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Fractional Fourier Transform of Modified Laguerre-Gaussian Beams

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Abstract The properties of Modified Laguerre-Gaussian beams (MLGBs) propagating through a fractional Fourier transform (FRFT) optical system are investigated. The analytical transformation formula for MLGBs propagation through an FRFT optical system is derived based on definition of the FRFT in the cylindrical coordinate system. By using the derived formula, the normalized intensity distribution of MLGBs in the FRFT plane is graphically illustrated with numerical examples, and the influence of different parameters on the normalized intensity distribution is discussed.

Key words physical optics; modified Laguerre-Gaussian beams; fractional Fourier transformation; intensity distribution

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1 Introduction

Optical vortices have drawn increasing attention due to their various applications, such as trapping and manipulation of small particles, metrology, microlithography, medical imaging, and surgery[1~3]. Many different vortex beams, such as Bessel-Gaussian, Laguerre-Gaussian, Hermite-Gaussian, hypergeometric-Gaussian (HyGG) beams[4~9], have been studied theoretically and experimentally. Modified Laguerre-Gaussian beams (MLGBs), special cases of HyGG beams introduced by Karimi *et al.*, have attracted much interest in recent years[9,10].

In the past decades, fractional Fourier transform (FRFT) has been widely studied because of its wide application in signal processing, beam shaping, beam analysis, optical security, etc.. FRFT was first introduced into optics in 1993[11]. Since then, much work has been done on its properties, optical implementations and applications[12~15]. Recently, much work has been done about different kinds of beams passing through both apertured and unapertured optical FRFT systems[16~23], such as hollow Gaussian[17], elliptical Gaussian[18], Hermite-Gaussian[19], Hermite-Laguerre Gaussian[20], Bessel-Gaussian[21], and partially coherent Gaussian Schell-model beams[22,23]. However, to the best of our knowledge, MLGBs have never been so considered. For the properties and wide application of MLGBs, the study of the behavior of MLGBs through FRFT optical systems would be of practical interest and importance. In this paper, based on the definition of FRFT, an analytical transformation formula for a MLGB passing through the FRFT system is derived, and the properties of a MLGB in the FRFT plane are investigated with numerical examples.

2 FRFT of MLGBs

The electric field of a MLGB at $z_1 = 0$ in the cylindrical coordinate system is defined as follows[10]:

$$E_{mn}(r_1, \theta_1, 0) = C_0 \left(\frac{r_1}{w_0}\right)^{m+|n|} \exp\left(-\frac{r_1^2}{w_0^2} + \mathrm{i}n\theta_1\right), \tag{1}$$

where C_0 is an arbitrary amplitude constant, r_1 and θ_1 are the radial and the azimuthal coordinates, w_0 is the Gaussian waist width, m is a nonnegative integer, n is the beam order.

The two kinds of optical setups introduced by Lohmann for implementing FRFT are shown in Fig. 1.

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The Lohmann I system and the Lohmann Ⅱ system are equivalent, where f is the standard focal length, f_1 is the focal length of the lens, d is the distance between the input plane P_1 and the fractional Fourier plane P_2, p is the fractional order of the FRFT, and φ is given by $\varphi = p \cdot (\pi/2)$. Obviously, the FRFT system on the fractional order is periodic with the period of 2. For the case $p = 4k$ (k is an arbitrary integer), $d = 0$. For the case $p = 4k+2$, $d \rightarrow \infty$. When $p = 4k+1$, the FRFT reverts to the conventional Fourier transform.

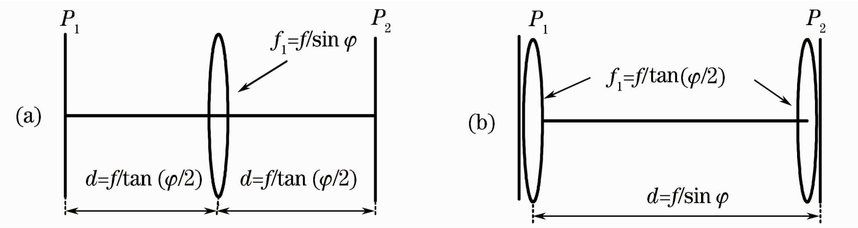

Fig.1 Optical system for performing the FRFT. (a) Lohmann I system, (b) Lohmann Ⅱ system.

Based on the well-known Collins integral formula, the propagation of a MLGB through the FRFT of a p-order optical system can be treated as[17]

$$E_2(r_2,\theta_2) = \frac{1}{\mathrm{i}\lambda f \sin\varphi}\int_0^{+\infty}\int_0^{2\pi} E_1(r_1,\theta_1)\exp\left[\frac{\mathrm{i}\pi(r_1^2+r_2^2)}{\lambda f \tan\varphi}\right]\exp\left[-\mathrm{i}\frac{2\pi r_1 r_2}{\lambda f\tan\varphi}\cos(\theta_2-\theta_1)\right] r_1 \mathrm{d}r_1 \mathrm{d}\theta_1, \quad (2)$$

where λ is the wavelength of the input beam, r_1, θ_1, and r_2, θ_2 are the radial and azimuthal coordinates in the input and output planes, respectively.

Substituting Eq.(1) into Eq.(2), one can obtain

$$E(r_2,\theta_2) = \frac{2\pi C_0}{\mathrm{i}\lambda f\sin\varphi}\int_0^{\infty}\left(\frac{r_1}{w_0}\right)^{m+|n|}\exp\left(-\frac{r_1^2}{w_0^2}\right)\exp\left[\frac{\mathrm{i}\pi(r_1^2+r_2^2)}{\lambda f\tan\varphi}\right] r_1 \mathrm{d}r_1 \times \int_0^{2\pi}\exp(\mathrm{i}n\theta)\exp\left[-\mathrm{i}\frac{2\pi r_1 r_2}{\lambda f\sin\varphi}\cos(\theta_2-\theta_1)\right]\mathrm{d}\theta_1. \quad (3)$$

The following integral formula[24] is used:

$$\int_0^{\infty} x^{\mu}\exp(-\alpha x^2)\mathrm{J}_v(\beta x)\mathrm{d}x = \frac{\beta^v\Gamma(v/2+\mu/2+1/2)}{2^{v+1}\alpha^{v/2+\mu/2+1/2}\Gamma(v+1)}\times {}_1F_1\left(\frac{v}{2}+\frac{\mu}{2}+\frac{1}{2};v+1;-\frac{\beta^2}{4\alpha}\right), \quad (4)$$

where $\Gamma(\cdot)$ is the gamma function and ${}_1F_1(\cdot)$ is the confluent hypergeometric function.

After tedious but straightforward integration, we obtain the output field distribution at the FRFT plane as

$$E(r_2,\theta_2) = \frac{\pi C_0(-\mathrm{i})^{|n|+1}\left(\frac{2\pi r_2}{\lambda f\sin\varphi}\right)^{|n|}}{2^{|n|}\lambda f\sin\varphi w_0^{m+|n|}}\frac{\Gamma\left(\frac{1}{2}m+|n|+1\right)}{\alpha^{\frac{1}{2}m+|n|+1}\Gamma(|n|+1)}\times {}_1F_1\left\{\frac{1}{2}m+|n|+1,|n|+1,-\frac{[2\pi r_2/(\lambda f\sin\varphi)]^2}{4\alpha}\right\}\times\exp\left(\frac{\mathrm{i}\pi r_2^2}{\lambda f\tan\varphi}+\mathrm{i}n\theta_2\right), \quad (5)$$

where

$$\alpha = \frac{1}{w_0^2}-\frac{\mathrm{i}\pi}{\lambda f\sin\varphi}. \quad (6)$$

Equation (5) is the analytical formula of the MLGBs through the FRFT optical system.

3 Numerical Calculations

The propagation properties of the MLGBs through FRFT optical systems are studied by using Eq.(5). In the following calculation, we assume that $f = 100$ mm, $w_0 = 1$ mm, $\lambda = 632.8$ nm, $m = 6$, and $n = 2$. It should be noted that the curves plotted are normalized to the peak intensity. In Fig. 2, the intensity distribution of the MLGBs through the FRFT optical system is plotted against different fractional orders. It is found that when the fractional order $0 < p < 1$, the intensity distribution of the output beam becomes more and more convergent with increasing fractional order. When $1 < p < 2$, the intensity distribution of the output beam becomes more and more divergent with increasing fractional order. Furthermore, the dependence of the intensity distribution of MLGBs in the fractional plane on the fractional order p is periodic, with a period of 2. Thus we can control the intensity distribution of the MLGB by properly choosing the fractional order p between 0 and 2. Finally, we want to point out that the on-axis intensity ($r = 0$) in all cases is always zero in the FRFT plane, which stems from the optical vortex embedded on the axis of a laser beam.

The normalized intensity distribution of the MLGBs against the beam order is shown in Fig.3. The other parameters are the same as those in Fig.2 except that $p = 1.01$. When the order $n = 0$, the intensity distribution shows that MLGBs become hollow-Gaussian beams, which agrees with Fig.4 in Ref.[17]. It can be shown that the radius of the bright ring of the intensity distribution of the MLGBs increases when n increases. It also means that the dark region of the MLGBs increases as n increases. Evolution of the intensity distribution of a MLGB with different positive integers m in the FRFT plane is shown in Fig.4. The other parameters are also the same as those in Fig.2 except that $p = 1.01$. It can be shown that the dark region of the intensity distribution of the MLGBs varies little with m, but the width of the bright ring increases as m increases.

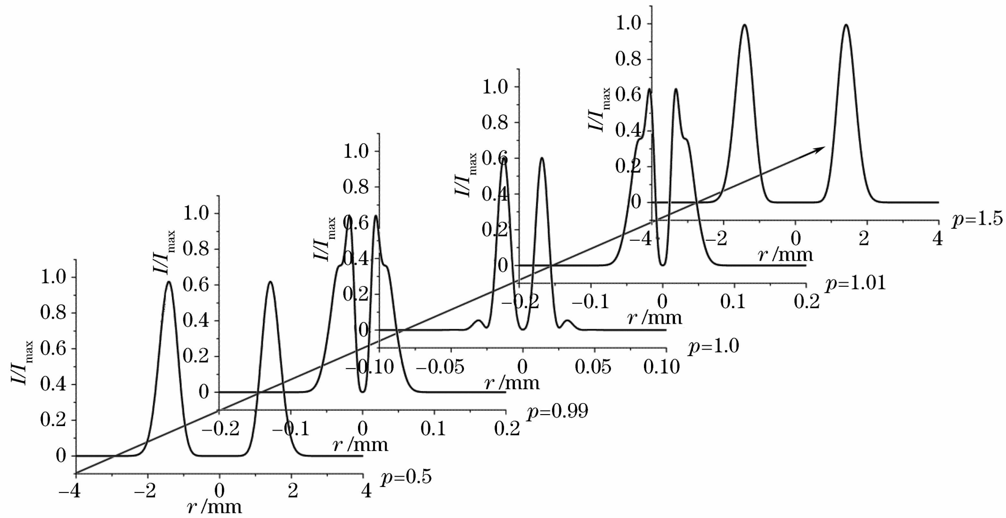

Fig.2 Evolution of the intensity distribution of a MLGB with different fractional orders p in the FRFT plane.

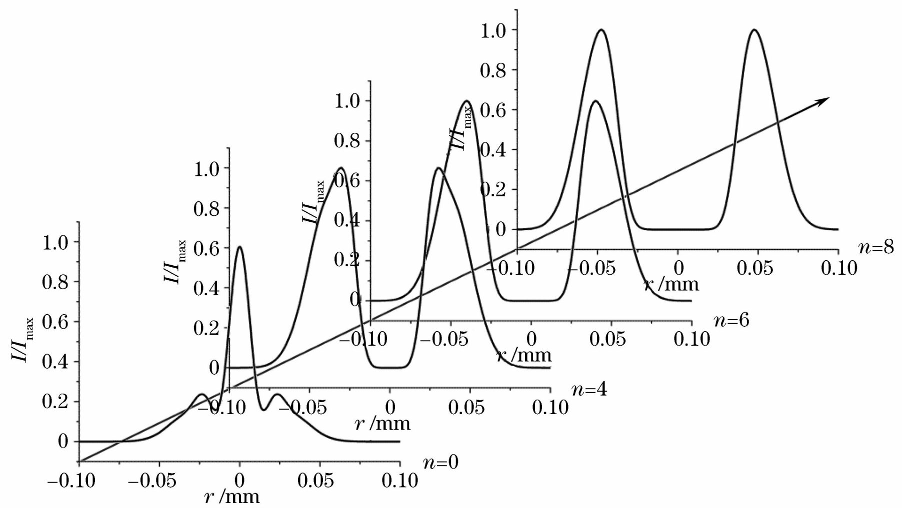

Fig.3 Evolution of the intensity distribution of a MLGB with different beam orders n in the FRFT plane.

From what have been discussed above, we find that we can conveniently control the intensity distribution properties of MLGBs in the FRFT plane by properly choosing the fractional order of the FRFT system and the initial beam parameters.

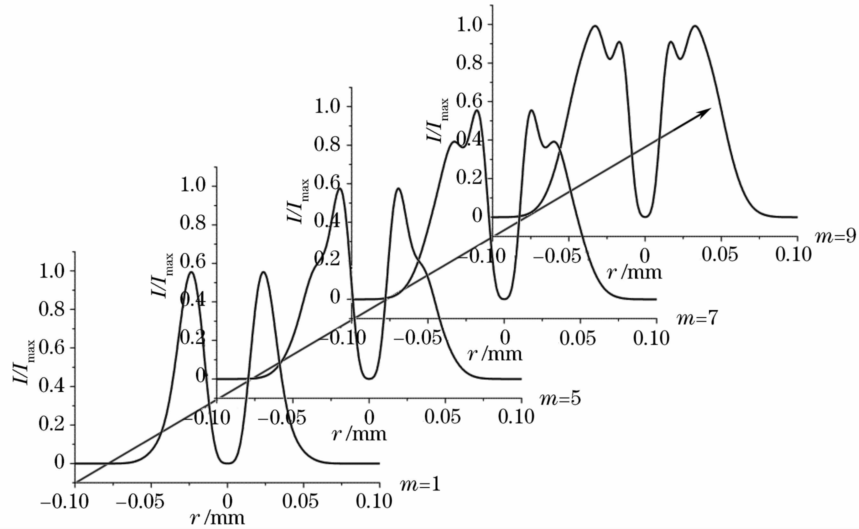

Fig. 4 Evolution of the intensity distribution of a MLGB with different positive integers m in the FRFT plane.

4 Conclusion

We have studied the propagation of MLGBs through an FRFT optical system based on the definition of FRFT in the cylindrical coordinate system. An analytical formula is derived for the FRFT of MLGBs. The results show that the intensity distribution properties of MLGBs in the FRFT plane are closely related to the fractional order and the initial beam parameters. The derived formulas provide a powerful tool for analyzing and calculating the FRFT of MLGBs. The FRFT optical system provides a convenient way for controlling the properties of MLGBs. Our results have potential application in guiding and manipulating cooling atoms and microparticles.

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