda\sqrt{\rho \cdot r_0}}{\sqrt{\pi}}, \tag{5}$$

where δ is the imaginary part of the deviation of the refractive index from unity, ρ is the electron number density, and r_0 is the electron radius. Since the critical angle θ_c is of the order of a milli-radian, we let

$$\sin(\theta_c/2) \approx \theta_c/2. \tag{6}$$

With Eqs.(5) and (6), we have the resolution limit

$$S = \frac{1}{2\sqrt{\pi \cdot r_0 \cdot \rho}}. \tag{7}$$

It can be obtained from Eq.(7) that the resolution limit of total-reflection X-ray optics depends on the material, and is independent of energy. For example, for the nickel total-reflection X-ray optics,

$$\theta_c = \frac{61}{E}\text{mrad}, \tag{8}$$

here $E = \frac{12.4}{\lambda}$ is X-ray photon energy in keV. Therefore, for the nickel total-reflection X-ray optics,

$$S = \frac{\lambda}{4\pi \cdot \sin\frac{\theta_c}{2}} = \frac{12.4 \times 10^{-7}}{2\pi \times 61} \text{ m} = 3.2 \times 10^{-9} \text{ m} = 3.2 \text{ nm}. \quad (9)$$

For the lead glass and borosilicate glass total-reflection X-ray optics[20] with θ_c of $46/E$ and $30/E$, the corresponding S values are 4.2 nm and 6.6 nm, respectively.

Table 1 shows the S values of total-reflection X-ray optics made of different materials.

Table 1 S values of total-reflection X-ray optics made of different materials

Material	C	Be	Al	Rh	Pt	Au	W	Ag	Si	Ge
Density /(g/cm^3)	2.27	1.85	2.70	12.41	21.45	19.30	19.25	10.49	2.33	5.32
S /nm	6.6	7.3	6.0	2.8	2.1	2.2	2.2	3.0	6.5	4.3

To our knowledge, there are not currently any theoretical and experimental results reaching our conclusions of the resolution limit of the total-reflection X-ray optics. For example, our result of S about glass total-reflection X-ray optics is smaller than that obtained from the point of view of the waveguide X-ray optics by a factor of about 2. Taking such X-ray optics made of SiO_2 as an example, our result is 6.9 nm, and the result from the point of view of the waveguide X-ray optics is 13.4 nm when the incoming beam consists of only the lowest mode[14].

3 Conclusion

The resolution limit of total-reflection X-ray optics could be obtained with the Heisenberg uncertainty principle. For the total-reflection X-ray optics made of nickel, lead glass and borosilicate glass, the resolution limits are 3.2 nm, 4.2 nm and 6.6 nm, respectively. The resolution limit of the total-reflection X-ray optics depends on their materials.

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