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Reasearch on the Interplay Between Disorder and Effective Three-Body Interaction of Bosons in a Double-Well Potential

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Abstract We investigate the phase coherence of bosons in a double-well trap with a random energy mismatch that is used to mimic disorder in the presence of effective three-body interaction. It is found that, in such a double-well disordered Bose-Hubbard model, the renormalized three-body interaction can enhance the phase coherence of the bosonic system in the presence of disorder. The dependence of the average phase coherence on the effective three-body interaction in the situation of strong disorder is also studied. To our best knowledge, it is the first time that the effective three-body interaction is investigated in the context of disordered quantum gases.

Key words atomic and molecular physics; quantum gas; phase coherence; disorder; three-body interaction

OCIS codes 020.1335; 020.1670; 270.1670

双阱玻色气体中的无序与有效三体相互作用研究

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摘要 研究了存在原子间有效三体相互作用的情况下双阱势场中超冷玻色气体的相位相干性，其中引入无序分布的阱深差来模拟无序势。研究发现，在无序 Bose-Hubbard 模型中，重整化的三体相互作用能有效增强玻色气体的相位相干性。同时，在强无序势条件下，讨论了平均相位相干性对三体相互作用的依赖关系。据我们所知，这是首次对无序量子气体中的有效三体相互作用进行研究。

关键词 原子与分子物理；量子气体；相位相干性；无序；三体相互作用

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1 Introduction

Since Anderson[1] predicted the effect of disorder on non-interacting particles in 1958, quantum localization has been intensively investigated in a variety of physical systems[2]. Thanks to rapid experimental development of producing, probing and manipulating ultracold atoms with unprecedented versatility and tunability, Anderson localization of the matter wave has been observed in Bose-Einstein condensates (BECs) irradiated by laser speckles and quasi-periodic optical lattices[3-4]. Moreover, more and more theoretical and experimental efforts have been made to understand the interplay between disorder and interaction in the weak-interacting and strong-correlated quantum gases[5-10].

On the other hand, with the ever-increasing precision in recent experiments with ultracold atoms[11-12], it becomes possible to observe effects beyond the standard Bose-Hubbard (BH) model. Conventionally, only binary interaction resulted from the collision between two atoms in the lowest band has been considered in a standard Bose-Hubbard model. However, multi-band effect has been measured in quantum phase revival spectroscopy[13-14] and photon-

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assisted tunneling[15]. In these experiments, single-band assumption should be checked carefully because virtual transitions of the atoms between the ground and excited bands have a non-trivial contribution to the many-body system. In fact, the multi-band correction can be renormalized into effective multi-body interaction of the atoms in the ground band[16-17]. To our knowledge, many concerns have been given to investigate the interplay between atomic binary interaction and disorder in the context of quantum degenerated gas[5-8], however, multi-body interactions have not been discussed by now.

In this work, we investigate the interplay between disorder and density-dependent three-body interaction in the framework of a simplified double-well model. We provide a thorough analysis of phase coherence by solving the two-site BH problem with random disordered potential firstly presented by Zhou *et al.*[18].

2 Theoretical model

Following the routine of Zhou *et al.*[18], we consider the effective three-body interaction of bosons in a randomly tilted double-well potential. The Hamiltonian can be written as

$$H = -t(b_L^\dagger b_R + b_R^\dagger b_L) + \frac{U_2}{2}\sum_{i=L,R} n_i(n_i-1) + \frac{U_3}{6}\sum_{i=L,R} n_i(n_i-1)(n_i-2) + \frac{\varepsilon}{2}(n_R - n_L), \quad (1)$$

where $b_L^\dagger$ (b_L) and $b_R^\dagger$ (b_R) are the creation (annihilation) operators in the left and right wells, respectively, $n_L = b_L^\dagger b_L$ ($n_R = b_R^\dagger b_R$) is the number operator in the left (right) well, t is the tunneling amplitude between the two wells, U_2 and U_3 are the on-site two-body and effective three-body interaction strengths, respectively, and ε is the energy mismatch between the subwells. In our case, ε is randomly distributed according to a certain probability function $P(\varepsilon)$, thus simulating disorder. Here we focus on the uniform disorder case $P(\varepsilon) = 1/(2\Delta)(-\Delta \leqslant \varepsilon \leqslant \Delta)$, where Δ characterizes the disorder strength. Other distribution functions for ε do not change the qualitative conclusion presented in this paper. The above equation can be viewed as a two-site version of the extensively studied BH model with random on-site energies.

To solve the problem, we start from the case of N atoms in the double-well with a fixed ε. We write the Schrödinger equation $H|\Psi\rangle = E|\Psi\rangle$ in the Fock space. The basis states are defined by $|l\rangle = |N_L, N_R\rangle = |\frac{N}{2}-l, \frac{N}{2}+l\rangle$, where $l = 0, \pm 1, \pm 2, \cdots, \pm\frac{N}{2}$. For simplicity, we assume N is a large even number. Expanding $|\Psi\rangle = \sum_l \psi_l |l\rangle$, we obtain the eigen equation given by

$$(E - E_l)|l\rangle = -tM_{l,l+1}|l\rangle - tM_{l-1,l}|l-1\rangle, \quad (2)$$

where $E_l = U_2 l^2 + \varepsilon l + \frac{U_2}{2}(N^2/2 - N) + \frac{U_3}{6}[3(N-2)l^2 + N^2/4 - 3N^2/2 + 2N]$, and $M_{l,l+1} = M_{l+1,l} = \sqrt{N/2(N/2+1) - l(l+1)}$. The eigen energies and eigen functions can be easily calculated by exact diagonalization. At 0K, the phase coherence between the subwells can be characterized as $C_\varepsilon = \frac{1}{N}\langle b_L^+ b_R\rangle_\varepsilon = \frac{1}{N}\sum_l M_{l,l+1}\psi_l^0\psi_{l+1}^0$, where ψ_l^0 is the ground-state wavefunction in the Fock space. $\langle O\rangle$ is the expectation value of the operator O in the ground state, and the subscript implies the fixed ε. As mentioned above, given a certain possibility on distribution of disorder $P(\varepsilon)$, the total effect can then be obtained by averaging those results for each ε as follows:

$$\langle O\rangle = \int_{-\Delta}^{\Delta} d\varepsilon P(\varepsilon)\langle O\rangle_\varepsilon. \quad (3)$$

In this work, following the line of Ref. [18], we will focus on the phase coherence of the bosons in such a double-well in the presence of the effective three-body interaction.

3 Numerical results and discussions

Now we investigate the effects of three-body interaction on the phase coherence (C_ε and $\bar{C}$) of bosons in optical lattice, respectively. As is well known, in a clean lattice, phase coherence is completely lost once the lattice depth is large enough and all atoms are isolated in the lattice sites, because the atom number in each site is fixed. On the contrary, the atoms can tunnel between neighbor sites in a shallow lattice and the phase coherence is restored and the superfluid phase appears[19]. The phase transition from a superfluid condensate to a Mott insulator is intrinsically interaction-induced localization, because mutual interaction between atoms predominates the tunneling and results in the localization of the atoms in each site and the phase coherence information are erased. Different from Mott insulator, Anderson localization is related to strong disorder experienced by the atoms even in the absence of interaction[20]. As $U_2 \neq 0$ ($U_3 = 0$), the phase coherence C_ε does not monotonously decrease as ε increases, while it has oscillating characteristics, which is induced by the discretization of E_l. The oscillating behavior of C_ε becomes more obvious as U_2 increases remarkably. So in the presence of interaction, weak disorder could strengthen the phase coherence in a bosonic system[18].

First we calculate the phase coherence for a fixed mismatch ε in the absence and presence of the effective three-body interaction, as shown in Fig. 1. For simplicity, binary interaction strength $U_2/(tN) = 1$ and the total atom number $N = 100$ are chosen to illustrate the physics. The curve of the case $U_3 = 0$ reproduces the key result in Ref. [18], which shows a non-monotonic decrease of the phase coherence in an interacting quantum gas. Furthermore, in the presence of the effective three-body interaction, we find several interesting phenomena that may reflect some complicated physics. Similar to the case of $U_3=0$, one can be also observed a remarkable oscillation of C_ε, however, the oscillation period grows as U_3 increases. As we know, local maxima of the curve emerge at certain values of ε, where $E_{l^*}=E_{l^*}-1$ is satisfied[18], and we find the local maxima $\varepsilon^*=[U_2+\frac{U_3}{2}(N-2)](1-2l^*)$. It is obvious that the distance between two neighbor local maxima will increase when U_3 becomes larger. Unexpectedly, the trend of slow decrease of C_ε as ε increases is not satisfied in the situation of strong three-body interaction strength. When U_3 is large (e. g., $U_3=0.1$), the phase coherence does not have the descending tendency with oscillating characteristics. However, in the absence of three-body interaction, the phase coherence will decline with oscillating behavior. So three-body interaction can maintain the phase coherence and superfluid state of the system, and prevent the phase transition to Mott insulator.

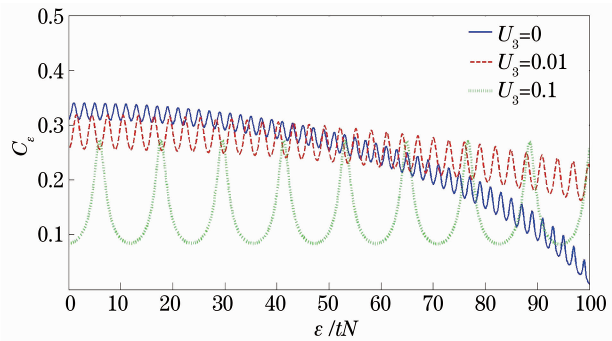

Fig. 1 C_ε as a function of ε/tN for different three-body interaction strengths U_3 of 0 (solid), 0.01 (dashed), and 0.1 (dotted). $U_2/(tN)$ is set to 1 for all curves. The total atom number is $N=100$.

Now we turn to the average phase coherence $\bar{C}=\int_{-\Delta}^{\Delta}\mathrm{d}\varepsilon P(\varepsilon)C_\varepsilon$ and depict the influence of the three-body interaction on $\bar{C}$ (shown in Fig. 2). As shown by the solid curve in Fig. 2, $\bar{C}$ drops down monotonously as U_3/U_2 increases for a weak disorder potential (i. e., small Δ). Here both of binary and three-body interactions contribute to the trend of localization in the strong interaction limit because they favor the on-site interaction and are unfavorable to the hopping of the atoms between neighbor sites. As the strength of disorder increases, such monotonicity of the curve is not obvious. As shown by the two curves of $\Delta=10$ and $\Delta=50$ in Fig. 2, although the declining tendency of $\bar{C}$ still appears, however, twists and turns emerge remarkably in the curves. In addition, in the limit of large U_3 and with a strong disorder (for example $\Delta=50$), $\bar{C}$ tends to a larger value than that in the case of weak disorder. Particularly, when three-body interaction and disorder strength are both significant, $\bar{C}$ does not disappear completely, which means that the atoms have not been completely isolated and the localization is absent. This finding strongly supports the "two negatives make a positive" effect predicted in Ref. [18], while three-body interaction and disorder coexist, the system cannot run to the entire localization.

Furthermore, in order to analyze the influence of disorder on the coherence in the presence of the effective three-body interaction, we calculate $\bar{C}$ as a function of the disorder strength Δ as shown in Fig. 3. Apparently, when three-body interaction does not exist, $\bar{C}$ decreases monotonously with increasing disorder strength Δ/tN, and the localization of particles is enhanced. As U_3 increases, $\bar{C}$ performs some oscillating behavior and the resonance feature of the curves becomes significant, $\bar{C}$ even does not tend to localization when the disorder is strong enough. And this result directly reveals "two negatives make a positive" effect described in Fig. 2. We also note that the larger U_3 is, the more minimum phase coherence of $\bar{C}$ the system can maintain, even though $\bar{C}$ could not go to zero. It is also quite clear that $\bar{C}$ first increases when the disorder strength Δ grows from zero ($\Delta<10$), and then the peaks of $\bar{C}$ become wiggles on the top of the slowly decaying curves. This fact shows the significant difference of the disorder effect between the presence and absence of three-body interaction. In the presence of three-body interaction, the disorder effect is non-monotonic.

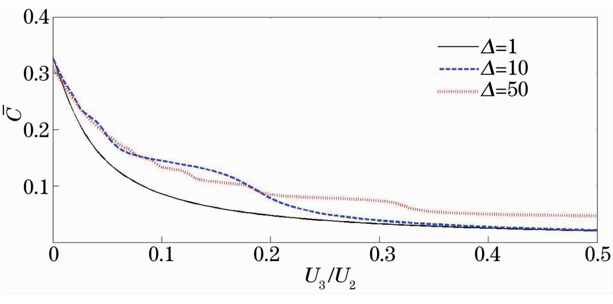

Fig. 2 Average phase coherence $\bar{C}$ as a function of U_3/U_2 at different disorder strengths. $U_2/(tN)=1$ is fixed.

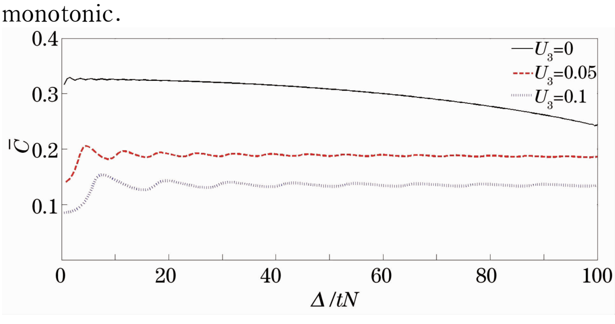

Fig. 3 $\bar{C}$ as a function of Δ/tN for different disorder strengths of the effective three-body interaction. $U_2/(tN)=1$ is also fixed.

At last, we turn around to review the influence on phase coherence by two-body interaction characterized by the strength U_2. Zhou *et al.*[18] have pointed out that in a disordered double-well, with increasing U_2, $\bar{C}$ firstly increases monotonously and then decreases, and this phenomenon is totally different from that in a clear system. We calculate $\bar{C}$ as a function of U_2 for $U_3 = 0$, 0.05, 0.1, respectively, where the disorder strength Δ is fixed to 20, and the corresponding calculation results are shown in Fig. 4. From the curves in Fig. 4, we find immediately that when the three-body interaction exists, with increasing U_2/tN, the monotonic increasing tendency of $\bar{C}$ cannot be found, instead of decreasing. It directly reveals that the three-body interaction destroys the phase coherence of the system, i. e., in some extent, high order interactions can result in localization. Furthermore, we also note that in the presence of significant strength of two-body interaction, although the strength of three-body interaction is different, the average phase coherence $\bar{C}$ still tends to a universal value close to 0.1. In general, the effective many-body interactions are much weaker than the binary one in this situation, and the competition between interaction and disorder is resolved primarily by two-body interaction.

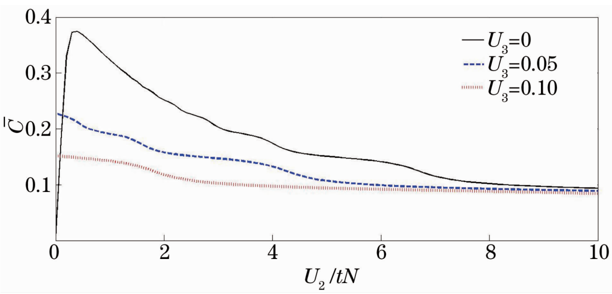

Fig. 4 $\bar{C}$ as a function of U_2 for different three-body interaction strengths. The disorder strength Δ is fixed to 20.

4 Conclusion

In conclusion, we have studied the coherent property of a disordered quantum gas in a double-well when the binary and effective three-body interactions are present. We find that the multi-orbital energy plays an essential role on the average phase coherence of the system. We expect this work will be helpful to understand more intricate phenomena in the context of interacting disordered quantum gases.

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