文章编号： 0258-7025(2009)Supplememt 1-0096-04

Ultraslow solitons and soliton pairs in asymmetric quantum wells

Yihong Qi（祁义红） Hui Sun（孙　辉） Yueping Niu（钮月萍）
Ni Cui（崔　妮）and Shangqing Gong（龚尚庆）

（*State Key Laboratory of High Field Laser Physics*，*Shanghai Institute of Optics and Fine Mechanics*，*Chinese Academy of Science*，*Shanghai* 201800，*China*）

Corresponding author：*qiyihong@siom.ac.cn*

Abstract Generation of ultraslow solitons and dynamic behaviors of soliton pairs with low intensity input pulse in semiconductor asymmetrical quantum wells (QWs) are investigated. It shows that the QWs structure can provide giant Kerr-type nonlinearity via resonant tunneling，therefore the group velocity dispersion can be balanced，which results in the generation of the ultraslow solitons and soliton pairs，and the initial relative phase of the soliton pairs plays an important role in the propagation of soliton pairs in such a semiconductor QW system. These properties are very useful in practical applications，such as quantum information processing and optical buffers.

Key words ultraslow soliton；asymmetric quantum well；Resonant tunneling；Kerr nonlinearity；group velocity dispersion

CLCN： TN253　　**Document Code**： A　　**doi**：10.3788/CJL200936s1.0096

As a fascinating phenomenon of nonlinearities，solitons have attracted extensive attention in various scientific fields，such as nonlinear optics[1] and Bose-Einstein condensates[2,3]. While optical waves propagate in the nonlinear media，the balance of the interplay between nonlinearity and dispersion (diffraction) results in the shape-preserving propagation phenomena. In early researches，most optical solitons are produced with intense electromagnetic fields，and far-off resonance excitation schemes are generally employed，which results in optical solitons propagation at a speed close to light velocity c in vacuum. But wave propagation in highly resonant media via electromagnetically induced transparency (EIT) has a significant feature of the reduction of the propagation velocity[4]. Such an ultraslow propagation of optical waves brings about some new interesting propagation effects[5~9]，which may be applied in modern optical and telecommunication engineering，such as high fidelity optical buffers，switches，and wavelength converters. Similar ultraslow propagation of optical pulses in semiconductor quantum well (QW) systems has also drawn great attention due to the potentially important applications[10~16]. In asymmetric QWs，resonant tunneling induced quantum interference causes giant Kerr-type nonlinear enhancement[17]，which can balance the group velocity dispersion，then resulting in the generation of the solitons.

This paper considers an n-doped asymmetric AlGaAs/GaAs double QWs system. The basic idea is to combine resonant tunneling induced constructive interference in cross phase modulation (XPM) and tunneling induced transparency (TIT). The band structure is shown in Fig. 1，which is designed at small electron decay rates to reduce the linear absorption effectively. An $Al_{0.07}Ga_{0.93}As$ layer with thickness of 8.3 nm is separated from a 6.9-nm GaAs layer by a 4.8-nm $Al_{0.32}Ga_{0.68}Al$ potential barrier. On the right side of the right well，these is a thin (3.4 nm) $Al_{0.32}Ga_{0.68}As$ barrier. In this structure，one can observe the ground subbands $|1\rangle$ and $|2\rangle$ with energies of 51.53 meV and 97.78 meV，respectively. Two new subbands $|3\rangle$ and $|4\rangle$ are created by mixing the ground subband of the shallow well $|se\rangle$ and the first excited subband of the right deep well $|de\rangle$ by tunneling，which have energies of 191.30 meV and 203 meV，respectively. The low intensity light pulse propagates in the z direction along the growth axis of the QWs and the polarization does likewise. As in Ref.[10]，a transverse magnetic polarized probe incident at an angle of 45° with respect to the growth axis is considered

This work was supported by the National Natural Science Foundation of China (60708008)，the Project of Academic Leaders in Shanghai (07XD14030)，and the Key Basic Research Foundation of Shanghai (08JC1409702).

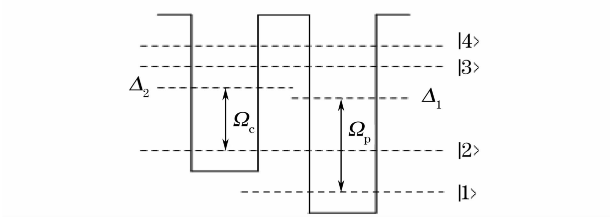

Fig.1 Energy level diagram and excitation scheme of a asymmetric QW with energies E_1 = 51.53 meV, E_2 = 97.78 meV, E_3 = 191.30 meV, and E_4 = 203.06 meV

so that all transition dipole moments include a factor $1/\sqrt{2}$ as the intersubband transition is polarized along the growth axis.

The weak probe field propagating in such a QWs system can be described by the following coupled Shrödinger-Maxwell equations, which manifests the media response and time-dependence of the probe field:

$$\frac{\partial A_1}{\partial t} = \mathrm{i}\Omega_p^*(A_3 + kA_4), \tag{1a}$$

$$\frac{\partial A_2}{\partial t} = \mathrm{i}(\Delta_1 - \Delta_2)A_2 + \mathrm{i}\Omega_s^*(A_3 + qA_4), \tag{1b}$$

$$\frac{\partial A_3}{\partial t} = \mathrm{i}\Omega_p A_1 + \mathrm{i}\Omega_s A_2 + (\mathrm{i}\Delta_1 - \gamma_3)A_3 + \kappa A_4, \tag{1c}$$

$$\frac{\partial A_4}{\partial t} = \mathrm{i}\Omega_p kA_1 + \mathrm{i}\Omega_s qA_2 + \kappa A_3 + [\mathrm{i}(\Delta_1 - \delta) - \gamma_4]A_4, \tag{1d}$$

$$\frac{\partial \Omega_p}{\partial z} + \frac{1}{c}\frac{\partial \Omega_p}{\partial t} - \mathrm{i}k_{13}(A_3A_1^* + kA_4A_1^*) = 0, \tag{1e}$$

where $A_j (j = 1,2,3,4)$ is the probability amplitude of the finding electron in subband level $|j\rangle$, $2\Omega_p$ and $2\Omega_s$ are the Rabi frequencies for the relevant transitions, $2\gamma_j (j = 3,4)$ is the decay rate of level $|j\rangle$, and $k_{13} = N|\mu_{13}|^3\omega_p/(2\hbar\varepsilon_0 c)$ is a constant related to media and probe field with N being electron density of the QWs and μ_{13} being the dipole moment of the $|1\rangle \to |3\rangle$ transition. The cross coupling term κ is added phenomenologically in $1c$ and $1d$, representing resonant tunneling induced Fano interference[15,18]. To obtain those equations mentioned above, we define $\Delta_1 = \omega_p - (\omega_3 - \omega_1)$ [$\Delta_2 = \omega_s - (\omega_3 - \omega_2)$] as probe (control) field detuning and use the rotating wave approximation and the slowly varying amplitude approximation.

To obtain the dispersion relation of this system, we apply perturbation treatment to the system responding to the first order of the weak probe field. We assume $A_j = \sum_k A_j^{(k)}$ with $A_j^{(k)}$ being the kth order part of A_j in terms of Ω_p. Under the adiabatic following regime, we prepare all the electrons on the ground state $|1\rangle$, thus $A_1^{(0)} = 1$, $A_j^{(0)}(j \neq 1) = 0$, and $A_1^{(1)} = 0$. Taking the Fourier transformation of Eqs.(1b)～(1e), we derive

$$(\omega + \Delta_1 - \Delta_2)\beta_2^{(1)} + \Omega_s^*(\beta_3^{(1)} + q\beta_4^{(1)}) = 0, \tag{2a}$$

$$\Omega_s\beta_2^{(1)} + (\omega + \Delta_1 + \mathrm{i}\gamma_3)\beta_3^{(1)} - \mathrm{i}\kappa\beta_4^{(1)} = -\Lambda_p, \tag{2b}$$

$$q\Omega_s\beta_2^{(1)} - \mathrm{i}\kappa\beta_3^{(1)} + (\omega + \Delta_1 - \delta + \mathrm{i}\gamma_4)\beta_4^{(1)} = -k\Lambda_p, \tag{2c}$$

$$\frac{\partial \Lambda_p}{\partial z} - \mathrm{i}\frac{\omega}{c}\Lambda_p - \mathrm{i}k_{13}(\beta_3^{(1)} + k\beta_4^{(1)}) = 0, \tag{2d}$$

with $\beta_3^{(1)} = \frac{D_3}{D}\Lambda_p$ and $\beta_4^{(1)} = \frac{D_4}{D}\Lambda_p$, where $\beta_j^{(1)}$ and Λ_p are the Fourier transformation terms of $A_j^{(1)}$ and Ω_p, respectively. To simplify the above equations, some new variables are defined as

$$\begin{aligned} D_3 = {} & k(\omega + \Delta_1 - \Delta_2)[|\Omega_s|^2 q + \mathrm{i}\kappa(\omega + \Delta_1 - \Delta_2)] - \\ & (\omega + \Delta_1 - \Delta_2)[|\Omega_s|^2 q - (\omega + \Delta_1 - \Delta_2)(\omega + \Delta_1 - \delta + i\gamma_4)], \end{aligned} \tag{3a}$$

$$\begin{aligned} D_4 = {} & (\omega + \Delta_1 - \Delta_2)[|\Omega_s|^2 q + \mathrm{i}\kappa(\omega + \Delta_1 - \Delta_2)] - \\ & k(\omega + \Delta_1 - \Delta_2)[|\Omega_s|^2 - (\omega + \Delta_1 - \Delta_2)(\omega + \Delta_1 + \mathrm{i}\gamma_3)], \end{aligned} \tag{3b}$$

$$\begin{aligned} D = {} & -[\kappa^2 + (\omega + \Delta_1 + \mathrm{i}\gamma_3)(\omega + \Delta_1 - \delta + \mathrm{i}\gamma_4)](\omega + \Delta_1 - \Delta_2) + \\ & [2\mathrm{i}\kappa q + q^2(\omega + \Delta_1 + \mathrm{i}\gamma_3) + (\omega + \Delta_1 - \delta + \mathrm{i}\gamma_4)]|\Omega_s|^2, \end{aligned} \tag{3c}$$

where D, D_3, and D_4 are all related to ω. Then we can rewrite Eq. (2d) as

$$\frac{\partial \Lambda_p}{\partial z} = \mathrm{i}\left[\frac{\omega}{c} + k_{13}\left(\frac{D_3}{D} + k\frac{D_4}{D}\right)\right]\Lambda_p = \mathrm{i}K(\omega)\Lambda_p, \quad (4)$$

where $K(\omega) = K_0 + K_1\omega + K_2\omega^2 + O(\omega^3)$, $K_0 = \phi + \mathrm{i}\alpha/2$ describes the phase shift ϕ per unit length and absorption coefficient α of the probe field, whereas $K_1 = 1/V_g$ offers a group velocity of the probe field, and K_2 represents the group velocity dispersion. Note that we neglecte the terms of higher orders than those of the third order. This approximation is sufficiently accurate for the problem we consider[19]. Thus we look for the solution of the form $\Omega_p \rightarrow \Omega_p \exp(\mathrm{i}K_0 z)$ to the wave propagation. Using the method in Ref. [20], we get the nonlinear wave equation of the slowly varying envelope Ω_p as

$$\frac{\partial \Omega_p}{\partial z} + K_1\frac{\partial \Omega_p}{\partial t} + \mathrm{i}K_2\frac{\partial^2 \Omega_p}{\partial t^2} = -\mathrm{i}k_{13}\left[(A_3^{(1)} + kA_4^{(1)})\sum_{j=2}^{4}|A_j|^2\right]\exp(-\mathrm{i}K_0 z), \quad (5)$$

where $A_2^{(1)} = -\frac{D_{30} + qD_{40}}{D_0(\Delta_1 - \Delta_2)}\Omega_s^*\Omega_p$, $A_3^{(1)} = \frac{D_{30}}{D_0}\Omega_p$ and $A_4^{(1)} = \frac{D_{40}}{D_0}\Omega_p$ with $D_0 = D(\omega = 0)$ and $D_{j0}(j = 3,4) = D_j(\omega = 0)$. In Eq. (5), the terms on the right side represent the nonlinearity resulting from self-phase modulation and cross-phase modulation. To make analysis simple, we use the variable transformation $\xi = z$ and $\eta = t - z/V_g$, thus Eq. (5) is reduced to the nonlinear Shrödinger equation with plural coefficient K_2 and W:

$$\mathrm{i}\frac{\partial \Omega_p}{\partial \xi} - K_2\frac{\partial^2 \Omega_p}{\partial \eta^2} = W|\Omega_p|^2\Omega_p\exp(-\mathrm{i}\alpha\xi), \quad (6)$$

where K_2 is the coefficient of the third term in the series expansion of the $K(\omega)$ about ω, and W is the nonlinear coefficient being independent of probe field Ω_p:

$$W = k_{13}\frac{D_{30} + kD_{40}}{D_0}\left[\frac{|D_{30}|^2 + |D_{40}|^2}{|D_0|^2} + \frac{|D_{30} + qD_{40}|^2|\Omega_s|^2}{|D_0(\Delta_1 - \Delta_2)|^2}\right]. \quad (7)$$

We notice that, if a reasonable and realistic set of parameters can be selected to make almost vanishing absorption α, $K_2 = K_{2r} + \mathrm{i}K_{2i}$ and $W = W_r + \mathrm{i}W_i$ with $K_{2r} \geqslant K_{2i}$ and $W_r \geqslant W_i$, Eq. (6) can be changed to the standard nonlinear Shrödinger equation:

$$\mathrm{i}\frac{\partial \Omega_p}{\partial \xi} - K_{2r}\frac{\partial^2 \Omega_p}{\partial \eta^2} = W_r|\Omega_p|^2\Omega_p, \quad (8)$$

which has the bright or dark soliton solution depending on the sign of $K_{2r}W_r$[21]. The fundamental bright soliton solution of Eq. (8) is

$$\Omega_p = \Omega_{p0}\,\mathrm{sech}(\eta/\tau)\exp(-\mathrm{i}\xi W_r|\Omega_{p0}|^2/2), \quad (9)$$

where arbitrary amplitude Ω_{p0} and pulse width τ, must satisfy the relation of $|\Omega_{p0}\tau|^2 = 2K_{2r}/W_r$, which the hyperbolic secant function.

Using the realistic parameters, we investigate propagation of the ultraslow bright soliton and soliton pairs in this QWs system. Considering the decay rates $\gamma_3 = 2.9947\times10^{10}\ \mathrm{s}^{-1}$, $\gamma_4 = 2.8431\times10^{10}\ \mathrm{s}^{-1}$, and the single photon detuning $\Delta_1 = 4.5001\times10^{10}\ \mathrm{s}^{-1}$, $\Delta_2 = 4.5\times10^{10}\ \mathrm{s}^{-1}$ with the wavelength of the probe field $\lambda_p = 8896$ nm. We obtain the very weak absorption $\alpha = 0.000493\ \mu\mathrm{m}^{-1}$, ultraslow group velocity of the probe field $V_g/c = 4.92\times10^{-7}$. Figure 2(a) is $|\Omega_p/\Omega_{p0}|\exp(-\mathrm{i}\alpha\xi)$ as a function of η/τ and ξ/L with $\tau = 10^{-5}$ s and $L = 1\ \mu$m, which is governed by Eq. (6), while Fig. 2(b) shows the evolution of $|\Omega_p/\Omega_{p0}|$ in Eq. (8). Comparing them, we find that they are almost identical only except a slight loss in Fig. 2(a). The resonant tunneling provides giant Kerr nonlinearity for balancing the group velocity dispersion of the pulse, which results in the formation of the ultraslow solitons.

We also investigate the propagation of the ultraslow soliton pairs in such a medium by only changing the input pulse but with the same parameters mentioned above. The soliton pairs with the same initial phase experience constructive interference in interfering regions, whereas they perform destructive interference with opposite relative phase, as shown in Figs. 2(c) and (d). But in our observing length, they hardly perform other interactions, which may be an important advantage in optical communication.

In conclusion, we investigate the propagation of ultraslow soliton and soliton pairs with weak input intensity in asymmetric double QW systems. Owing to the balance between the dispersion and giant Kerr-type nonlinearity provided by the resonant tunneling, ultraslow solitons and soliton pairs are generated and kept stable propagation in such a semiconductor system. With different initial relative phases, soliton pairs can experience constructive or destructive interference in the interfering regions. These properties of ultraslow soliton and soliton pairs are useful in the applications of optical communication, quantum information processing, and also optical buffer.

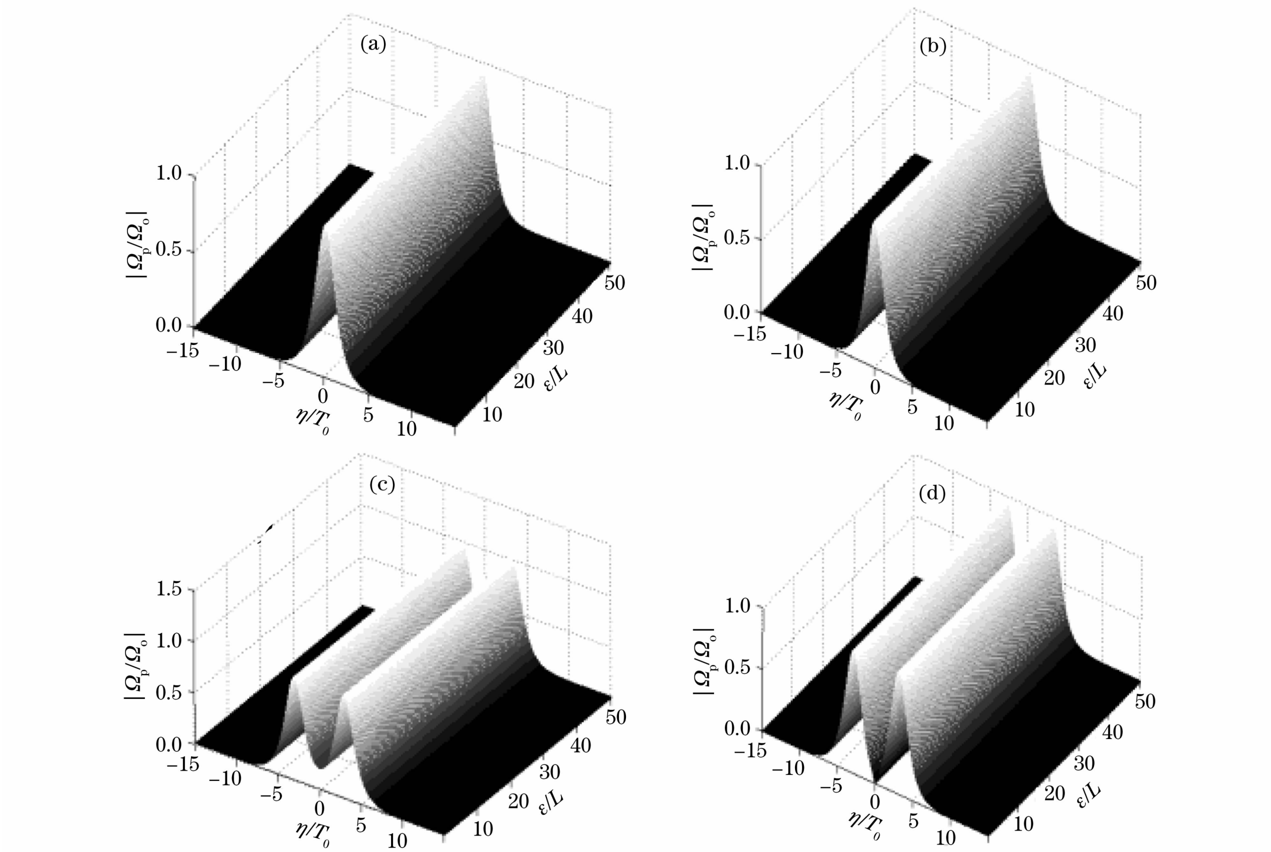

Fig. 2 Evolution of the ultraslow solitons and soliton pairs in semiconductor double QWs systems. (a) Numerical soliton solution of Eq. (6), (b) soliton solution given in Eq. (8), (c) soliton pairs with identical initial phase, (d) soliton pairs with adverse phase

References

1 A. Hasegawa, M. Matsumoto. Optical Solitons in Fibers[M]. Berlin: Springer, 2003

2 J. Denschlag *et al.*. Generating solitons by phase engineering of a Bose-Einstein condensate[J]. *Science*, 2000, **287**: 97～101

3 G. Huang, J. Szeftel, S. H. Zhu. Dynamics of dark solitons in quasi-one-dimensional Bose-Einstein condensates[J]. *Phys. Rev. A*, 2002, **65**: 053605

4 G. P. Agrawal. Nonlinear Fiber Optics[M]. New York: Academic, 2001

5 H. Schmidt, A. Imamoglu. Giant Kerr nonlinearities obtained by electromagnetically induced transparency[J]. *Opt. Lett.*, 1996, **21**: 1936

6 S. E. Harris, L. V. Hau. Nonlinear optics at low light levels[J]. *Phys. Rev. Lett.*, 1999, **82**: 4611～4614

7 M. D. Lukin, A. Imamoglu. Nonlinear optics and quantum entanglement of ultraslow single photons[J]. *Phys. Rev. Lett.*, 2000, **84**: 1419～1422

8 L. Deng *et al.*. Opening optical four-wave mixing channels with giant enhancement using ultraslow pump waves[J]. *Phys. Rev. Lett.*, 2002, **88**: 143902

9 Y. Wu, L. Wen, Y. Zhu. Efficient hyper-Raman scattering in resonant coherent media[J]. *Opt. Lett.*, 2003, **28**: 631～633

10 J. Faist, F. Capasso, C. Sirtori *et al.*. Controlling the sign of quantum interference by tunnelling from quantum wells[J]. *Nature*, 1997, **390**: 589～591

11 H. Schmidt, K. L. Campman, A. C. Gossard *et al.*. Tunneling induced transparency: Fano interference in intersubband transitions[J]. *Appl. Phys. Lett.*, 1997, **70**: 3455～3457

12 M. Phillips, H. Wang. Electromagnetically induced transparency due to intervalence band coherence in a GaAs quantum well[J]. *Opt. Lett.*, 2003, **28**: 831～833

13 X. D. Hu, W. Potz. Coherent manipulation of phonon emission rates in semiconductor heterostructures[J]. *Phys. Rev. Lett.*, 1999, **82**: 3116～3119

14 T. Muller, W. Parz, G. Strasser *et al.*. Influence of carrier-carrier interaction on time-dependent intersubband absorption in a semiconductor quantum well[J]. *Phys. Rev. B*, 2004, **70**: 155324

15 J. H. Wu, J. Y. Gao, J. H. Xu *et al.*. Ultrafast all optical switching via tunable Fano interference[J]. *Phys. Rev. Lett.*, 2005, **95**: 057401

16 J. H. Wu, J. Y. Gao, J. H. Xu *et al.*. Dynamic control of coherent pulses via Fano-type interference in asymmetric double quantum wells[J]. *Phys. Rev. A*, 2006, **73**: 053818

17 H. Sun, S. Q. Gong, Y. P. Niu *et al.*. Enhancing Kerr nonlinearity in an asymmetric double quantum well via Fano interference [J]. *Phys. Rev. B*, 2006, **74**: 155314

18 J. Luo, S. Gong, Y. Niu *et al.*. Phase evolution of the reemitted field in the semiconductor quantum wells under the femtosecond pulse train intersubband excitation[J]. *Chin. Opt. Lett.*, 2007, **5**(5): 304～307

19 L. Wang, K. Xia, H. Sun *et al.*. Femtosecond laser pulse propagation in a metallic nan-slit array[J]. *Chin. Opt. Lett.*, 2007, **5**(suppl.): S126～S128

20 W. X. Yang, J. M. Hou, R. K. Lee. Ultraslow bright and dark solitons in semiconductor quantum wells[J]. *Phys. Rev. A*, 2008, **77**: 033838

21 C. Hang, G. X. Huang, L. Deng. Generalized nonlinear Schrödinger equation and ultraslow optical solitons in a cold four-state atomic system[J]. *Phys. Rev. E*, 2006, **73**: 036607

22 H. A. Haus, W. S. Wong. Solitons in optical communications[J]. *Rev. Mod. Phys.*, 1996, **68**: 423～444

23 Y. Wu, L. Deng. Ultraslow optical solitons in a cold four-state medium[J]. *Phys. Rev. Lett.*, 2004, **93**: 143904