Image denoising algorithm based on even step-length generalized cross validation model

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Abstract Generalized cross validation (GCV) is a significant mean square error (MSE) estimator. It is widely used for image denoising because it can provide an optimal denoising threshold for these wavelet coefficients of noise image. However, the computational complexity of GCV is higher than that of the universal threshold denoising algorithm. In this study, an efficient and fast image denoising algorithm is proposed based on even step-length (ESL) GCV model. In ESL-GCV model, only the thresholds on even points are calculated from four to the maximum wavelet coefficient. In addition, the ESL-GCV model is optimized using the integer wavelet transform (IWT). These experimental results show that the IWT-based ESL-GCV model can provide lower computational complexity and the better peak signal-to-noise ratio (PSNR) than those of the traditional GCV. The proposed algorithm has important theoretical and practical value for image denoising in the future.

Key words image processing; image denoising; integer wavelet transform; generalized cross validation

OCIS codes 100.7410; 100.0100

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1 Introduction

The image is often corrupted by noise in its collection, acquisition, or transmission. Because the noise is the main factor that influences image quality and greatly affects to extract the information, it must be removed. Some images such as the remote sensing images consist of a great mount of data. So an excellent image denoising algorithm must have both good performance and good efficiency[1].

Image denoising is one of the main researches in the field of image processing. According to the different characteristics of the image and noise, the researchers have made a lot of image denoising algorithms. These algorithms can be divided into two categories: spatial domain denoising and transform domain denoising. Spatial domain denoising algorithm is a classic denoising algorithm which appears earlier and has more complete theoretical basis. Pixels of the image are processed directly in this algorithm. Representative algorithms are mean filtering algorithm and median filtering algorithm[2]. Transform domain denoising algorithm uses the different characteristics of the useful signal and noise signal in the transform domain to remove the noise effectively. Representative algorithms are denoising algorithms based on Fourier transform, independent component analysis (ICA), and wavelet[3].

Wavelet shrinkage is an effective method to reduce Gaussian noise. The principle is to set a threshold. Those coefficients less than the threshold are set to zero, for they are greatly affected by noise and contain little information. Those coefficients carry the main information of the original image while the noise is relatively small[4]. So those coefficients will be kept or shrinked if they are greater than the threshold, and the noise is reduced while the main information of the original image can be retained. The selection of optimal threshold is crucially important compared with other factors involved in the process. A large threshold might remove important image features and cause a serious distortion of the signal. A small threshold might remain noisy that cannot be accepted by the observer. The optimal threshold minimizes the error of the denosing result compared with the unknown, noise-free data. The universal threshold by Donoho *et al.*[4] is widely known and states that the optimal value is proportional to amount of noise. In fact, the universal threshold is an upper bound for optimal threshold for finite number of coefficients. Jansen *et al.*[5] applyed the generalized cross validation (GCV) function as a mean square error (MSE) estimator to select the optimal threshold through a minimize MSE process. However, the computational cost of GCV algorithm is much too high. Jansen *et al.*[5] pointed out that the implementation of GCV algorithm was rather slow. On one hand the scale and spatial resolution of the image, especially the remote sensing image are increasing quickly. On the other hand, the real-time image processing is highly demanded. This bottleneck greatly limits the application of GCV methods in image restoration.

In this study, an efficient and fast image denoising algorithm is proposed based on even step-length (ESL) GCV model. These experimental results show that the integer wavelet transform (IWT)-based ESL-GCV model can provide lower computational complexity and the better peak signal-to-noise ratio (PSNR) than that of the traditional GCV.

2 Image denoising model based on the DWT

There is a variety of classification methods of noise in digital image processing. From the statistics, they can be divided into smooth and non-stationary noises. From the noise amplitude, they can be divided into Gaussian noise, Rayleigh noise, Erlang (gamma) noise, exponential noise, uniform noise, impulse noise (salt & pepper noise), and so on. From the impact on the signal, they can be divided into additive noise and multiplicative noise.

We denote $f(x,y)$ is the ideal image, $n(x,y)$ is the noise, and $g(x,y)$ is the actual output image. For the additive noise, its characteristic is that $n(x,y)$ has nothing to do with image light levels. That is

$$g(x,y) = f(x,y) + n(x,y). \tag{1}$$

For the multiplicative noise, its characteristic is that $n(x,y)$ has something to do with image light levels. That is

$$g(x,y) = f(x,y) \times [1 + n(x,y)]. \tag{2}$$

The discrete wavelet transform (DWT)-based image denoising is the image denoising in wavelet domain. Thus, image threshold denoising algorithm based on DWT has three basic steps: 1) calculate the wavelet transform of the image; 2) set threshold for the wavelet coefficients and estimate the true wavelet coefficients; 3) do inverse wavelet transform after the treatment of wavelet coefficients. The DWT-based image denoising process is shown in Fig. 1.

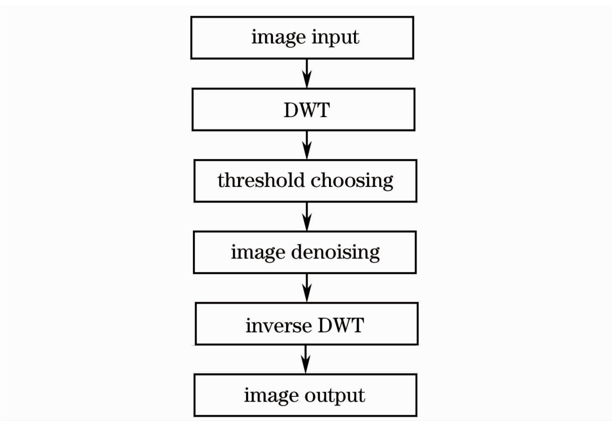

Fig. 1 DWT-based image denoising.

Hard threshold method is very good to retain the local characteristics of the image edge. But, the image will appear visual distortion like ringing, the pseudo-Gibbs effects. The results of the soft thresholding method are relatively smoother, but it will result in the distortion like edge blur. Semi-soft and semi-hard threshold method is able to achieve a good compromise between the method of soft threshold and hard threshold. Therefore, the different approaches should be taken to different parts of the image. For example, low-frequency part is smooth at the edges, so use a soft threshold and there is no blur problem. The high-frequency, high-frequency subband (HH) part of high-frequency part is the most high-frequency part (the most dramatic changes on the edge) and the coefficient is very small, so use a hard threshold for processing. It can retain the local characteristics of the edge of the original image well and there is no image distortion. The high-frequency, low-frequency subband (HL) and low-frequency, high-frequency subband (LH) parts of high-frequency part are between the low-frequency part and the most high-frequency part. So, the approach between the soft threshold and hard threshold, which is semi-soft and semi-hard threshold method, can be used.

3 GCV model

In the proposed denoising algorithm based on the DWT[4], the universal threshold is an upper bound for optimal threshold for finite number of coefficients. So Jansen *et al.*[5] applied the GCV function as a MSE estimator to select the optimal threshold through a minimize MSE process. The GCV is derived from cross validation (CV). Assume that the original signal is regular to some extent, which means that the value of one pixel could be approximated by a linear combination of its neighbors. So we can eliminate the noise in this particular component by considering the replacement of a weighted average of its neighbor pixels. A relatively clean, noise-independent value is acquired, which could be used in the computation of an approximation for MSE.

If we definite δ as a selected threshold, then the GCV function can be described as

$$\mathrm{GCV}_j^c(\delta) = \frac{1}{N_j^c}\left\| \omega_j^c - \omega_{j,\delta}^c \right\|^2 \Big/ \left(\frac{N_{j0}^c}{N_j^c}\right)^2, \tag{3}$$

where ω_j^c is the vector of wavelet coefficients at the resolution level j and for component c; $\omega_{j,\delta}^c$ is the vector of shrunk coefficients for a given threshold δ and N_{j0}^c is the number of zero elements in this vector. For a large number of coefficients, the minimizer of $\mathrm{GCV}_j^c(\delta)$ also can minimize the mean square error of the threshold coefficients. For finite N_j, it gives better estimates of the optimal threshold than the universal threshold.

4 ESL GCV model

When the GCV function is taken to the minimum, the calculated result is the requested GCV threshold. When the threshold is δ_i, let $\delta_i = i$ $(i = 1, 2, \cdots, N)$, the GCV function value is GCV[δ_i]. It is easy to know that the step is 1. GCV values are specific when $i=1$ and 2. So i is selected from 4, that is $i=4,5,\cdots,N$. Evaluate the value of GCV[δ_i] and make a comparison. Finally, threshold is determined to $\tilde{\delta}=\operatorname{argmin}\mathrm{GCV}(\delta_i)$.

In order to improve the efficiency of the algorithm, the ESL-GCV model is proposed. It can reduce the amount of computation for the calculated value of GCV function by five steps.

1) Based on Eq.(3), we can calculate GCV using

$$\mathrm{GCV}_j^c(\delta)=\frac{\frac{1}{N_j^c}\left\|\omega_j^c-\omega_{j,\delta}^c\right\|^2}{\frac{(N_{j0}^c)^2}{(N_j^c)^2}}=\frac{N_j^c\left\|\omega_j^c-\omega_{j,\delta}^c\right\|^2}{(N_{j0}^c)^2}. \quad (4)$$

Because N_j^c is the number of wavelet coefficients at the resolution level j and for component c, we can use the result of GCV'^c_j instead of the result of GCV_j^c in Eq.(4) as

$$\mathrm{GCV}'^c_j(\delta)=\frac{\left\|\omega_j^c-\omega_{j,\delta}^c\right\|^2}{(N_{j0}^c)^2}. \quad (5)$$

In fact, the computational complexity of Eq.(5) is lower than that of Eq.(3).

2) Let $\delta_i=i$ and $i=2r$, $(r=2,3,\cdots,N/2)$, calculate the result of $\mathrm{GCV}'^c_j(\delta_i)$ and find the minimizer of $\mathrm{GCV}'^c_j(\delta_i)$, It is easy to know that the Eq.(6) is correct.

$$\operatorname{argmin}[\mathrm{GCV}_j^c(\delta)]=\operatorname{argmin}[\mathrm{GCV}'^c_j(\delta)]. \quad (7)$$

3) Calculate the results of $\mathrm{GCV}'^c_j(\delta_{i-1})$ and $\mathrm{GCV}'^c_j(\delta_{i+1})$;

4) Compare $\mathrm{GCV}'^c_j(\delta_i)$ with the $\mathrm{GCV}'^c_j(\delta_{i-1})$ and $\mathrm{GCV}'^c_j(\delta_{i+1})$, find the minimum value $[\mathrm{GCV}'^c_j(\delta_i)]_{\min}$;

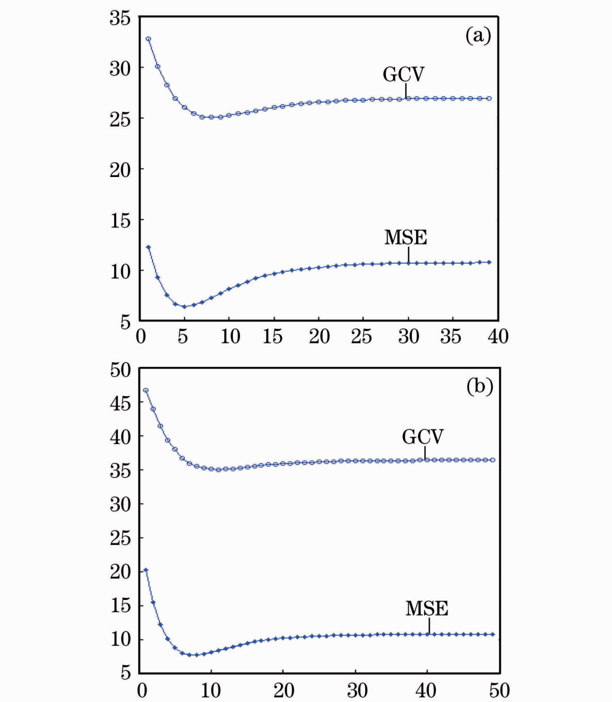

Fig.2 Comparison between GCV and MSE. Noise standard deriatiations are (a) 4 and (b) 5

5) Use the following equation to require the denoising threshold δ.

$$\delta=\operatorname{argmin}\{[(\mathrm{GCV}'^c_j(\delta_i)]_{\min}\}. \quad (8)$$

Compared with GCV, the computational efficiency of the ESL-GCV model is less than 50% of that of GCV. However, there is no change for the ESL-GCV model in GCV theoretical knowledge.

The threshold which minimizes the GCV function is the right asymptotically optimal threshold for image denoising. The experiential results in Fig.2 also testify this theory. The X-coordinate is the threshold of wavelet coefficients and the Y-coordinate is the output values of GCV and MSE functions. The monotonicity of GCV and MSE are quite similar which provide a concrete prove that the GCV and MSE reach their minimum at pretty much the same threshold.

5 IWT for image denoising

The DWT is widely used in image denoising. Donoho *et al*.[4] proposed a soft threshold method based on wavelet transform. By setting the threshold of wavelet coefficients, these wavelet transform coefficients interfered by noise can be weakened. After wavelet inverse transform reconstruction, noise is suppressed to a certain extent. The edge information can be reserved better and there is no pseudo-edge. However, the DWT has three main disadvantages.

1) These coefficients of the reconstruction image are floating-point and the reconstruction image is lossy by the DWT.

2) Transformation process requires a large number of convolution operations and it leads to high computational complexity. But the image data of SAR is very large, so the workload is very large.

3) In order to ensure the integrity of the transformation, it must do boundary extension for the image, increase the complexity of the codec, and add storage expenses before the wavelet transform.

Using reversible IWT for compression of image has two advantages. Firstly, IWT has lower computational complexity than DWT because of the lifting scheme (LS). Second, the use of IWT is also a means to reduce the memory demands of the denoising algorithm as integers are used instead of real numbers[6,7].

The LS presented by Sweldens *et al*.[6], supports the low-complexity and efficient IWT scheme. Using IWT for image denoising can reduce the memory demands of the denoising algorithm as integers are used instead of real numbers, which is very significant for medical and remote sensing image processing. Two IWTs known to be effective for image denoising are evaluated in Table 1.

In Table 1, we use the notation (x,y) to indicate that the underlying filter bank has lowpass and highpass analysis filters of lengths x and y, respectively. In the forward transform equations, the input signal, lowpass subband signal, and highpass subband signal are denoted as $x[n]$, $s[n]$ and $d[n]$, respectively. For convenience, we also define the quantities $s_0[n] = x[2n]$ and $d_0[n] = x[2n+1]$.

Table 1 Several Forward Transform of IWTs

Name (x, y)	Forward Transform of IWT
(5,3)	$\begin{cases} d[n] = d_0[n] - [1/2(s_0[n+1] + s_0[n])] \\ s[n] = s_0[n] + [1/4(d[n] + d[n-1]) + 1/2] \end{cases}$
(13,7)	$\begin{cases} d[n] = d_0[n] + [1/16((s_0[n+2] + s_0[n-1]) - 9(s_0[n+1] + s_0[n]) + 1/2)] \\ s[n] = s_0[n] + [1/32(9(d[n] + d[n-1]) - (d[n+1] + d[n-2]) + 1/2] \end{cases}$

6 Experimental results

Experiments are carried out on Matlab. Two level 5/3 IWT is applied on remote sensing image (1024 × 1024 Stockton) with additive white Gaussian noise. First, we compare the subjective effects of denoising. Figures 3(a) and (b) are the remote sensing images with additive white Gaussian noise which the standard deviations are 20 and 30, respectively. Figures 3(c) and (d) are respectively denoising results using the ESL-GCV model and IWT.

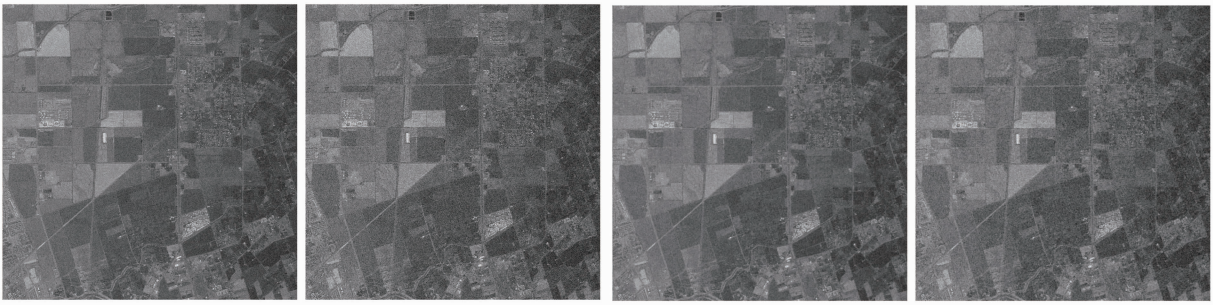

Fig.3 Remote sensing images with (c) $\sigma = 20$ and (b) $\sigma = 30$; (c) and (d) are denoising images of (a) and (b).

Table 2 Optimal Coefficient Scaling Factors of Several IWTs

	GCV			ESL-GCV		
Subband Component	HH	HL	LH	HH	HL	LH
Time Spending (s)	1.598	1.584	1.923	0.706	0.637	0.924

7 Conclusion

An efficient and fast image denoising algorithm is proposed based on ESL-GCV model. In ESL-GCV model, only the thresholds on even points are calculated. In addition, the ESL-GCV model is optimized using the IWT. These experimental results show that the new denoising algorithm can provide lower computational complexity than that of the traditional GCV. The proposed algorithm has important theoretical and practical value for image denoising in the future.

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