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有偏压光伏光折变晶体中低振幅非相干耦合孤子对

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摘要 从理论上研究了有偏压的光伏光折变晶体中低振幅屏蔽光伏孤子对的特性,给出了明-明、暗-暗屏蔽光伏孤子对的单孤子解。它们起源于对外电场的非均匀空间屏蔽和光伏效应,不同于起源于对外电场非均匀空间屏蔽孤子和起源于光伏效应的光伏孤子,具有有趣的特性。当光伏效应可忽略时,它们的非线性波动方程就转化为低振幅屏蔽孤子对的非线性波动方程,它们就转化为低振幅屏蔽孤子对;当外偏压为零时,它们的非线性波动方程就转化为低振幅闭路和开路光伏孤子对的非线性波动方程,它们就转化为低振幅闭路和开路光伏孤子对。如果载体光束具有相同的偏振和波长以及互不相干时,这些孤子对就能得到,相关的例子在光伏光折变晶体铌酸锂($LiNbO_3$)中给出。

关键词 光电子学;光折变晶体;光孤子;光伏效应

中图分类号 O 437.5 **文献标识码** A

Incoherently Coupled Low-Amplitude Soliton Pairs in Biased Photorefractive-Photovoltaic Crystals

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Abstract A theory on incoherently coupled low-amplitude soliton pairs for screening-photovoltaic solitons in biased photorefractive-photovoltaic crystals is investigated that gives rise to the analytical solutions of bright-bright and dark-dark soliton pairs, which are due to both the spatially nonuniform screening of the external electric field and the photovoltaic effect. These soliton pairs, differ from screening soliton pairs originating from the spatially nonuniform screening of the external electric field and photovoltaic soliton pairs originating from the photovoltaic effect, are of interesting properties. When the photovoltaic effect is neglected, their nonlinear wave equations change into that for screening solitons, and these soliton pairs change into screening soliton pairs. When the external bias field is absent, their nonlinear wave equation change into that for photovoltaic solitons in closed and open circuits, and these soliton pairs change into photovoltaic soliton pairs in closed and open circuits. If the carrier beams share the same polarization, wavelength, and are mutually incoherent, these soliton pairs can be obtained. Relevant examples are provided where the photorefractive-photovoltaic crystal is lithium niobate ($LiNbO_3$) crystal.

Key words optoelectronics; photorefractive crystal; optical soliton; photovoltaic effect

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1 引 言

光折变空间光孤子是近年来人们一直感兴趣的研究领域[1~9]。已经发现自陷光束发生在两个横向上，并且在微瓦功率的水平上能观察到自陷光束形成的空间光孤子。这样，光折变空间光孤子对研制所有光开关和光处理器显示了重要的希望。目前，已预言并验证在介电晶体内存在着两类稳态的光折变空间孤子：屏蔽孤子[1,2]和光伏孤子[3,4]。特别有趣的稳态空间光孤子是我们近来提出的有偏压光伏光折变晶体中的屏蔽光伏孤子[5,6]，当外偏压为零时，屏蔽光伏孤子的物理系统就转化为光伏孤子的物理系统，屏蔽光伏孤子就转化为光伏孤子；当光伏效应为零时，屏蔽光伏孤子的物理系统就转化为屏蔽孤子的物理系统，屏蔽光伏孤子就转化为屏蔽孤子。因此，屏蔽孤子和光伏孤子的研究可以转化为屏蔽光伏孤子的研究上。最近，人们已经预言了非相干耦合屏蔽孤子对、光伏孤子对、屏蔽光伏孤子对[7,8]，但只是研究了这些孤子对的数值解。另一方面，对于研究这些孤子对的解析解也具有十分重要的意义[2]。本文研究了低振幅非相干耦合屏蔽光伏孤子对的理论，得到了它们的解析解。在适当的条件下，这类孤子对的非线性波动方程就转化为低振幅非相干耦合屏蔽孤子对和闭路与开路光伏孤子对的非线性波动方程，这类孤子对就转化为低振幅非相干耦合屏蔽孤子对和闭路与开路光伏孤子对。

2 非线性波动方程

设沿光伏光折变晶体的晶轴 c 取为 x 方向施加外电压，让频率相同非相干的两束只在 x 方向偏振的光波 $\boldsymbol{E}_1$ 和 $\boldsymbol{E}_2$ 在该晶体中沿 z 方向传播。两光波的总电场分量 $\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{E}_2$ 满足下面的 Helmholtz 方程

$$\nabla^2\boldsymbol{E}+(k_0n'_e)^2\boldsymbol{E}=0 \tag{1}$$

式中，$k_0=2\pi/\lambda_0$，λ_0 为光波在真空中的波长，$n'_e=(n_e^2-n_e^4r_{33}E_{\mathrm{sc}})^{1/2}$ 为两光波沿 x 方向扰动的折射率，r_{33} 为电光系数，n_e 为晶体的非常折射率，$\boldsymbol{E}_{\mathrm{sc}}=E_{\mathrm{sc}}\boldsymbol{i}$ 为光波感应的空间电荷场，$\boldsymbol{i}$ 为沿 x 轴的单位矢量。根据慢变化包络 ϕ 和 φ 与 $\boldsymbol{E}_1$ 和 $\boldsymbol{E}_2$ 的关系[7,8]，$\boldsymbol{E}_1=\boldsymbol{i}\phi(x,z)\exp(\mathrm{i}kz)$ 和 $\boldsymbol{E}_2=\boldsymbol{i}\varphi(x,z)\exp(\mathrm{i}kz)$，由(1)式得到包络 ϕ 和 φ 的演化方程

$$\boldsymbol{i}\phi_z+\frac{1}{2k}\phi_{xx}-\frac{k_0(n_e^3r_{33}E_{\mathrm{sc}})}{2}\phi=0 \tag{2}$$

$$\boldsymbol{i}\varphi_z+\frac{1}{2k}\varphi_{xx}-\frac{k_0(n_e^3r_{33}E_{\mathrm{sc}})}{2}\varphi=0 \tag{3}$$

式中，$k=k_0n_e$。另一方面，在强偏压的条件下，光波感应的空间电荷场为[6,8]

$$E_{\mathrm{sc}}=-(\varepsilon\eta+E_p\sigma\eta)\frac{I_\infty+I_d}{I+I_d}+E_p\frac{I_\infty-I}{I+I_d} \tag{4}$$

式中，$I=I(x,z)$ 为两光束的总强度；I_d 为所谓的暗匀辐射度；$I_\infty=I(x\to\pm\infty)$；$\varepsilon$ 为外电压，$\eta=1/\int_{-l/2}^{l/2}[(I_\infty+I_d)/(I+I_d)]\mathrm{d}x$；$\sigma=\int_{-l/2}^{l/2}[(I_\infty-I)/(I+I_d)]\mathrm{d}x$；$l$ 为晶体两电极之间的距离。$E_p=\kappa\gamma N_A/e\mu$ 为光伏场常数；κ 为光伏常数；γ 为载流子复合速率；N_A 为负电性受主密度；e 为基本电荷；μ 为电子迁移率。对于非相干的两光束，总光强为两光束的 Poynting 矢量的叠加，即 $I=(n_e/2\eta_0)(|\phi|^2+|\varphi|^2)$。式中，$\eta_0=(\mu_0/\varepsilon_0)^{1/2}$。为了便于分析，采用下列无量纲变量[7,8]：$\xi=z/(kx_0^2)$，$s=x/x_0$，$\phi=(2\eta_0I_d/n_e)^{1/2}U$，$\varphi=(2\eta_0I_d/n_e)^{1/2}V$。其中，$x_0$ 为任意空间宽度。这样，在低振幅($|U|^2\ll1$ 和 $|V|^2\ll1$)[2] 的条件下，将(4)式代入(2)和(3)式得到归一化光波包络 U 和 V 满足下列的动态演化方程

$$\boldsymbol{i}U_\xi+\frac{1}{2}U_{ss}+[(\alpha+\beta)(\rho+1)-\delta\rho]U+(\rho+1)(\delta-\alpha-\beta)(|U|^2+|V|^2)U=0 \tag{5}$$

$$\boldsymbol{i}V_\xi+\frac{1}{2}V_{ss}+[(\alpha+\beta)(\rho+1)-\delta\rho]V+(\rho+1)(\delta-\alpha-\beta)(|U|^2+|V|^2)V=0 \tag{6}$$

式中，$\rho=I_\infty/I_d$；$\beta=(k_0x_0)^2(n_e^4r_{33}\eta/2)\varepsilon$；$\alpha=(k_0x_0)^2(n_e^4r_{33}\sigma\eta/2)E_p$；$\delta=(k_0x_0)^2(n_e^4r_{33}/2)E_p$。

3 孤子对的解

3.1 明-明孤子对的解

令 $U=r^{1/2}y(s)\cos\theta\exp(\mathrm{i}\nu\xi)$ 和 $V=r^{1/2}y(s)\sin\theta\exp(\mathrm{i}\nu\xi)$。式中，$r=I(0)/I_d>0$；$\nu$ 为光波传播常数的非线性移动；θ 为一个任意的投影角；$y(s)$ 为一个归一化的实函数，$0\leqslant y(s)\leqslant1$。明-明孤子对的边界条件为：$y(0)=1$；$y'(0)=0$；$y(s\to\pm\infty)=0$。由 $y(s\to\pm\infty)=0$，得 $\rho=I_\infty/I_d=0$。这样，由(5)和(6)式得到

$$y''-2(\nu-\beta-\alpha)y+2(\delta-\beta-\alpha)y^3=0 \tag{7}$$

由边界条件，积分(7)式得到

$$\nu=\frac{1}{2}r(\delta-\beta-\alpha)+\beta+\alpha \tag{8}$$

$$y = \mathrm{sech}\{[r(\delta-\beta-\alpha)]^{1/2}s\} \tag{9}$$

这样，归一化光波包络 U 和 V 有如下的形式

$$U = r^{1/2}\mathrm{sech}\{[r(\delta-\beta-\alpha)]^{1/2}s\}\times \cos\theta\exp\{\mathrm{i}[r(\delta-\beta-\alpha)/2+\beta+\alpha]\xi\} \tag{10}$$

$$V = r^{1/2}\mathrm{sech}\{[r(\delta-\beta-\alpha)]^{1/2}s\}\times \sin\theta\exp\{\mathrm{i}[r(\delta-\beta-\alpha)/2+\beta+\alpha]\xi\} \tag{11}$$

以铌酸锂晶体为例，晶体参数取为：$n_e = 2.2$；$r_{33} = 30\times10^{-12}$ m/V；$E_p = 40$ kV/cm。再取 $\lambda_0 = 0.5$ μm，$l = 1$ cm，$x_0 = 40$ μm，$r = 5$，$\theta = 30°$ 和 $\varepsilon = \pm 20000$ V。图 1 给出了在这些条件下归一化低振幅明-明屏蔽光伏孤子对的强度随 s 变化的理论曲线。

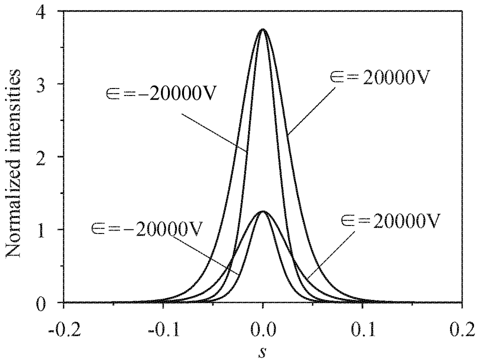

图 1 归一化低振幅明-明屏蔽光伏孤子对的强度随 s 变化的理论曲线。$r = 5$；$\theta = 30°$；$\varepsilon = \pm 20000$ V

Fig. 1 Soliton components, $|U|^2$ and $|V|^2$ as a function of s, for bright-bright screening-photovoltaic soliton pairs in the low-amplitude case when $r = 5$, $\theta = 30°$, and $\varepsilon = \pm 20000$ V

3.2 暗-暗孤子对的解

令 $U = \rho^{1/2}y(s)\cos\theta\exp(\mathrm{i}\nu\xi)$ 和 $V = \rho^{1/2}y(s)\sin\theta\exp(\mathrm{i}\nu\xi)$。式中，$|y(s)|\ll 1$。暗-暗孤子对的边界条件为：$y(0) = 0$；$y''(\infty) = y'(\infty) = 0$；$y(s\rightarrow\pm\infty) = \pm 1$。将归一化光波包络 U 和 V 的这种形式代入(5)和(6)式得到

$$y'' - 2[\nu-(\beta+\alpha)(\rho+1)+\delta\rho]y + 2\rho(\rho+1)(\delta-\beta-\alpha)y^3 = 0 \tag{12}$$

使用边界条件，积分(12)式得

$$\nu = (1-\rho^2)(\beta+\alpha)+\rho^2\delta \tag{13}$$

$$y = \tanh\{[\rho(\rho+1)(\beta+\alpha-\delta)]^{1/2}s\} \tag{14}$$

这样，归一化光波包络 U 和 V 有如下形式

$$U = \rho^{1/2}\tanh\{[\rho(\rho+1)(\beta+\alpha-\delta)]^{1/2}s\}\times \cos\theta\exp\{\mathrm{i}[(1-\rho^2)(\beta+\alpha)+\rho^2\delta]\xi\} \tag{15}$$

$$V = \rho^{1/2}\tanh\{[\rho(\rho+1)(\beta+\alpha-\delta)]^{1/2}s\}\times \sin\theta\exp\{\mathrm{i}[(1-\rho^2)(\beta+\alpha)+\rho^2\delta]\xi\} \tag{16}$$

图 2 给出了铌酸锂晶体中归一化低振幅暗-暗屏蔽光伏孤子对的强度随 s 变化的理论曲线。

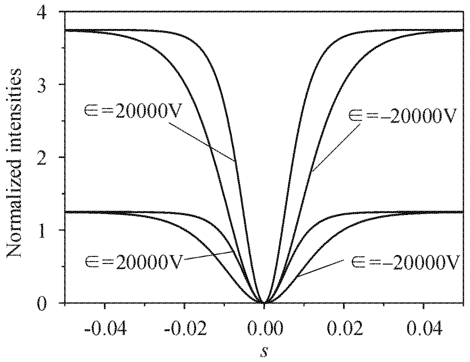

图 2 归一化低振幅暗-暗屏蔽光伏孤子对的强度随 s 变化的理论曲线。$\rho = 5$；$\theta = 30°$；$\varepsilon = \pm 20000$ V

Fig. 2 Soliton components, $|U|^2$ and $|V|^2$ as a function of s, for dark-dark screening-photovoltaic soliton pairs in the low-amplitude case when $\rho = 5$, $\theta = 30°$, and $\varepsilon = \pm 20000$ V

4 讨论

光伏孤子对和屏蔽孤子对都可以从有偏压的光伏光折变晶体中的屏蔽光伏孤子对中推得。当光伏效应为零(即 $\alpha = \delta = 0$)时，屏蔽光伏孤子对的物理系统就转化为屏蔽孤子对的物理系统。在这种情况下，由(5)，(6)，(10)，(11)，(15)和(16)式得到

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} + \beta(\rho+1)U - (\rho+1)\beta(|U|^2+|V|^2)U = 0 \tag{17}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} + \beta(\rho+1)V - (\rho+1)\beta(|U|^2+|V|^2)V = 0 \tag{18}$$

$$U = r^{1/2}\mathrm{sech}\{[r(-\beta)]^{1/2}s\}\times \cos\theta\exp\{\mathrm{i}[r(-\beta)/2+\beta]\xi\} \tag{19}$$

$$V = r^{1/2}\mathrm{sech}\{[r(-\beta)]^{1/2}s\}\times \sin\theta\exp\{\mathrm{i}[r(-\beta)/2+\beta]\xi\} \tag{20}$$

$$U = \rho^{1/2}\tanh\{[\rho(\rho+1)\beta]^{1/2}s\}\times \cos\theta\exp[\mathrm{i}(1-\rho^2)\beta\xi] \tag{21}$$

$$V = \rho^{1/2}\tanh\{[\rho(\rho+1)\beta]^{1/2}s\}\times \sin\theta\exp[\mathrm{i}(1-\rho^2)\beta\xi] \tag{22}$$

(17)～(22)式表示了低振幅屏蔽孤子对的非线性波动方程、明-明孤子对、暗-暗孤子对。图 3 给出了铌酸锶钡(SBN)晶体中归一化低振幅明-明屏蔽孤子对的强度随 s 变化的理论曲线。这里，铌酸锶钡晶体的参数取为[7]：$n_e = 2.33$，$r_{33} = 237\times10^{-12}$ m/V；外电压取为：$\varepsilon = 2000$ V。

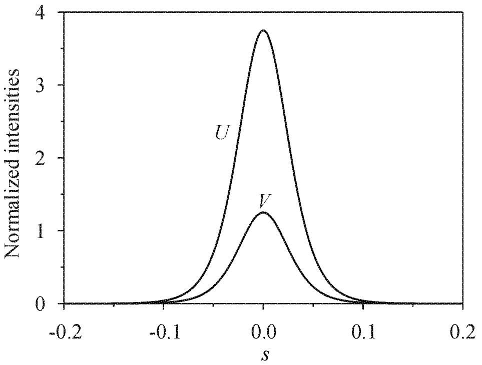

图 3 铌酸锶钡晶体中归一化低振幅明-明屏蔽孤子对的强度随 s 变化的理论曲线。$\rho = 5$；$\theta = 30^{\circ}$；$\beta = 176$

Fig. 3 Soliton components，$|U|^2$ and $|V|^2$ as a function of s，for bright-bright screening soliton pairs in a SBN crystal in the low-amplitude case when $\rho = 5$，$\theta = 30^{\circ}$，and $\beta = 176$

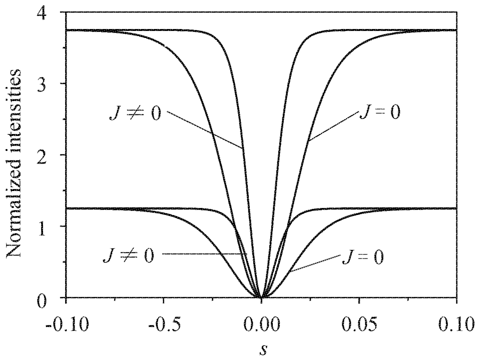

图 4 铌酸锂晶体中归一化低振幅暗-暗光伏孤子对的强度随 s 变化的理论曲线。$\rho = 5$；$\theta = 30^{\circ}$

Fig. 4 Soliton components，$|U|^2$ and $|V|^2$ as a function of s，for dark-dark photovoltaic soliton pairs in a $\mathrm{LiNbO_3}$ crystal in the low-amplitude case when $\rho = 5$ and $\theta = 30^{\circ}$

当外偏压为零(即 $\beta = 0$) 时，屏蔽光伏孤子对的物理系统就转化为光伏孤子对的物理系统。另一方面，外偏压为零意味着闭路($J \neq 0$) 或开路($J = 0$)。这里，$\boldsymbol{J} = J\boldsymbol{i}$ 为电流密度矢量，并且能够得到[8]

$$\hat{J} = \rho - \sigma\eta(\rho + 1) \tag{23}$$

式中，$\hat{J} = J/(\hat{s}I_d N_D\kappa)$；$\hat{s}$ 为光电离截面；N_D 为总施主密度。在这种情况下，将(23) 式代入(5)，(6)，(10)，(11)，(15) 和(16) 式得到

$$\begin{aligned} &\boldsymbol{i}U_{\xi} + \frac{1}{2}U_{ss} - \hat{J}\delta U + \\ &(\hat{J} + 1)\delta(|U|^2 + |V|^2)U = 0 \end{aligned} \tag{24}$$

$$\begin{aligned} &\boldsymbol{i}V_{\xi} + \frac{1}{2}V_{ss} - \hat{J}\delta V + \\ &(\hat{J} + 1)\delta(|U|^2 + |V|^2)V = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} U = &r^{1/2}\mathrm{sech}\{[r\delta(\hat{J} + 1)]^{1/2}s\} \times \\ &\cos\theta\exp\{\mathrm{i}\delta[(r/2 - 1)(\hat{J} + 1) + 1]\xi\} \end{aligned} \tag{26}$$

$$\begin{aligned} V = &r^{1/2}\mathrm{sech}\{[r\delta(\hat{J} + 1)]^{1/2}s\} \times \\ &\sin\theta\exp\{\mathrm{i}\delta[(r/2 - 1)(\hat{J} + 1) + 1]\xi\} \end{aligned} \tag{27}$$

$$\begin{aligned} U = &\rho^{1/2}\tanh\{[-\rho\delta(\hat{J} + 1)]^{1/2}s\} \times \\ &\cos\theta\exp\{\mathrm{i}\delta[1 - (1 - \rho)(\hat{J} + 1)]\xi\} \end{aligned} \tag{28}$$

$$\begin{aligned} V = &\rho^{1/2}\tanh\{[-\rho\delta(\hat{J} + 1)]^{1/2}s\} \times \\ &\sin\theta\exp\{\mathrm{i}\delta[1 - (1 - \rho)(\hat{J} + 1)]\xi\} \end{aligned} \tag{29}$$

(24) ～ (29) 式表示了低振幅闭路和开路光伏孤子对的非线性波动方程、明 - 明孤子对、暗 - 暗孤子对。图 4 表示了铌酸锂晶体中归一化低振幅暗 - 暗光伏孤子对的强度随 s 变化的理论曲线。

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